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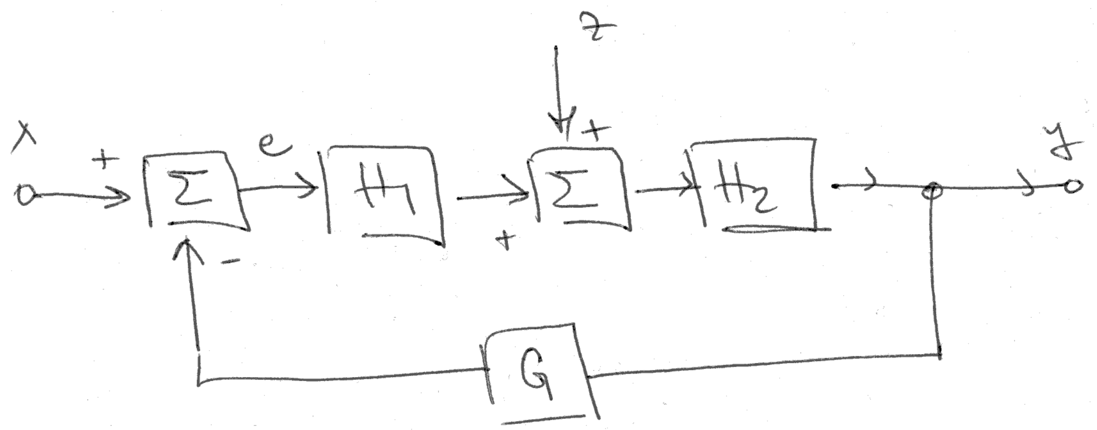


$$y = H_2 (z + H_1 e) \quad , \quad e = x - G y$$

$$y = H_2 (z + H_1 (x - G y))$$

$$y = H_2 z + H_1 H_2 x - H_1 H_2 G y$$

$$y (1 + \underbrace{H_1 H_2 G}_{T}) = H_2 z + H_1 H_2 x$$

OPEN LOOP:

$$y = H_2 z + H_2 H_1 e = H_2 z + H_1 H_2 x$$

$$\boxed{y = H_1 H_2 x + H_2 z}$$

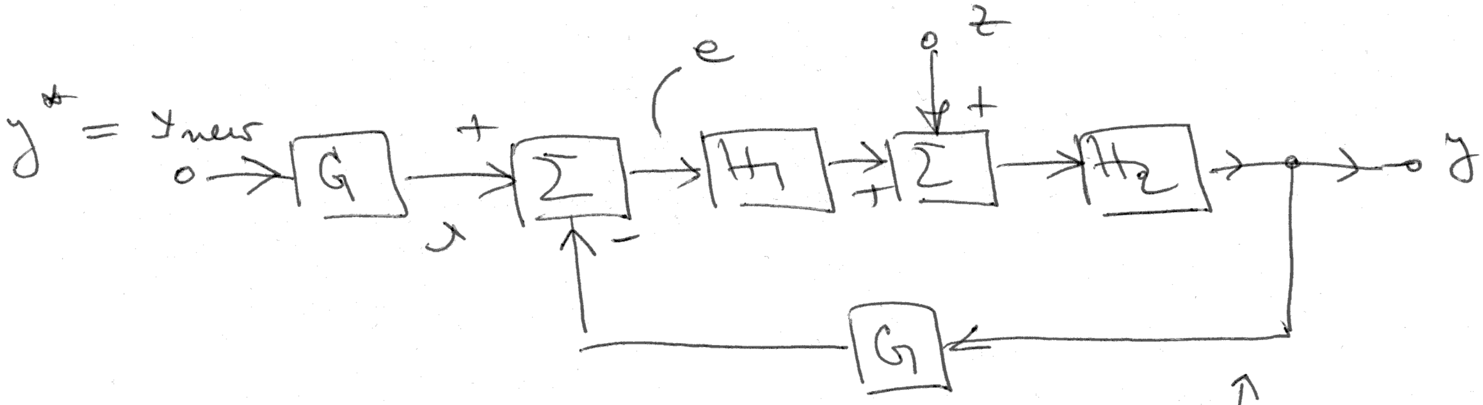
CLOSED LOOP

СВЕ ФУНКЦИЈЕ ПРЕНОСА ПОДЕЉЕНЕ СА $1+T$

$$\boxed{y = \frac{H_1 H_2}{1+T} \cdot x + \frac{H_2}{1+T} z}$$

Све функције преноса се у затвореној спрези деле са $1+T$; $|T| \to \infty$ све $\to 0$?

Не, једна сигурно не; увођење на tracking system:

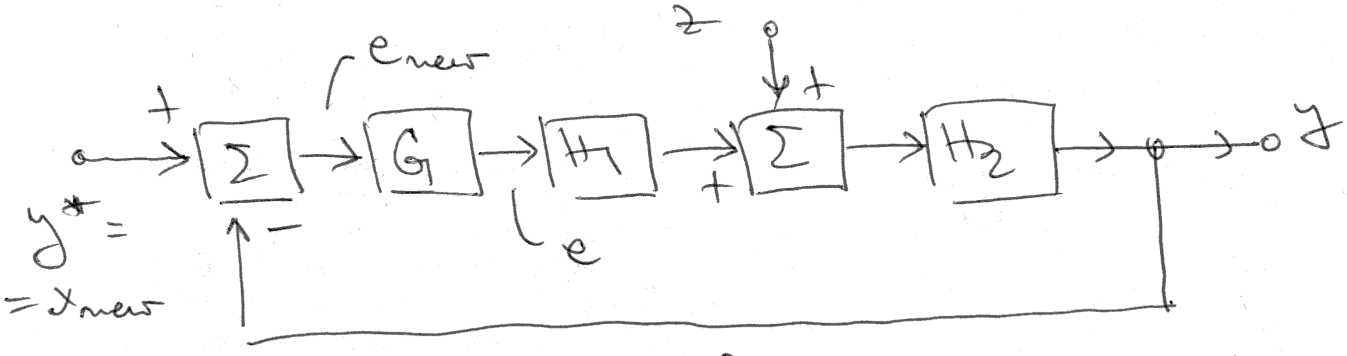


да овде изведем нови улаз

$$e = x - Gy =$$
$$= G x_{new} - G y =$$
$$= G(x_{new} - y)$$
$$= G(y^* - y)$$

извлачење пред заграду је измештање сабирка

услов: линеарност, задовољен !!!



tracking system, servosystem

$$e_{new} = x_{new} - y$$
$$y = H_2(z + H_1 G e_{new}) = H_2 z + H_1 H_2 G e_{new}$$
$$= H_2 z + H_1 H_2 G x_{new} - H_1 H_2 G y$$

$$y(1 + H_1 H_2 G_1) = H_1 H_2 G_1 x_{new} + H_2 z$$

$$\boxed{y = \frac{H_1 H_2 G_1}{1 + H_1 H_2 G_1} x_{new} + \frac{H_2}{1 + H_1 H_2 G_1} z}$$

$$\boxed{T \triangleq H_1 H_2 G_1}$$

– КРЪГОВО УСИЛВАНЕ, МНОЖЕНЕТО, ПОЛЮСИ И НУЛИ СЕ ДОБАВЯТ!!! НЕ МЕНЯТ СЕ, НЕКА "ЛИНЕАРНОСТ"

$$\boxed{y = \frac{T}{1+T} x_{new} + \frac{H_2}{1+T} z}$$

GVS 1

– ДВЕ ВАЖНИ ФУНКЦИИ: (IMPORTANT QUANTITIES)

$\frac{T}{1+T}$ – CLOSED LOOP TRANSFER FUNCTION, "ЗАТВАРЯ" ПОЛЮСИ И НУЛИ

$\frac{1}{1+T}$ – SCALING TRANSFER FUNCTION

– ПОЛЮСИ:

$T = \frac{N(s)}{D(s)} = \frac{N}{D}$ ← POLYNOMIALS OVER S, N(S) AND D(S)

$$\frac{T}{1+T} = \frac{\frac{N}{D}}{1 + \frac{N}{D}} = \frac{N}{N+D}$$

$$\frac{1}{1+T} = \frac{1}{1 + \frac{N}{D}} = \frac{D}{N+D}$$

— Нов карактеристични полином, после затварања повратне спреге:

$p(s) = D(s) + N(s)$ ← проблем: збир

— пре затварања повратне спреге:

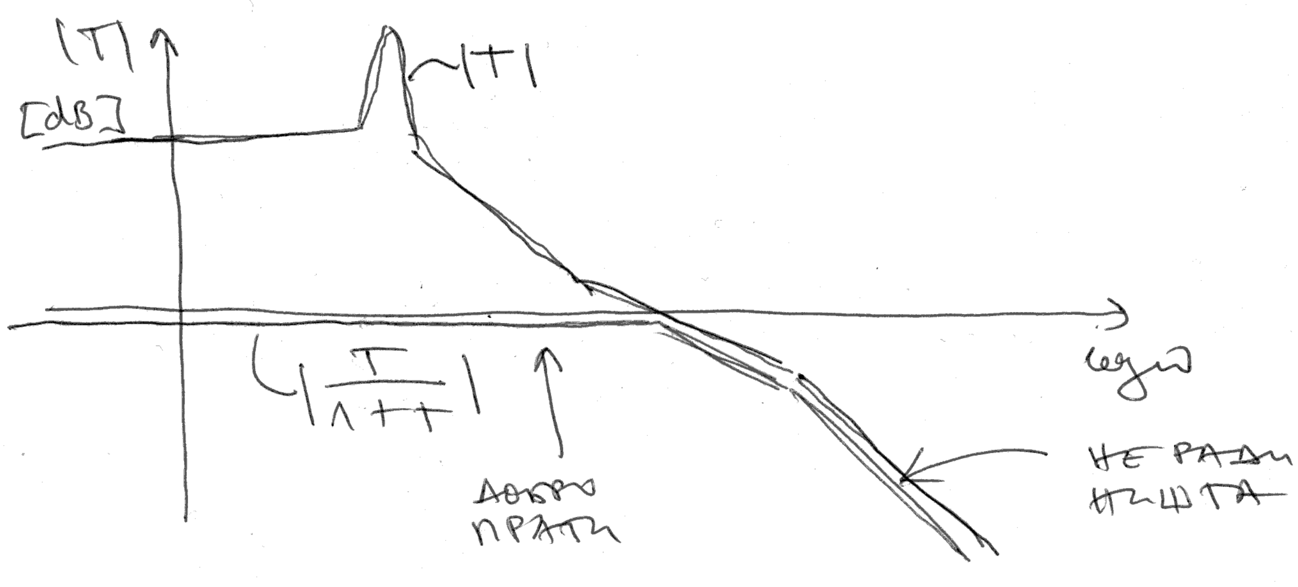
$p_0(s) = D(s)$

— потпуно нови корени, потпуно нове сопствене вредности, потпуно нов полином!!!

Конструкција амплитудске карактеристике за $\frac{T}{1+T}$ и $\frac{1}{1+T}$

① $\frac{T}{1+T}$

$$\frac{T}{1+T} \approx \begin{cases} 1, & |T| \gg 1 \\ T, & |T| \ll 1 \end{cases}$$



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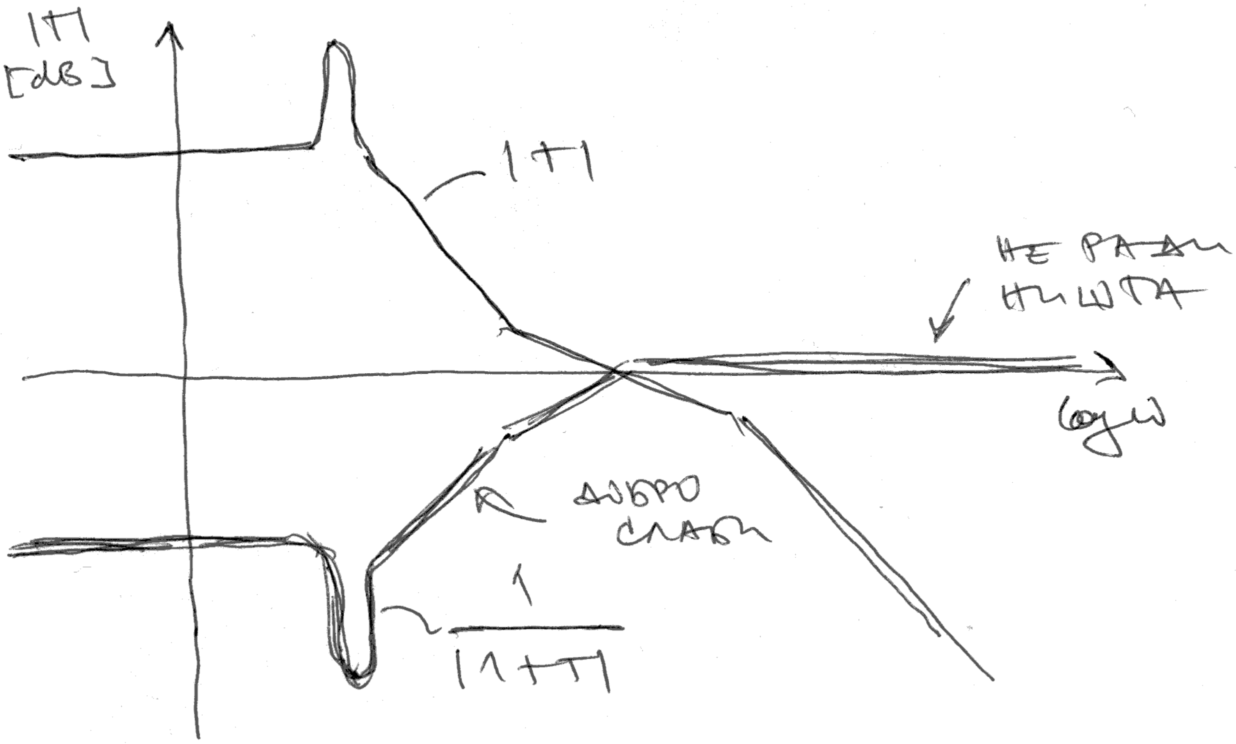
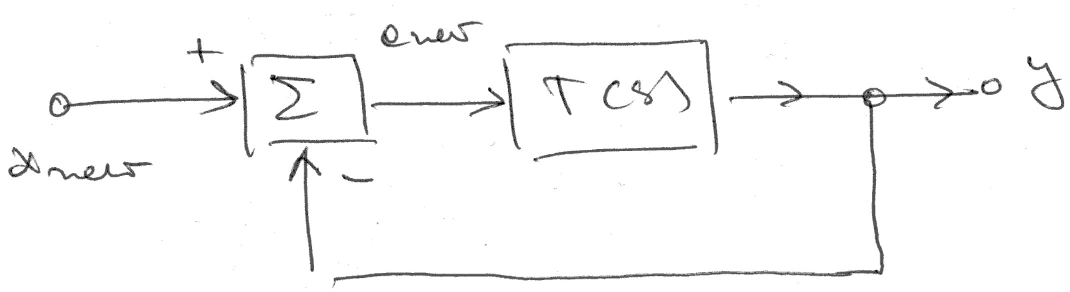
(2) $\dfrac{1}{1+T}$

