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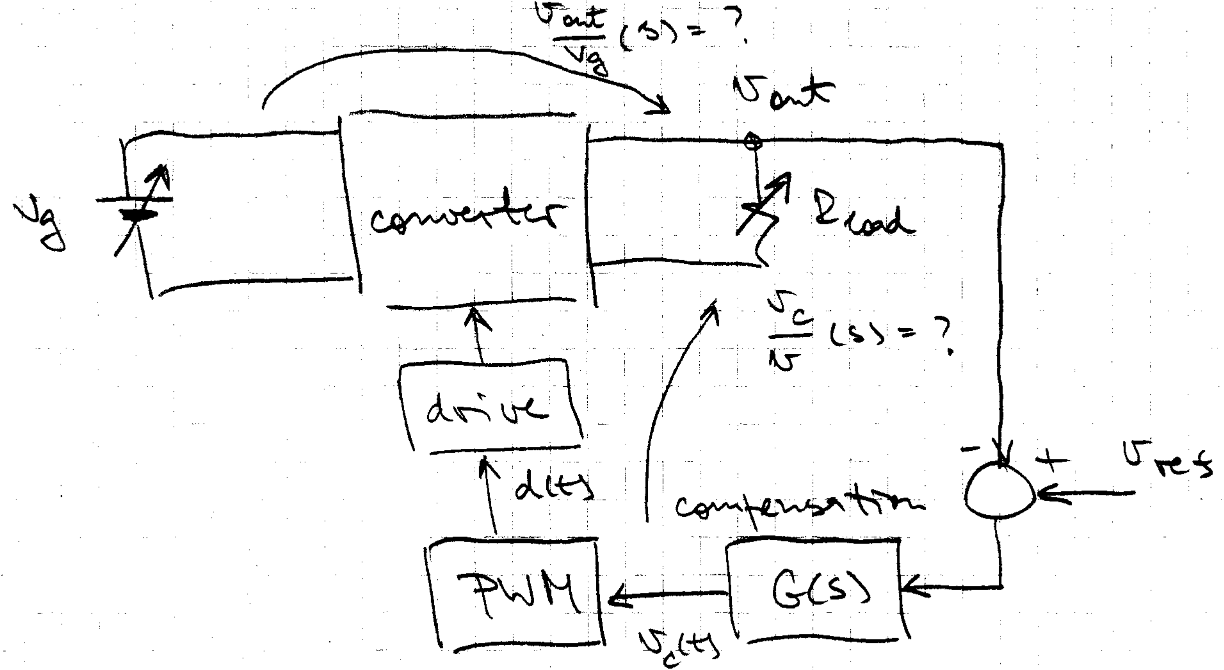
Управљање DC-DC конверторима

- Типична апликација

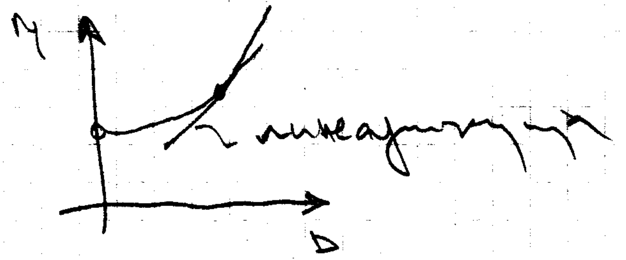


1 transient overshoot
2 setting time
3 steady-state regulation

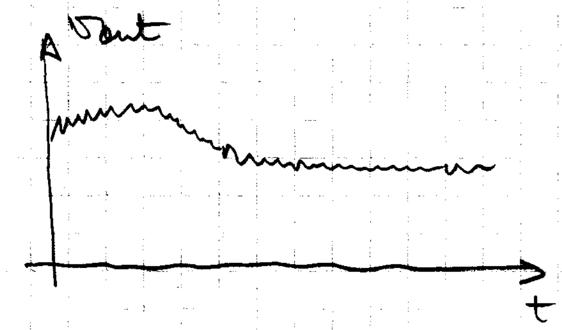
- Систем: нелинеаран | нестационаран
- У принципу - симулација - не вала за design, потребна анализа
- Аналитички проблеми:

1) нелинеарност

boost, $M(D) = \frac{1}{1-D}$ - ово је steady-state



2) ripple (високофреквентна динамика)