$$\frac{1}{1+T} \approx \begin{cases} \dfrac{1}{T}, & |T| \gg 1 \\ 1, & |T| \ll 1 \end{cases}$$





TRACKING SYSTEM

— ШТА ЈЕ $\frac{1}{1+T}$? ДОБИТАК !

$$e_{new} = x_{new} - y = x_{new} - T\,e_{new}$$

$$e_{new}(1+T) = x_{new}$$

$$\boxed{e_{new} = \frac{1}{1+T}\,x_{new}}$$

$\hat{C}$ ПРЕНОС ОД УЛАЗА ДО ГРЕШКЕ

T ВЕЛИКО, СИСТЕМ МАЛО ГРЕШИ,

$$|e_{new}| \approx \frac{1}{|T|}\,|x_{new}|$$

$|T|$ МАЊЕ ОД 1 (МНОГО МАЊЕ),

НИШТА СЕ НЕ ДОБИЈА

$$|e_{new}| \approx |x_{new}|$$

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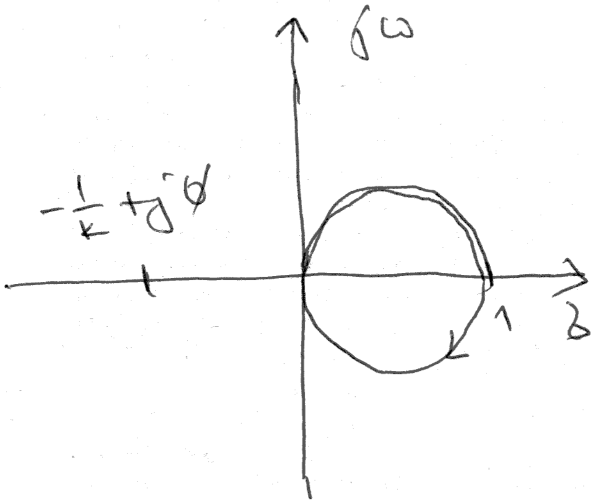
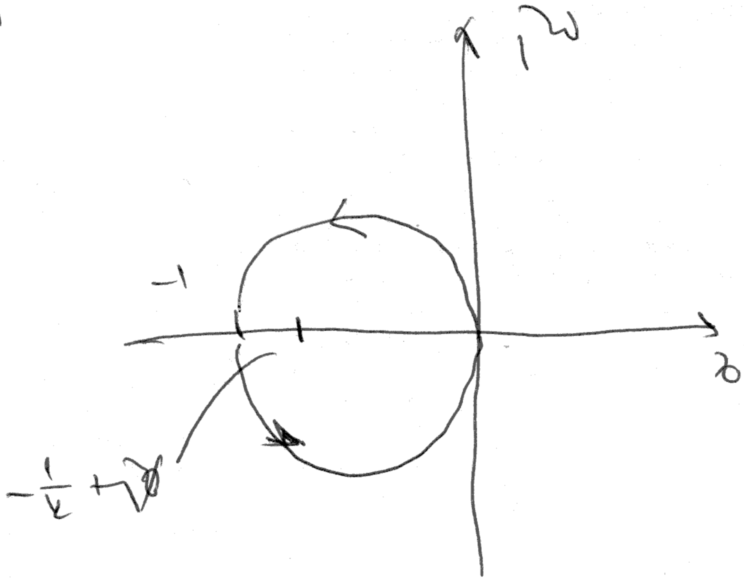
НИКВИСТОВ КРИТЕРИУМ, ПРИМЕРИ

$$H_1(s) = \frac{1}{s+1}$$



УВЕК СТАБИЛНО

$$H_2(s) = \frac{1}{s-1}$$



СТАБИЛНО ЗА $-\frac{1}{k} > -1$

$$\frac{1}{k} < 1$$

$$\boxed{k > 1}$$

ТРЕБА ДА ОБУХВАТИ Пр ПУТА У СМЕРУ ДИРЕКТНОМ, CCW

OPEN LOOP НЕСТАБИЛАН СИСТЕМ СЕ МОЖЕ УЧИНИТИ СТАБИЛНИМ!

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① $T(s) = k \dfrac{1}{s+1}$

$$\frac{T(s)}{1+T(s)} = \frac{\frac{k}{s+1}}{1+\frac{k}{s+1}} = \frac{k}{1+k+s}$$

$s_p = -(1+k)$

$k > 0 \rightarrow$ стабилно

② $T(s) = k \dfrac{1}{s-1}$

$$\frac{T(s)}{1+T(s)} = \frac{\frac{k}{s-1}}{1+\frac{k}{s-1}} = \frac{k}{s-1+k}$$

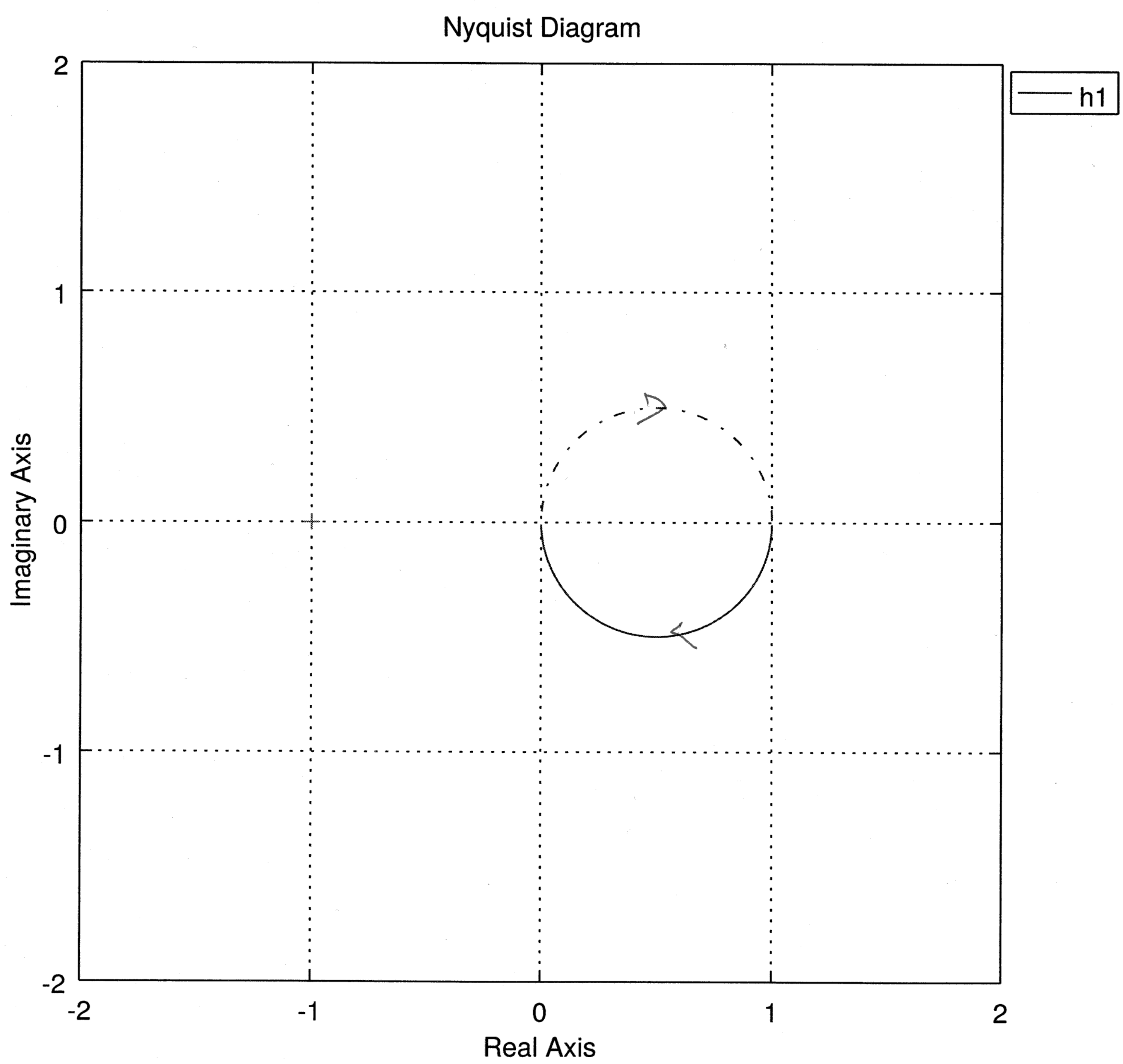
$s_p = 1-k$

$s_p < 0$

$1-k<0$

$\boxed{k>1}$ – стабилно!

$\boxed{0-1}$



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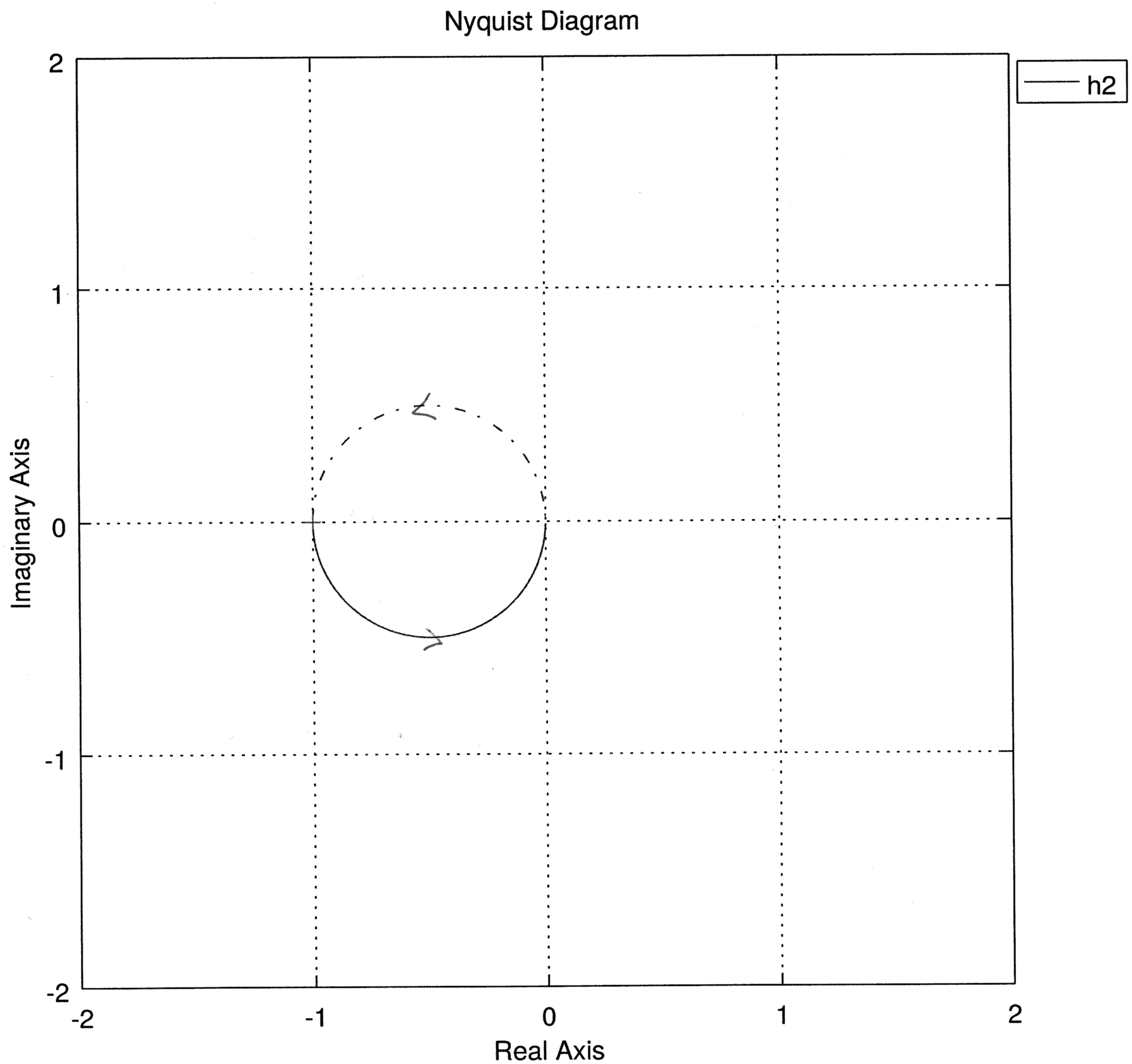
Nyquist Diagram
h2
2
1
0
-1
-2
Imaginary Axis
-2
-1
0
1
2
Real Axis

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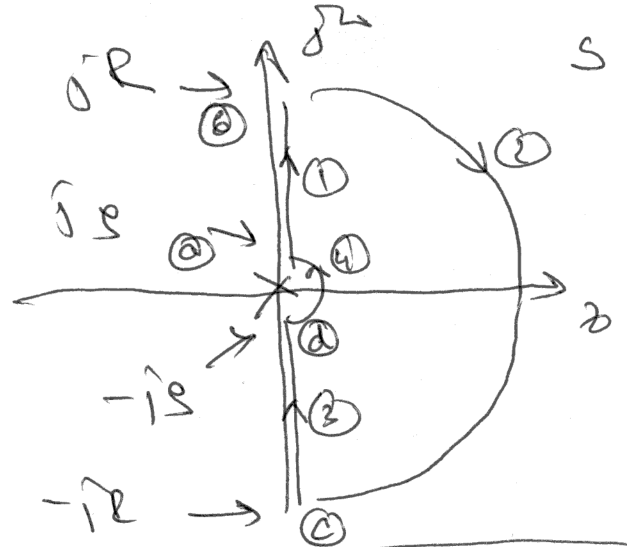
ШТА СА ПОЛОВИМА И НУЛАМА НА Im ОСИ ?

— РЕАЛНО, ВЕОМА РЕАЛНО, ИНТЕГРАТОР У ПЕТЉИ

— nyquist(tf([1], [1, 0]))



S — ОБИЛАЗАК СА ДЕСНЕ СТРАНЕ
— НИЈЕ ОБУХВАЋЕН НИ ЈЕДАН ПОЛ
— КАДА ИДЕ СЛИКА? ARGUMENT PRINCIPLE!

---

(1) : $s = j\omega$ $\qquad h(s) = \frac{1}{s} = \frac{1}{j\omega} = -j\frac{1}{\omega}$

$\rho^* < \omega < R$

(a) $h(j\rho) = -j\frac{1}{\rho} \to -j\infty$

(b) $h(jR) = -j\frac{1}{R} \to -j\emptyset$

---

(2) : $s = Re^{j\varphi}$

$\frac{\pi}{2} > \varphi > -\frac{\pi}{2}$

$h(Re^{j\frac{\pi}{2}}) = h(jR) = \frac{1}{jR} = -j\frac{1}{R}$

$h(Re^{-j\frac{\pi}{2}}) = h(-jR) = \frac{1}{-jR} = j\frac{1}{R}$

$h(s) = \frac{1}{Re^{j\varphi}} = \frac{1}{R}e^{-j\varphi}$

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(3) : $s = -j\omega$ $h(s) = \frac{1}{s} = \frac{1}{-j\omega} = j\frac{1}{\omega}$

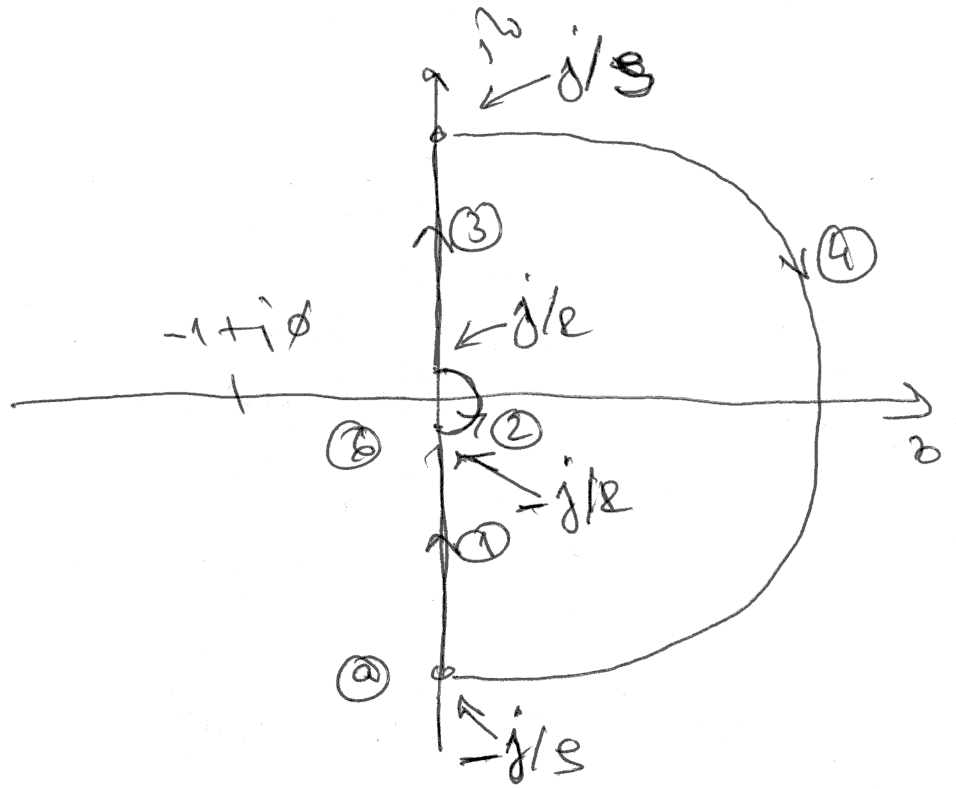
$R > \omega > \rho$ $h(-jR) = j\frac{1}{R}$

$h(-j\rho) = j\frac{1}{\rho}$

(4) : $s = \rho e^{j\varphi}$ $h(s) = \frac{1}{\rho e^{j\varphi}} = \frac{1}{\rho} e^{-j\varphi}$

$-\frac{\pi}{2} < \varphi < \frac{\pi}{2}$ $h(\rho e^{-j\frac{\pi}{2}}) = \frac{j}{\rho} = j\frac{1}{\rho}$

$h(\rho e^{j\frac{\pi}{2}}) = \frac{1}{\rho} e^{-j\frac{\pi}{2}} = -j\frac{1}{\rho}$



— ПРОГРАМИ ЦРТАЈУ САМО jω !!!

— R → ∞ НИКАКОВ ПРОБЛЕМ

— (4) СЕ НЕ ЦРТА

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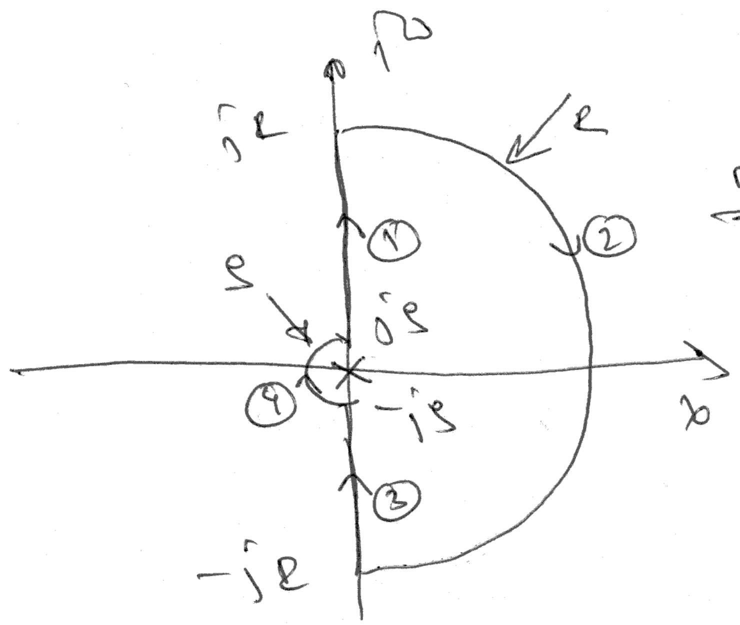
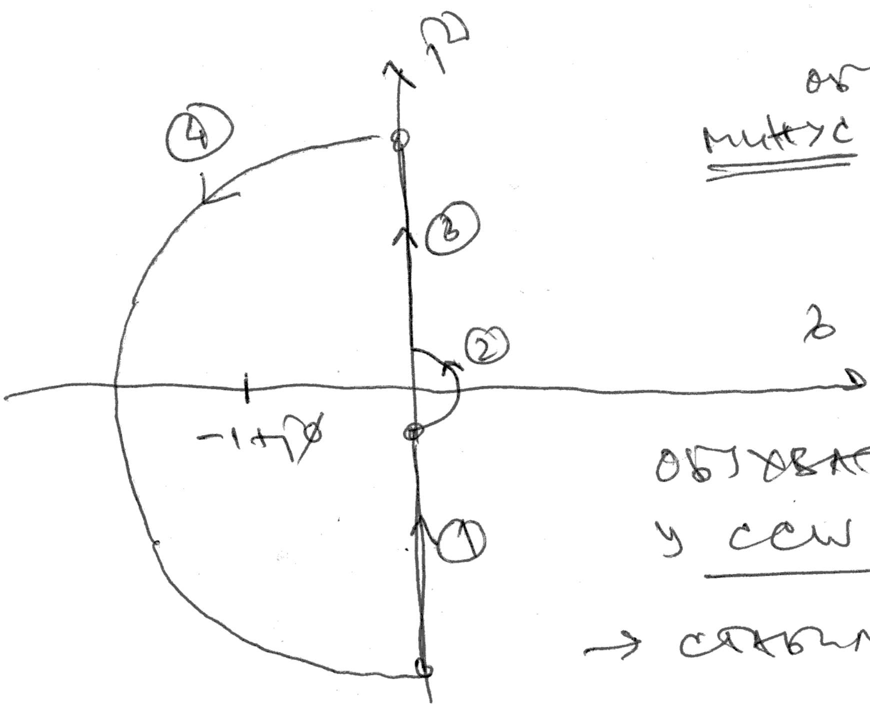
Други обилазак око пола!



обухвата
плус једном

- обухвата пол, $n_p = 1$!!!
- 1, 2 и 3 исто !!!
- $h(s) = \frac{1}{s} = \frac{1}{\rho e^{i\varphi}} = \frac{1}{\rho} e^{-i\varphi}$

$-\frac{\pi}{2} < \varphi < \frac{\pi}{2}$, али и $\varphi = \pi$!



обухвата
минус једном !!!

обухвата једном у CCW смеру

→ стабилно!

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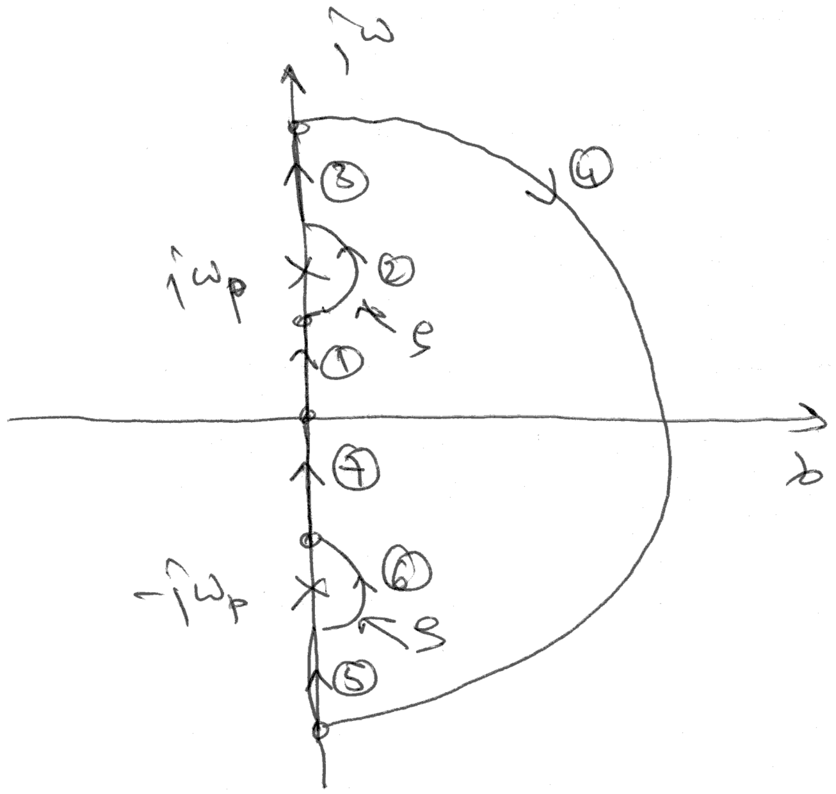
Другой случай!