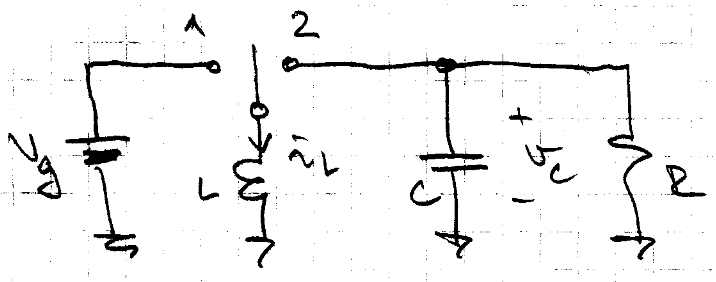
решение - neglect ripple

Процедура:

1) усреднување

2) линеаризација

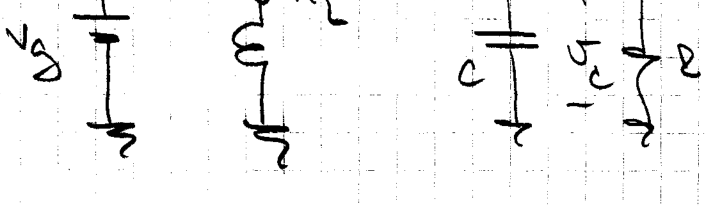
Пример: Buck-boost конвертор



T_s - switching period

D - duty ratio

position 1

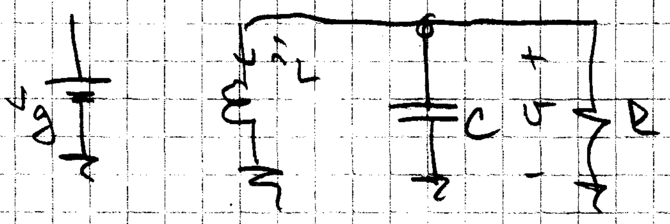


DT_s

$$v_L = L\frac{di_L}{dt} = V_g$$

$$i_C = C\frac{dv_C}{dt} = -\frac{v_C}{R}$$

position 2



$(1-D)T_s$

$$v_L = L\frac{di_L}{dt} = v_c$$

$$i_c = C\frac{dv_c}{dt} = -i_L - \frac{v_c}{R}$$

выражаем $i_L(T_s)$ и $v_c(T_s)$ как ф-ии от $i_L(0)$, $v_c(0)$ и D

$$i_L(DT_s) = i_L(0) + DT_s\frac{V_g}{L}$$

$$i_L(T_s) = i_L(DT_s) + D'T_s\frac{v_c}{L}$$

С одн. и применением linear ripple approximation

$$i_L(T_s) = i_L(0) + DT_s\frac{V_g}{L} + D'T_s\frac{v_c}{L}$$

$$\boxed{i_L(T_s) = i_L(0) + \frac{T_s}{L}\underbrace{(DV_g + D'v_c)}_{\overline{v_L}}}$$

После n периода

$$i_L((n+1)T_s) = i_L(nT_s) + \frac{T_s}{L}\big(D(nT_s)V_g(nT_s) +$$
$$+ D'(nT_s)v(nT_s)\big)$$

Претпостављено да се V_g и v споро (дискретно) мењају (тј. константни)

Апроксимација извода:

$$\frac{di_L}{dt} \cong \frac{\Delta i_L}{\Delta t} = \frac{i_L((n+1)T_s) - i_L(nT_s)}{T_s}$$

пошто се све споро мења $nT_s \to t$, континуализација

$$\frac{di_L}{dt} \cong \frac{1}{L}\big(D(t)V_g(t) + D'(t)v_c(t)\big)$$

односно:

$$\boxed{L\frac{di_L}{dt} = DV_g + D'v_c = \bar{v}_L}$$

овако ће се увек писати

Слично за $v_c(t)$!

$$v_c(DT_s) = v(0) - DT_s \frac{v_c}{RC}$$

$$v_c(T_s) = v_c(DT_s) - D'T_s \left(\frac{i_L}{C} + \frac{v_c}{RC} \right)$$

$$v_c(T_s) = v_c(0) - \frac{T_s}{C} (\underbrace{\frac{v}{R} + D' i_L)}_{= \bar{i}_c}$$

опет узима 0 n одбирака, апроксимација извода

$$\frac{dv_c}{dt} \cong \frac{\Delta v}{\Delta t} = \frac{v_c((n+1)T_s) - v_c(nT_s)}{T_s}$$

$$\frac{dv_c}{dt} = -\frac{1}{C} \left(\frac{v_c(t)}{R} + D'(t)\, i_L(t) \right)$$