$$H(s) = \frac{1}{1 + \frac{s^2}{\omega_p^2}}$$

① $s = j\omega$ $\qquad H(j\omega) = \dfrac{1}{1 - \frac{\omega^2}{\omega_p^2}}$

$s = j0 \qquad H(j0) = 1$

$$s = j(\omega_p - \rho) \qquad H(j(\omega_p - \rho)) = \frac{1}{1 - \frac{\omega_p^2 - 2\rho\omega_p + \rho^2}{\omega_p^2}}$$

$$\approx \frac{1}{\frac{2\rho}{\omega_p}} \approx \frac{\omega_p}{2\rho}$$

② : $s = j\omega_p + \rho e^{j\varphi}$

$-\frac{\pi}{2} < \varphi < \frac{\pi}{2}$

$$\frac{s^2}{\omega_p^2} = \frac{-\omega_p^2 + 2j\omega_p\rho e^{j\varphi} + \rho^2 e^{j2\varphi}}{\omega_p^2} =$$

$$= -1 + 2j\frac{\rho}{\omega_p}e^{j\varphi} + \frac{\rho^2}{\omega_p^2}e^{j2\varphi}$$

$$\approx -1 + 2j\frac{\rho}{\omega_p}e^{j\varphi}$$

② : $$H(s) \approx \frac{1}{1 - 1 + 2j\frac{\varrho}{\omega_p} e^{j\varphi}} = -j \frac{\omega_p}{2\varrho} e^{-j\varphi}$$

$$H(s) \approx \frac{\omega_p}{2\varrho} e^{-j\frac{\pi}{2}} e^{-j\varphi}$$

$$H(s) \approx \frac{\omega_p}{2\varrho} e^{-j(\varphi + \frac{\pi}{2})}$$

---

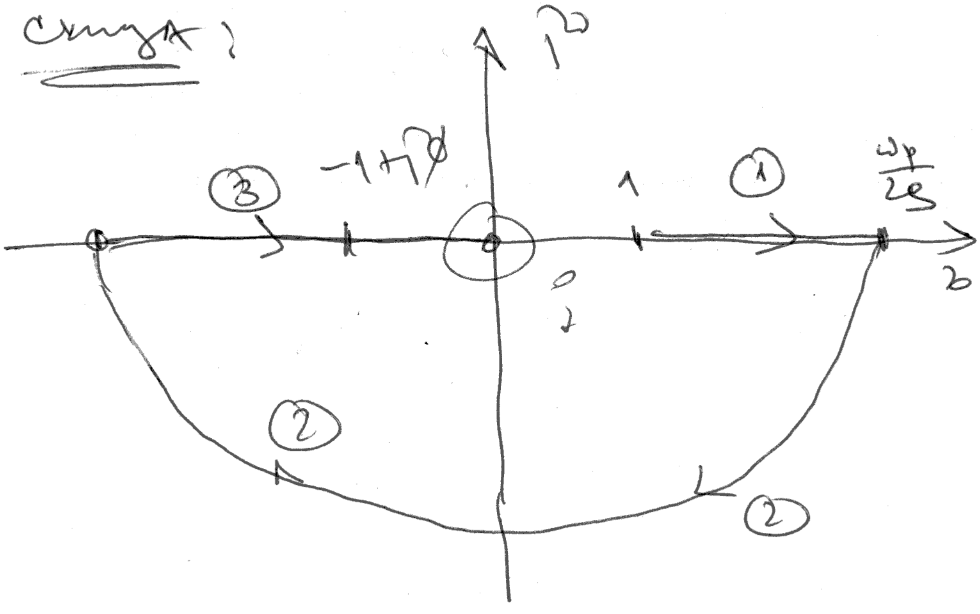
③ : $s = j\omega$

$$\omega_p + \varrho < \omega < R$$

$$H(j\omega) = \frac{1}{1 - \frac{\omega^2}{\omega_p^2}} =$$

$$= \frac{1}{1 - \frac{\omega_p^2 + 2\Delta\omega\,\omega_p + \Delta\omega^2}{\omega_p^2}} = \frac{1}{-2\frac{\Delta\omega}{\omega_p} - \frac{\Delta\omega^2}{\omega_p^2}}$$

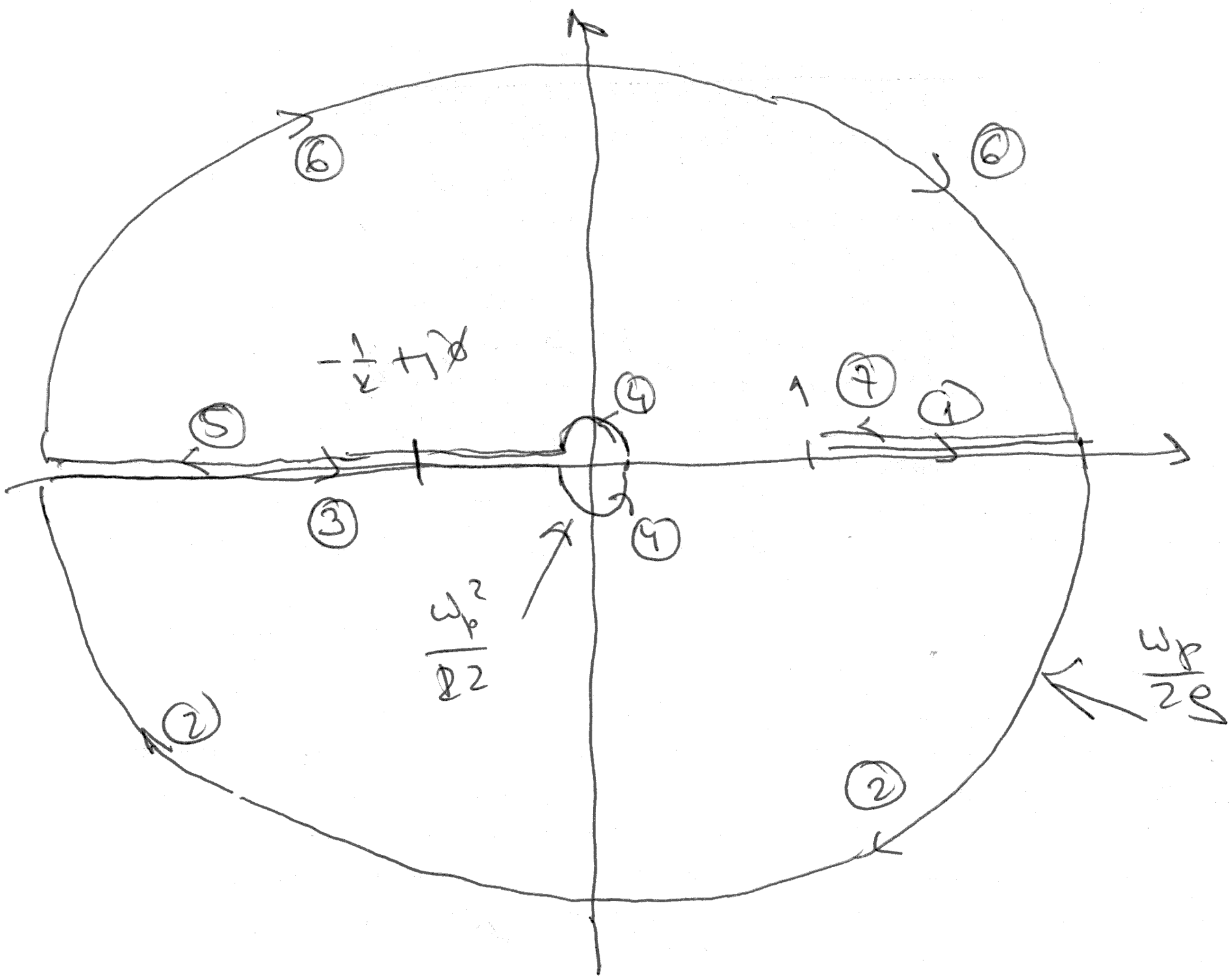
<u>Скица</u> :



4 : $s = R e^{j\varphi}$ $\qquad H(s) = \dfrac{1}{1 + \dfrac{R^2}{\omega_p^2} e^{j2\varphi}}$

$\dfrac{\pi}{2} > \varphi > -\dfrac{\pi}{2}$

$\approx \dfrac{\omega_p^2}{R^2} e^{-j2\varphi}$



- два пута прелази преко критичне тачке, преко Баркхаузенове тачке

$$\frac{T}{1+T} = \frac{\dfrac{k}{1+s^2/\omega_p^2}}{1+\dfrac{1}{1+s^2/\omega_p^2}} = \frac{K}{1+\dfrac{s^2}{\omega_p^2}+K} =$$

$$k=1 \quad \boxed{= \frac{1}{2+\dfrac{s^2}{\omega_p^2}}} \qquad = \frac{k}{1+k+\dfrac{s^2}{\omega_p^2}}$$

полови:

$$1+k+\frac{s^2}{\omega_p^2} = 0$$

$$s^2 = -\omega_p^2(1+k)$$

$$\boxed{s = \pm j\,\omega_p\sqrt{1+k}}$$

како год окренете полови су на Im оси, 2x пази по Баркхаузену

ТРИ ВАЖНЕ ОСОБИНЕ FREQUENCY RESPONSE

1) <u>LOW FREQUENCY GAIN</u>

- грешка устаљеног стања
- грешка на нижим фреквенцијама, до f_c
- "грешка" се односи на систем праћења

2) <u>CROSSOVER FREQUENCY</u>

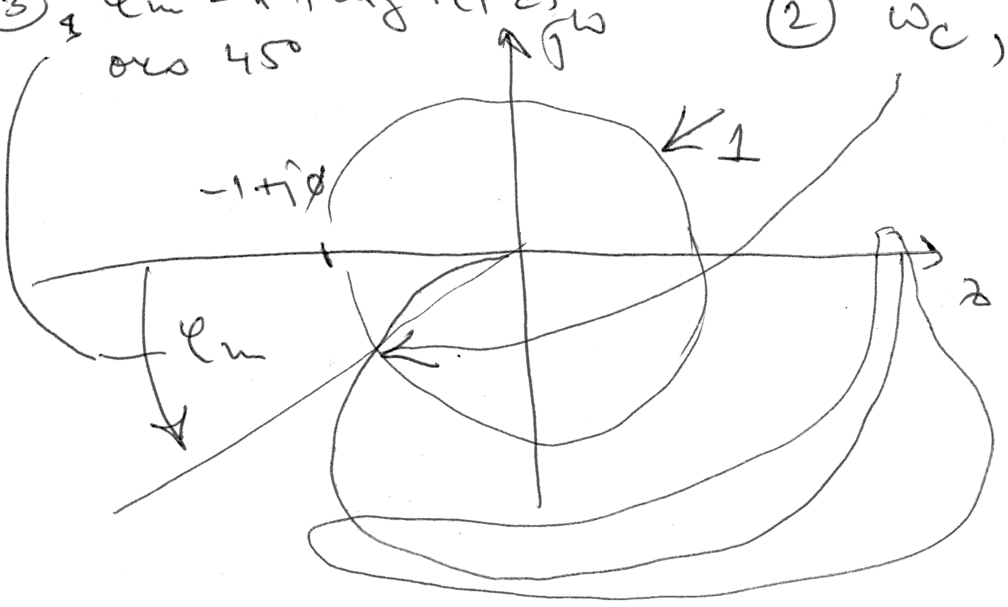
- closed loop bandwidth
- брзина реакције система, нпр. одзив на степ функцију

3) <u>PHASE MARGIN</u>

- стабилност
- пригушеност одзива

③ $\varphi_m = \pi + \arg T(j\omega_c)$ око 45°

② ω_c, $|T(j\omega_c)| = 1$ <u>што веће</u>



① LOW FREQUENCY GAIN <u>што веће</u>

ПРИМЕРИ:

1) LOW FREQUENCY GAIN:

$H_1 = \frac{1}{s+1}$; $A_0 = 1$, $\omega_p = 1$, 0dB

$H_{10} = \frac{10}{10s+1}$; $A_0 = 10$, $\omega_p = \frac{1}{10}$, 20dB

$H_{100} = \frac{100}{100s+1}$; $A_0 = 100$, $\omega_p = \frac{1}{100}$, 40dB

$H_{1000} = \frac{1000}{1000s+1}$; $A_0 = 1000$, $\omega_p = \frac{1}{1000}$, 60dB