односно

$$\boxed{C \frac{dv_c}{dt} \cong -\frac{v_c}{R} - D' i_L = \bar{i}_c}$$

- Kombinerat large-signal dynamical model

$$\boxed{\begin{aligned} L\frac{di_L}{dt} &= DV_g + D' v_C \\ C\frac{dv_C}{dt} &= -\frac{v_C}{R} - D' i_L \end{aligned}}$$

- Nonlinear: multiplication of time-varying quantities, $D(t)$, $v_c(t)$, $i_L(t)$

- Low-frequency behavior of $i_L(t)$ & $v_c(t)$, ripple filtered out

- Still nonlinear

- Next step: large signal DC and small signal AC models

DC magen beh. jobbet

- Quiescent operating point

$$v_g(t) = V_g + \hat{v}_g(t)$$
$$d(t) = D_0 + \hat{d}(t)$$
$$i_L(t) = I_{L0} + \hat{i}_L(t)$$
$$v_c(t) = V_{c0} + \hat{v}_c(t)$$

– DC model (large signal)
transients died out

$$0 = D_0 V_g + D_0' V_c \rightarrow V_c = -\frac{D_0}{D_0'} V_g$$

$$0 = -\frac{V_c}{R} - D_0' I_L \rightarrow I_L = -\frac{V_c}{R D_0'}$$

↑
versions of eqns
V sec & A sec balance

– Obs of steady-state transfer functions,
nonlinear, but who cares

– Perturbation of large-signal dynamical
model (≠ large signal DC model)

$$L\frac{d(I_L+\hat{i}_L)}{dt} = (D_0+\hat{d})(V_g+\hat{v}_g) +$$

$$(D_0'-\hat{d})(V_c+\hat{v}_c)$$

$$L\left(\cancelto{0}{\frac{dI_L}{dt}} + \frac{d\hat{i}_L}{dt}\right) = \underbrace{(\cancelto{0}{D_0 V_g + D_0' V_c})}_{\text{dc terms}} +$$

$$+ \underbrace{D_0 \hat{v}_g + \hat{d} V_g - \hat{d} V_c + D_0' \hat{v}_c}_{\text{1st order ac terms}} +$$

$$+ \underbrace{\hat{d}\hat{v}_g - \hat{d}\hat{v}_c}_{\text{2nd order ac terms}} \rightarrow \text{neglect}$$

$$L\frac{d\hat{i}_L}{dt} = D_0\hat{v}_g + D_0'\hat{v}_c + \hat{d}(V_g - V_c)$$

- исто за кондензатор

$$C\frac{d(V_c+\hat{v}_c)}{dt} = -\frac{V_c+\hat{v}_c}{R} - (D_0'-\hat{d})(I_L+\hat{i}_L)$$

$$C\frac{d\hat{v}_c}{dt} = \underbrace{\cancelto{0}{-\frac{V_c}{R} - D_0' I_L}}_{DC}$$

$$\underbrace{-\frac{\hat{v}_c}{R} + \hat{d} I_L - D_0'\hat{i}_L}_{\text{1st order ac}} + \underbrace{\hat{d}\,\hat{i}_L}_{\substack{\text{2nd order ac}\\ \to \text{neglect}}}$$

$$C\frac{d\hat{v}_c}{dt} = -\frac{\hat{v}_c}{R} - D_0'\hat{i}_L + \hat{d} I_L$$

$$\boxed{\begin{aligned} L\frac{d\hat{i}_L}{dt} &= D_0\hat{v}_g + D_0'\hat{v}_c + \hat{d}(V_g - V_c)\\ C\frac{d\hat{v}_c}{dt} &= -\frac{\hat{v}_c}{R} - D_0'\hat{i}_L + \hat{d} I_L \end{aligned}}$$

small-signal ac model

– Small signal ac models

1 линеарни

2 стационарни

3 ужасно зависи од параметара кола и радне тачке (одговара десно и лево 2!)

4 величине са ^ нису праве

5 праве се више не виде, одређене се са средњим вредностима

– Summary of the models

- large signal dynamic model
- large signal DC model
- small signal AC model

– Summary of Procedure

1° Изразити $i_L(T_s)$ и $v_C(T_s)$ у зависности од $i_L(0)$, $v_C(0)$ и D

2° Апроксимација извода $\frac{di_L}{dt} \approx \frac{i_L(T_s) - i_L(0)}{T_s}$
$\frac{dv_C}{dt} \approx \frac{v_C(T_s) - v_C(0)}{T_s}$; result: nonlinear state equations, nonlinear dynamic model

3° Perturbation and Linearization of nonlinear state equations – small signal ac model

- Систем једначина је по варијацијама линеаран и стационаран – linear control theory за пројектовање регулатора.