$H_0 = \frac{1}{s}$; $A_0 \to \infty$, $\omega_p \to 0$, $AB = 1$

ZEROGAIN

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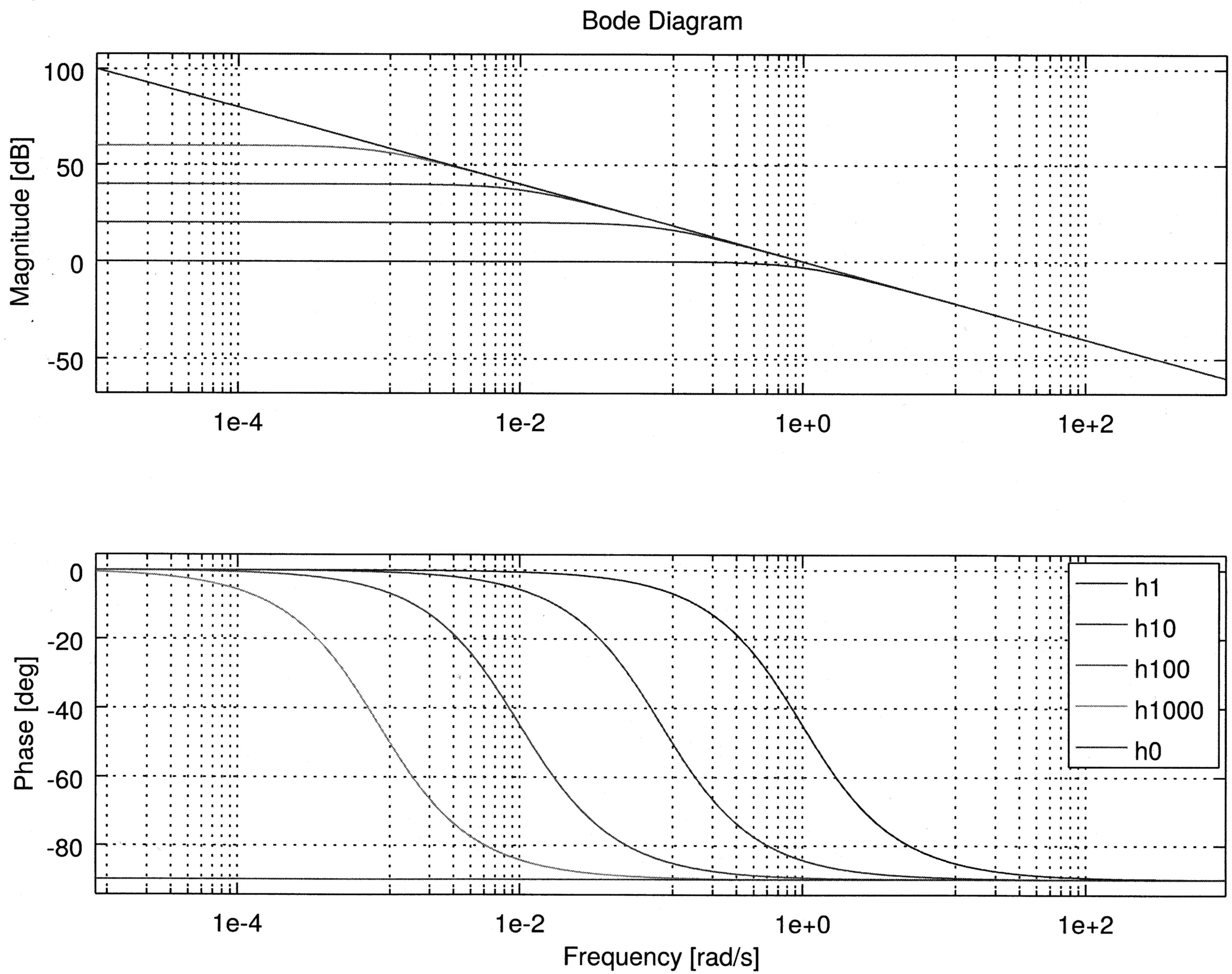

2-1

ZEROGAIN

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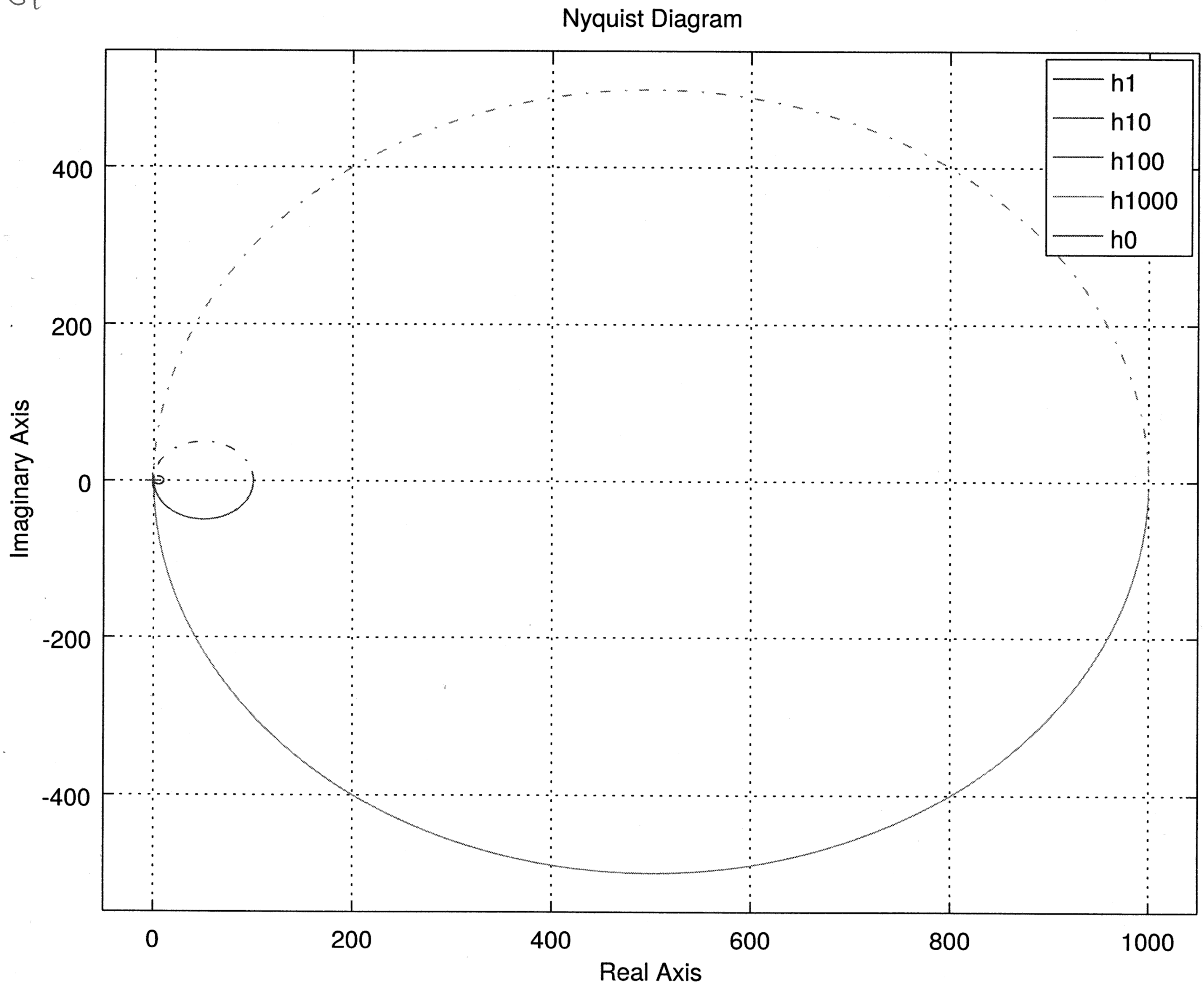

2-2

ZEROGAIN

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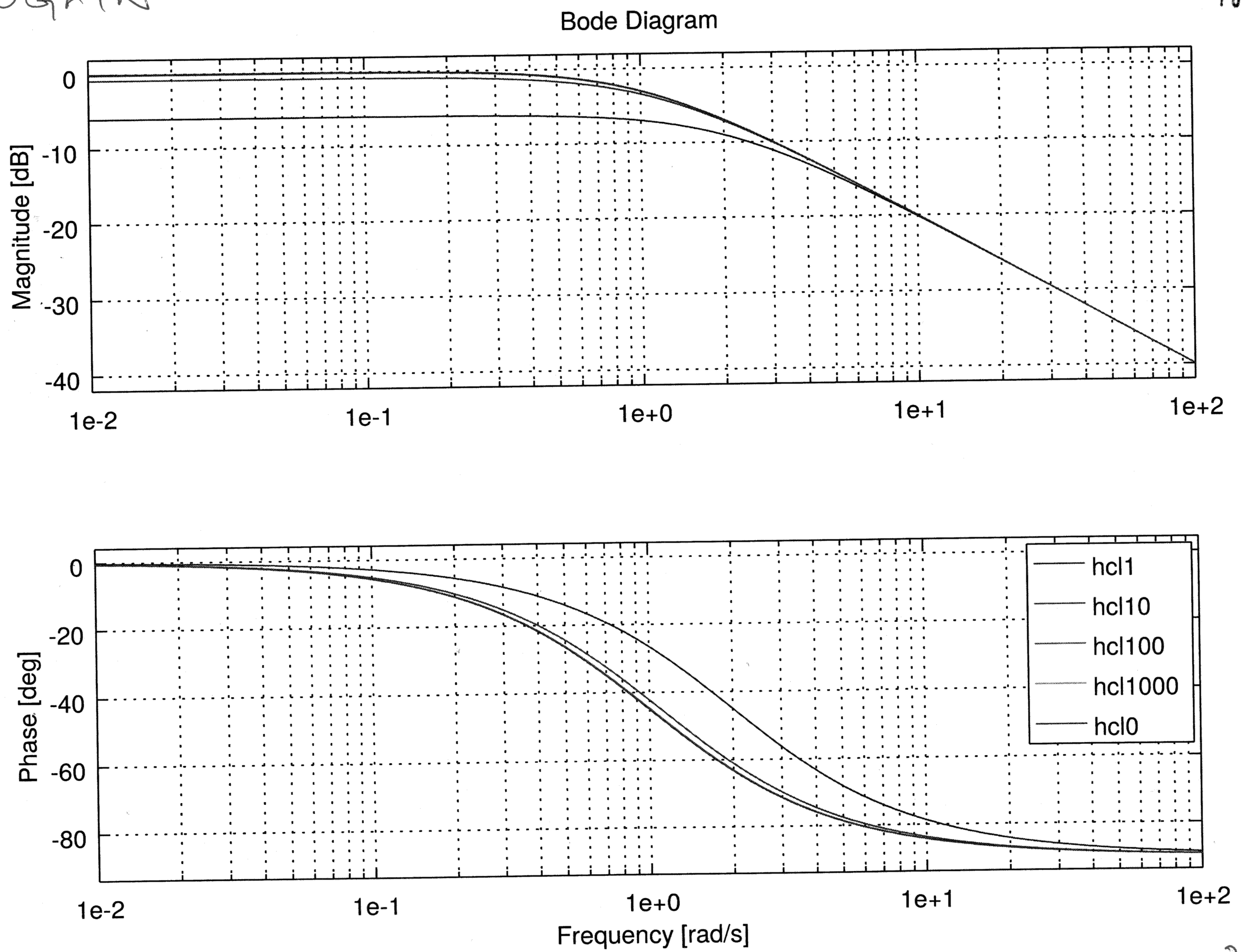

2-3

ZEROGAIN

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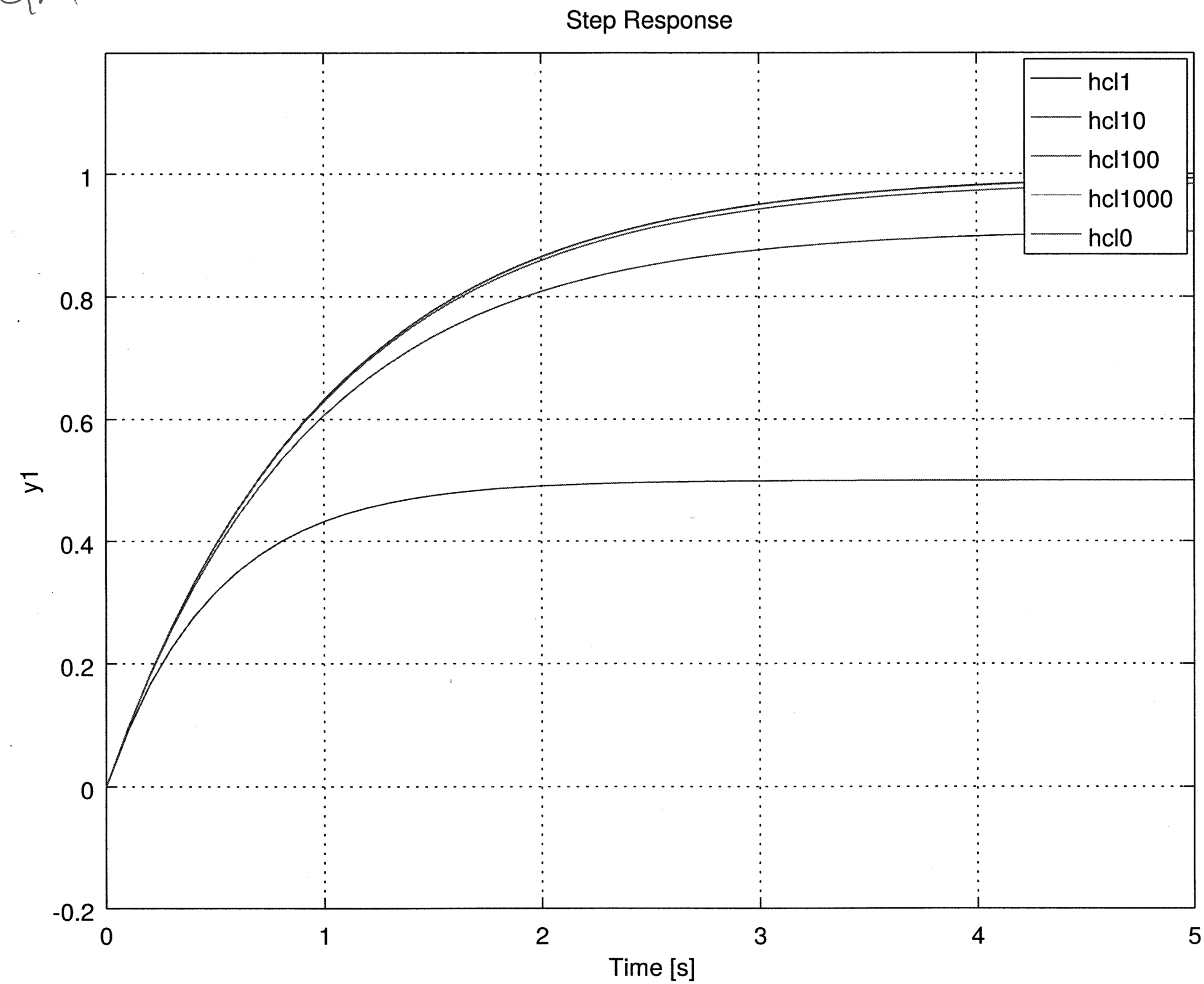

2-4

ZEROGAIN

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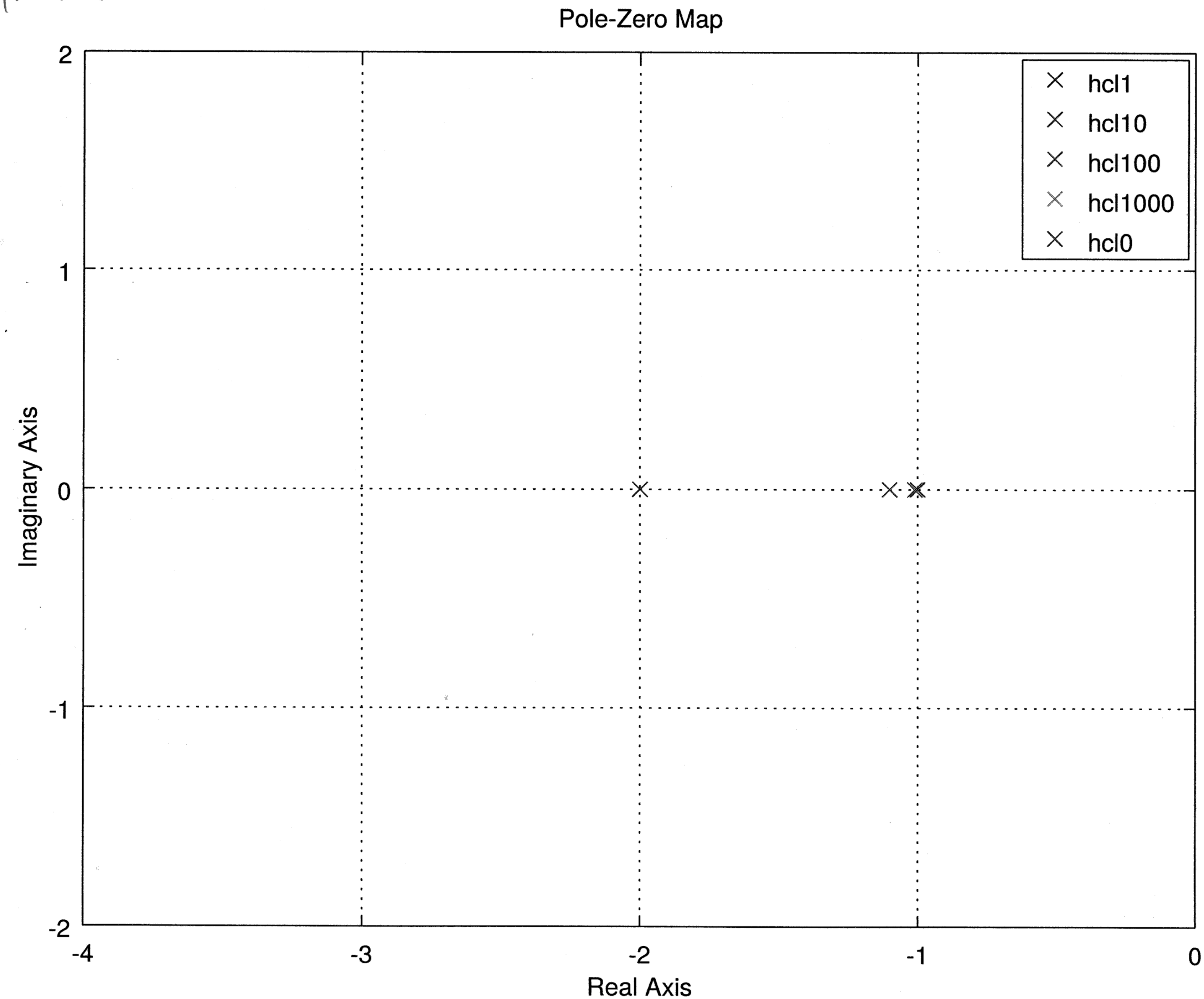

2-5

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ПРИМЕРИ:

2) CROSSOVER FREQUENCY:

$$H_1 = \frac{1}{s}, \quad \omega_c = 1$$

$$H_{10} = \frac{10}{s}, \quad \omega_c = 10$$

$$H_{100} = \frac{100}{s}, \quad \omega_c = 100$$

$$H_{1000} = \frac{1000}{s}, \quad \omega_c = 1000$$

CROSSOVER

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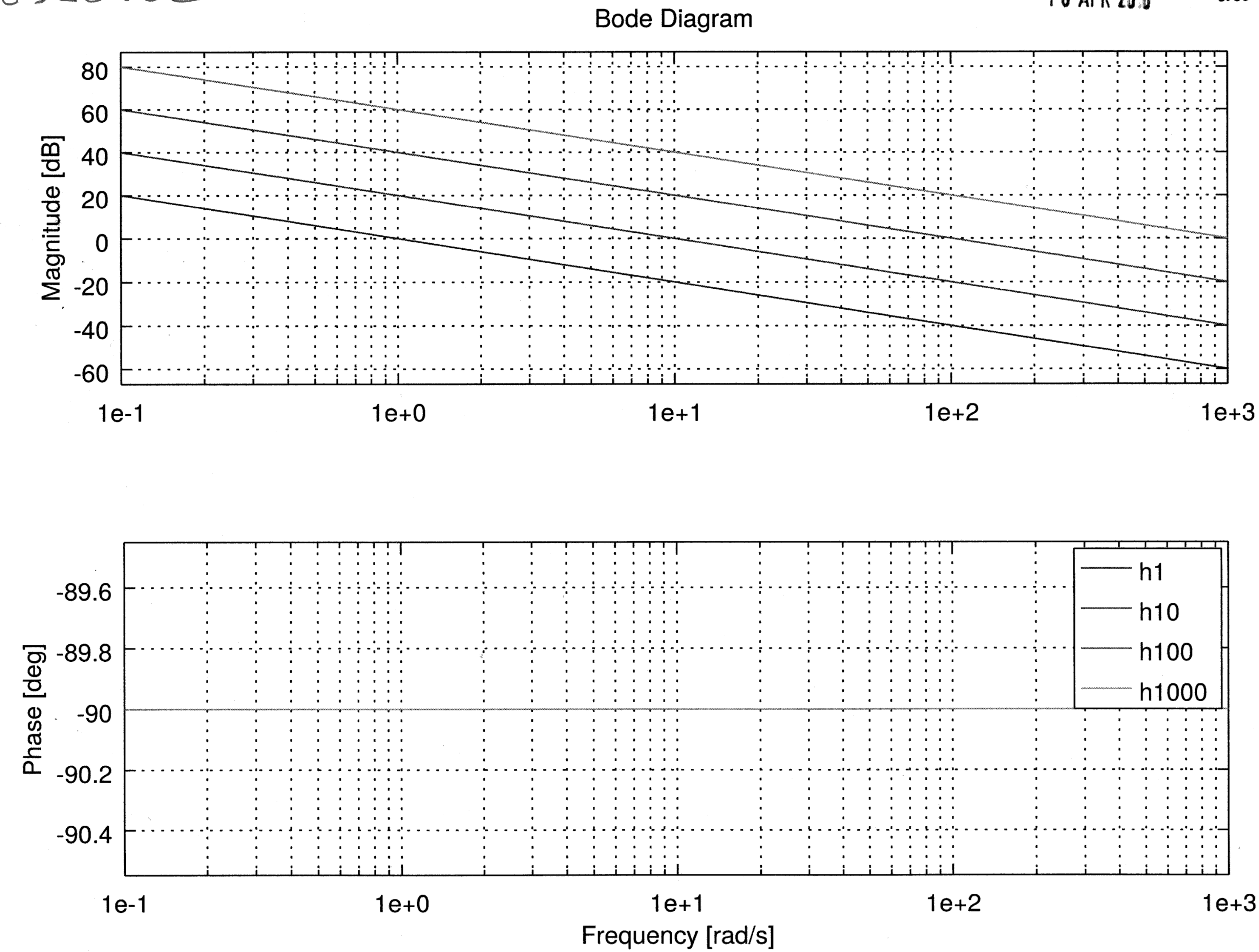

3-1

CROSSOVER

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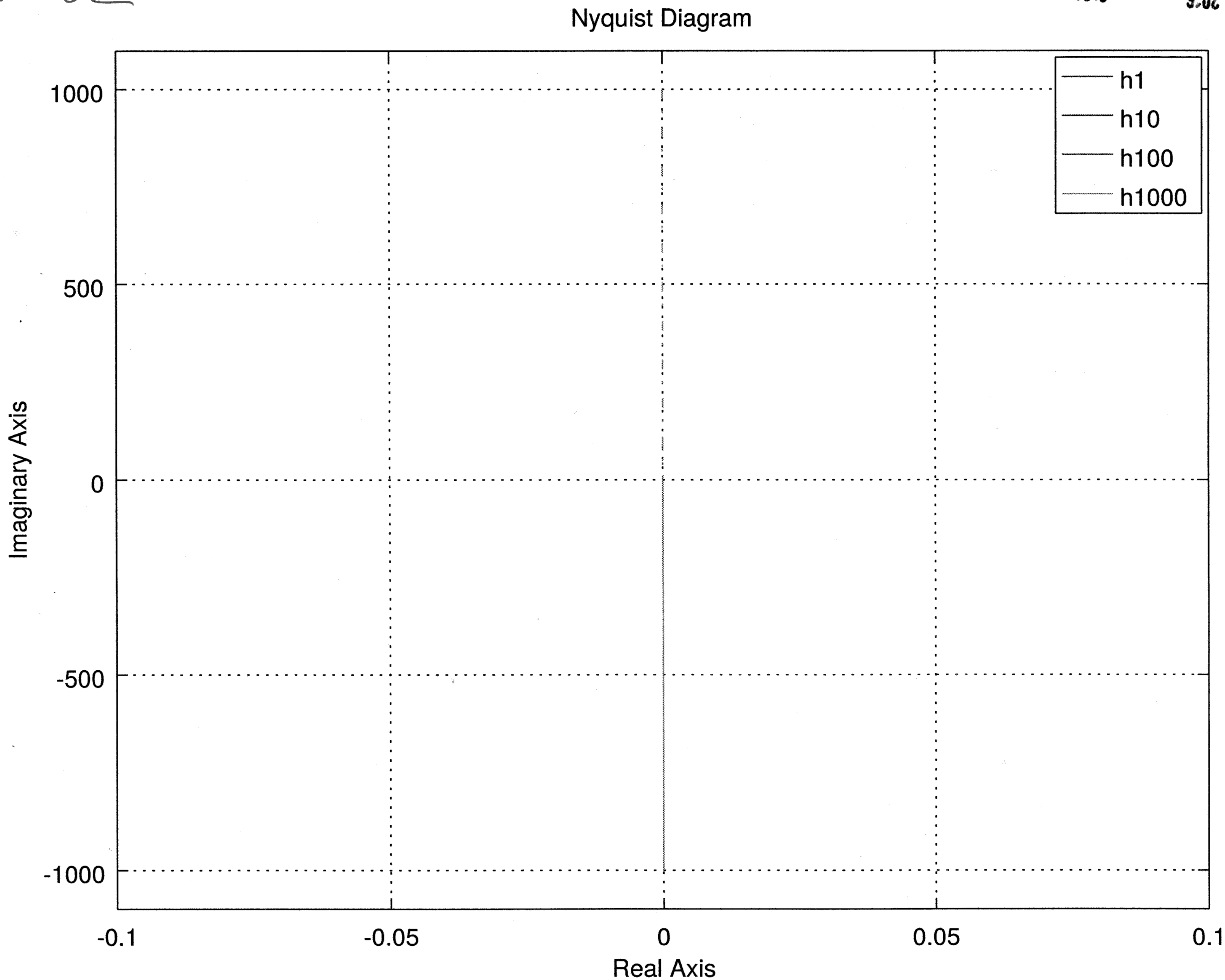

3-2

CROSSOVER

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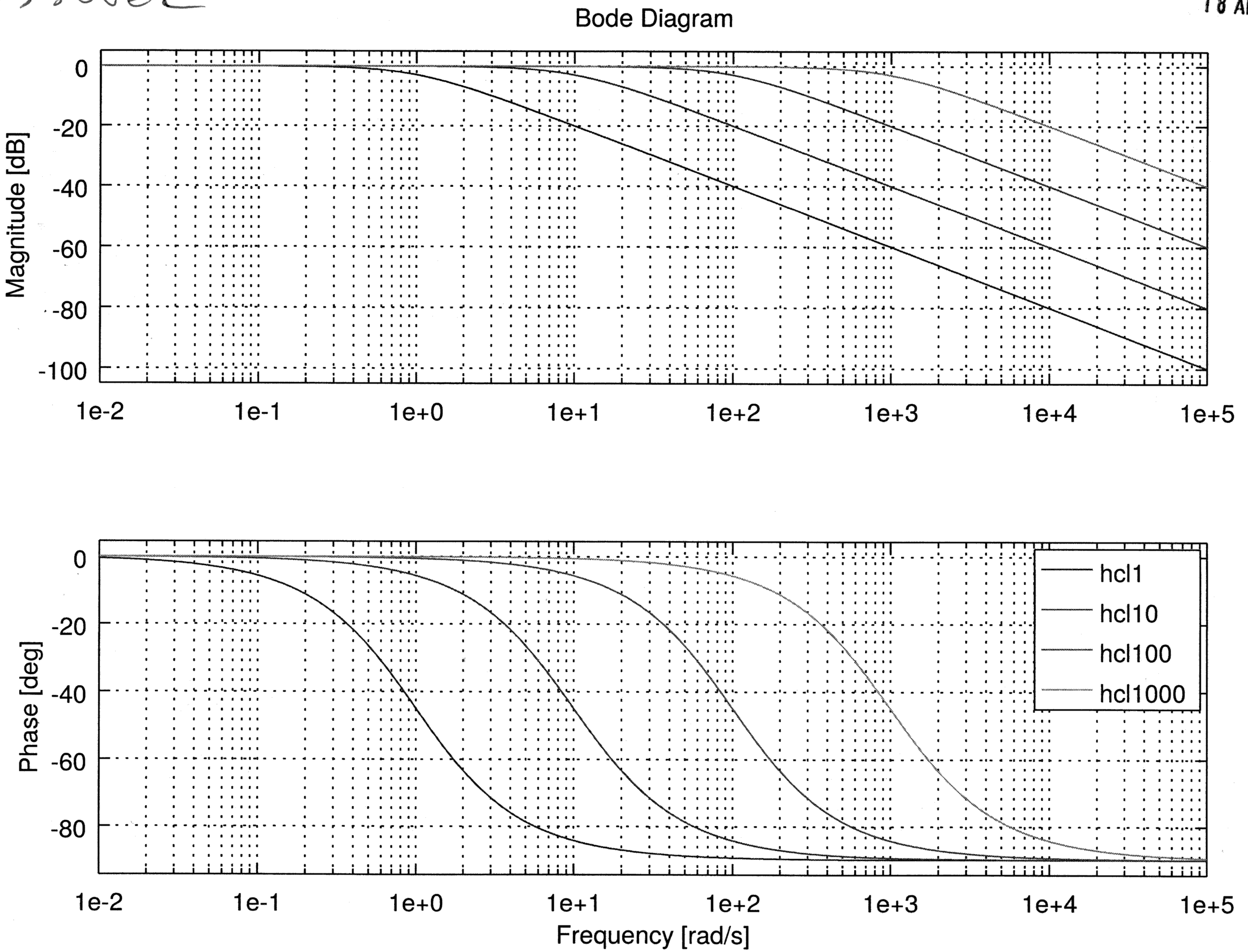

3-3

CROSSOVER

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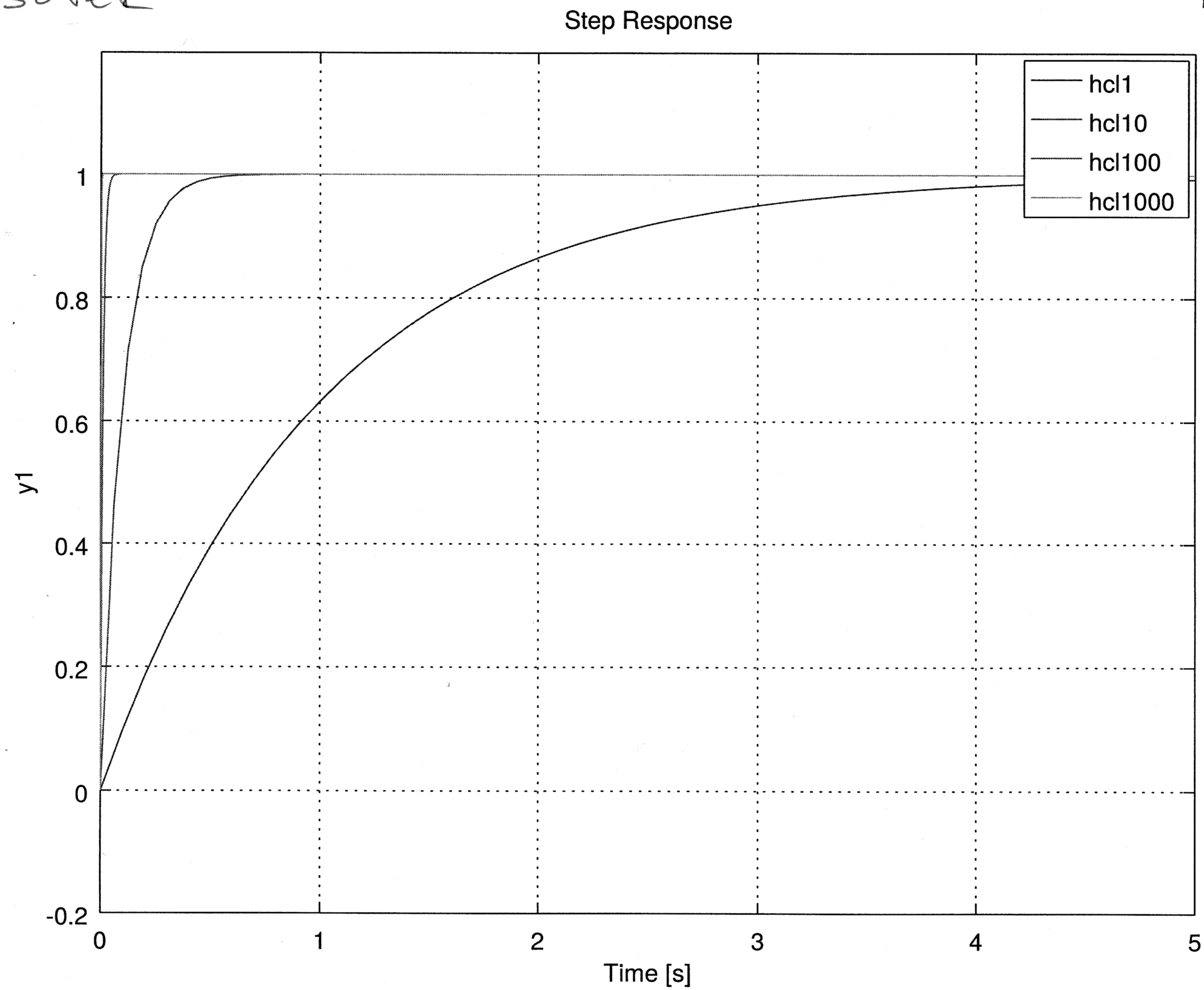

3-4

CROSSOVER

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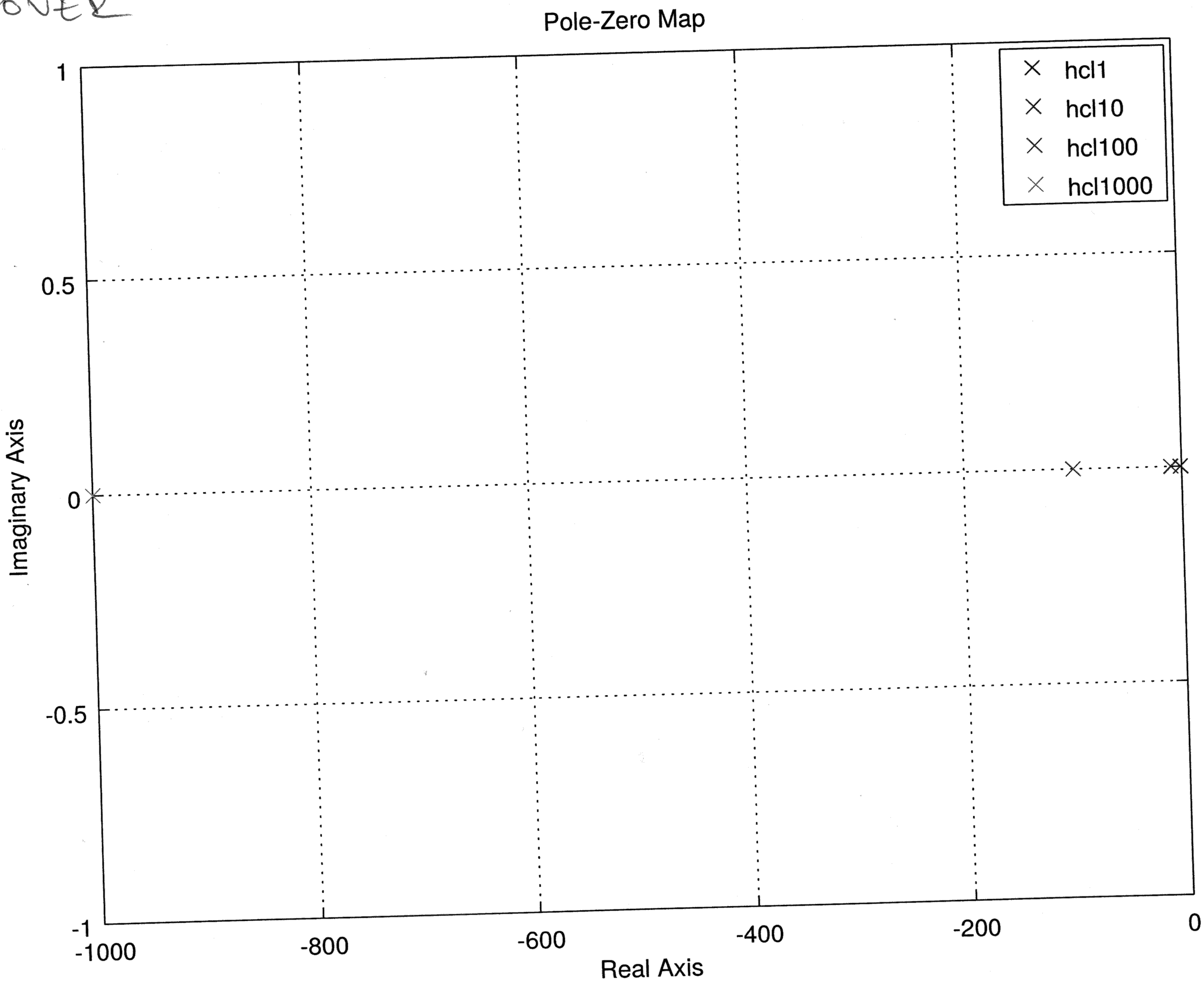

3-5

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примери :

3) PHASE MARGIN

$$H(s) = \left(\frac{\omega_c}{s}\right)^2 \cdot H_{LEAD}(p)$$

$$H_{LEAD}(p) = \frac{1}{p} \frac{1 + \frac{s}{\omega_c / p}}{1 + \frac{s}{p\omega_c}}$$

$$\omega_c = 100$$

$$H_1 = H(s)\, H_{LEAD}(1)$$

$$H_2 = H(s)\, H_{LEAD}(2)$$

$$H_5 = H(s)\, H_{LEAD}(5)$$

$$H_{10} = H(s)\, H_{LEAD}(10)$$

$$H_{100} = H(s)\, H_{LEAD}(100)$$

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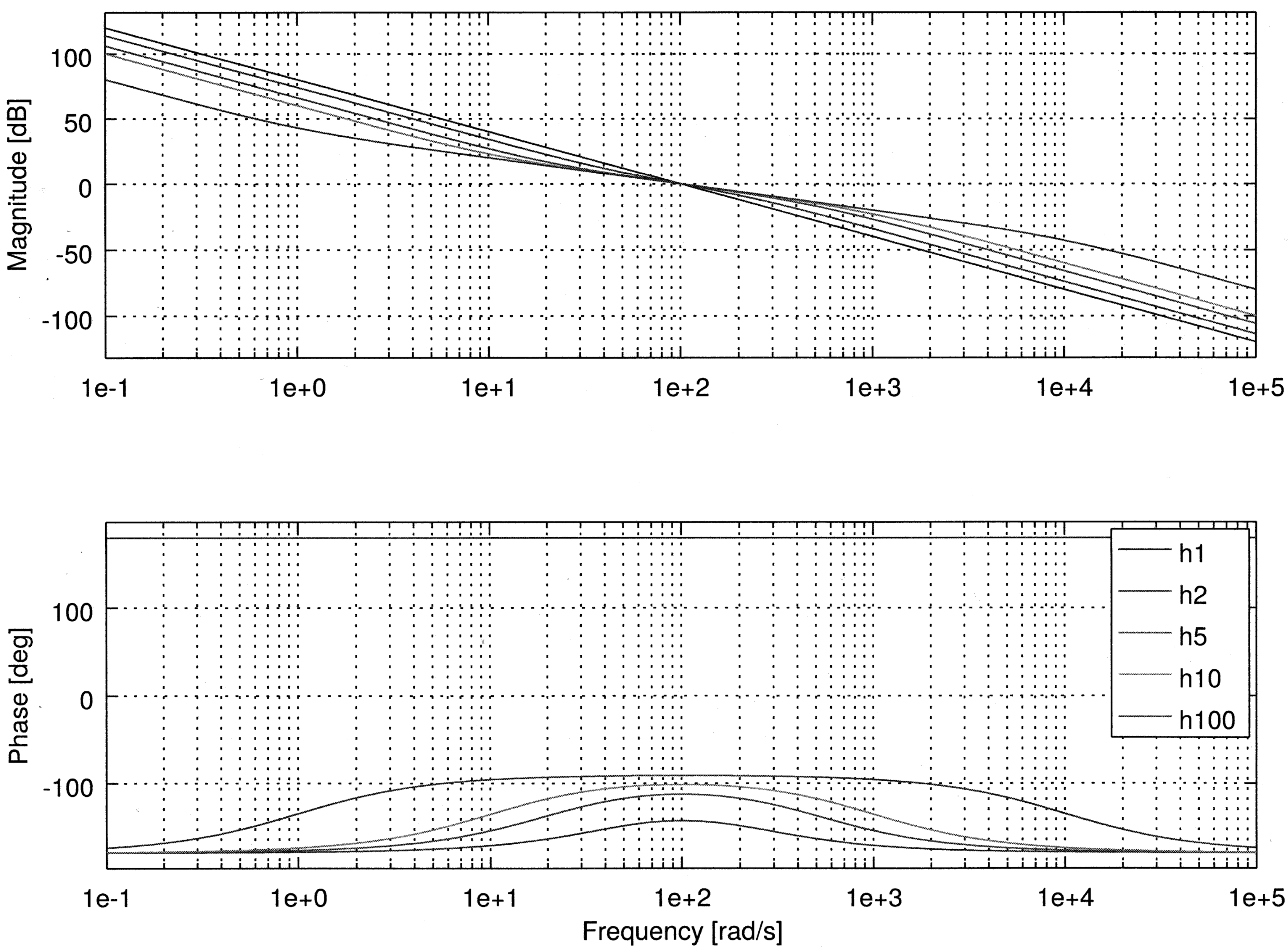
Bode Diagram



4-1

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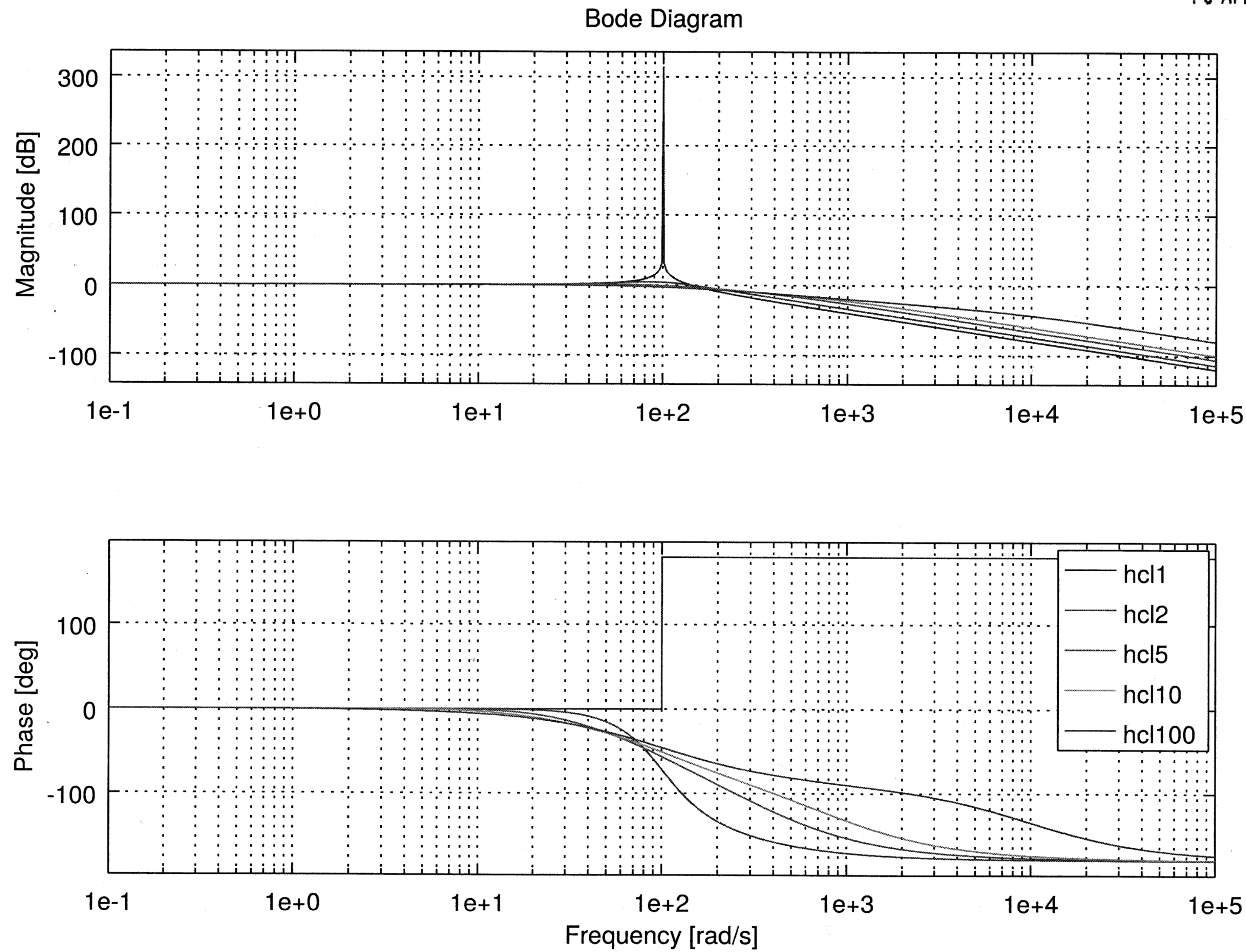

4-2

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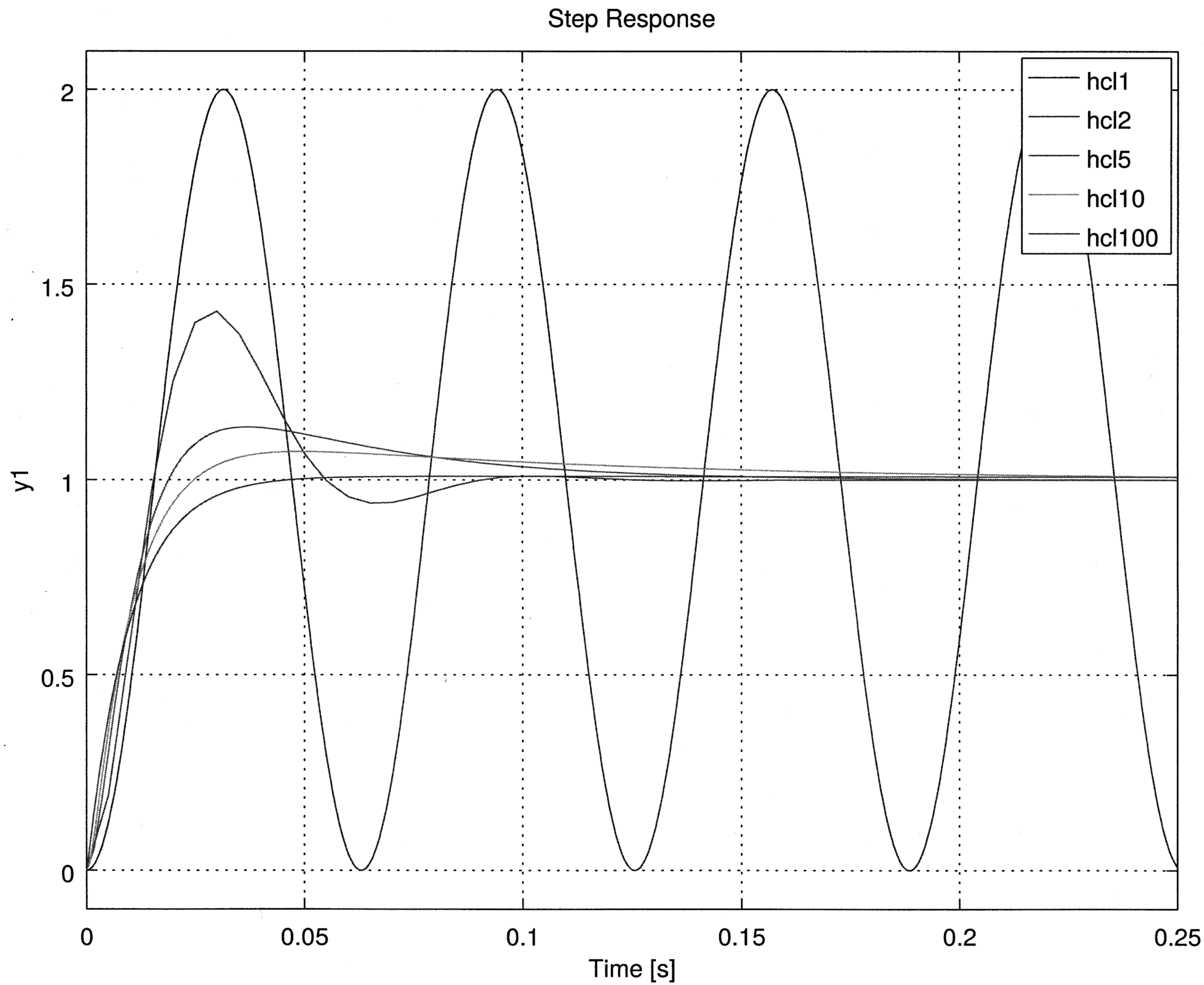
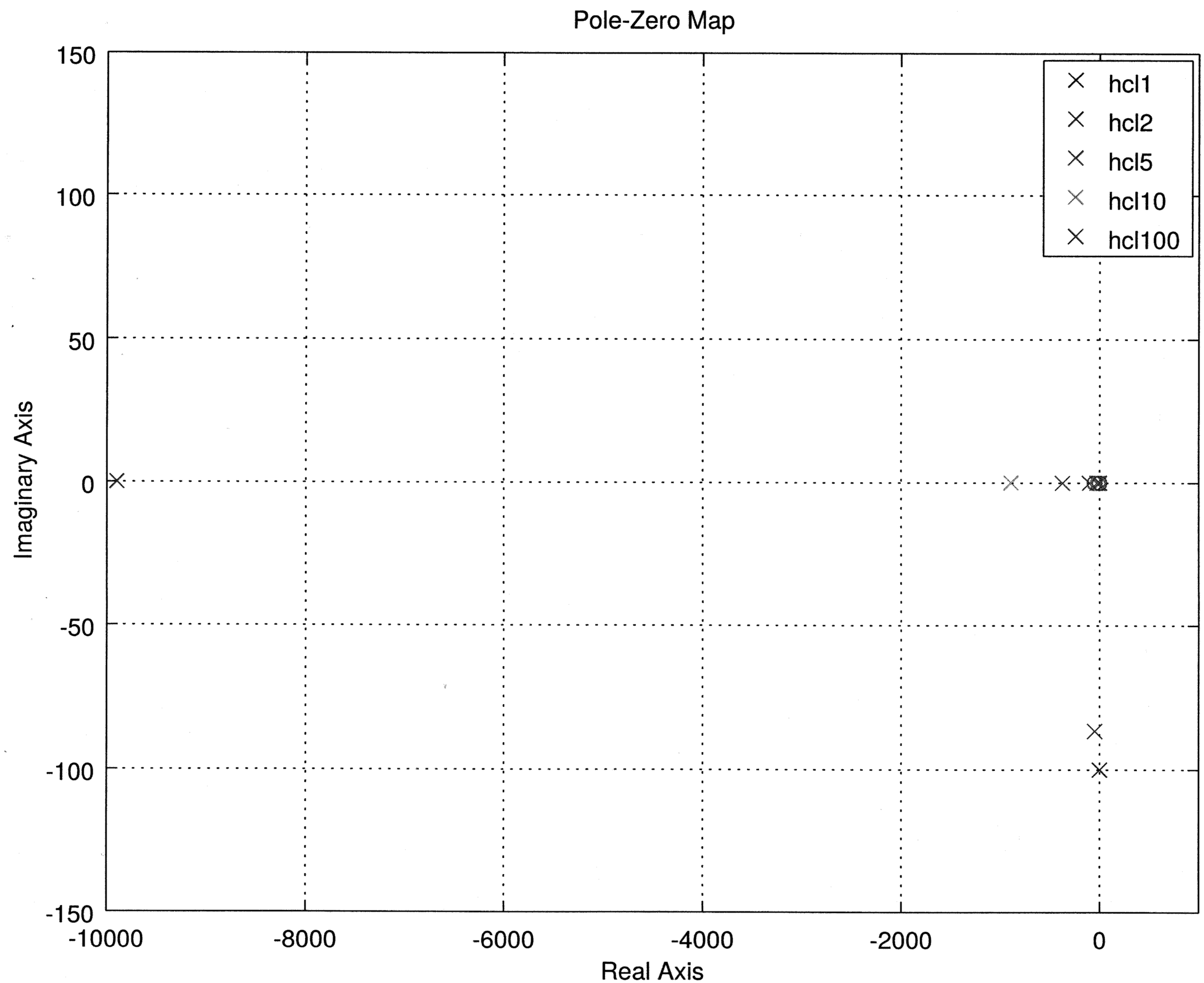


4-3

18 APR 2016



4-4