- Пример: преносна функција $\frac{\hat{v}_c(s)}{\hat{d}(s)}$

Лапласова трансформација за small signal ac model

$$sL\hat{i}_L(s) = D_0'\,\hat{v}_C(s) + (V_g - V_C)\,\hat{d}(s) + D_0\,\cancelto{0}{\hat{v}_g(s)}$$

$$sC\,\hat{v}_C(s) = -D_0'\,\hat{i}_L(s) - \hat{v}_C(s)/R + I_L\hat{d}(s)$$

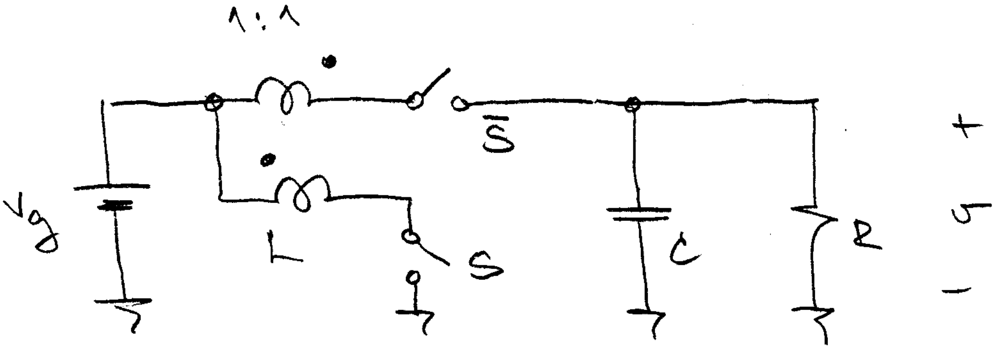
$\hat{v}_g(s) = 0$ – нема варијације улазног напона, линеаран систем, суперпозиција

елиминишемо $\hat{i}_L(s)$, решавамо по $\frac{\hat{v}_c(s)}{\hat{d}(s)}$

$$\boxed{\frac{\hat{v}}{\hat{d}}(s) = -\frac{V_g}{D_0'^2}\,\frac{\left(1 - s\frac{D_0 L}{D_0'^2 R}\right)}{\left(1 + s\frac{L}{D_0'^2 R} + s^2\frac{LC}{D_0'^2}\right)}}$$

Домаћи задатак:

Анализа Watkins-Johnson конвертора



1) DC модел
2) AC (small signal) модел
3) реализација S и $\bar{S}$ (кад и који од њих може бити диода)
4) преносне функције

$\frac{\hat{v}}{\hat{d}}(s)$ $\frac{\hat{v}}{\hat{v}_g}(s)$

The State-Space Averaging Method

- претходни метод – perturbation method
- нема битне разлике, ово је формалније
- генерални метод for ANY converter
- претпоставке
  1) прекидач је "брз", динамика осталих процеса у односу на прекидач је спора
  2) конвертор има два стања (може и за више, извођење за два)
- Position 1:

$$\frac{d\underline{x}}{dt} = A_1\underline{x} + B_1\underline{u}$$

$$\underline{y} = C_1\underline{x} + E_1\underline{u}$$

- Position 2:

$$\frac{d\underline{x}}{dt} = A_2\underline{x} + B_2\underline{u}$$

$$y = C_2\underline{x} + E_2\underline{u}$$

- natural frequencies of converter, regulator, line variations много мање од switching frequency

$$\left.\begin{aligned} \underline{0} &= A\,\underline{X}_0 + B\,\underline{U}_0 \\ \underline{Y}_0 &= C\,\underline{X}_0 + E\,\underline{U}_0 \end{aligned}\right\} \text{ steady state equations}$$

где је

$$A = D_0 A_1 + D_0' A_2$$

$$B = D_0 B_1 + D_0' B_2$$

$$C = D_0 C_1 + D_0' C_2$$

$$E = D_0 E_1 + D_0' E_2$$

$\underline{X}_0$, $\underline{U}_0$, $\underline{Y}_0$, D_0 – у мирној радној тачки

- Ово је нова математичка формулација пертурбационог метода, тиме су две исте резултате
- Small signal ac model

$$\frac{d\hat{x}}{dt} = A\hat{x} + B\hat{u} + \left((A_1 - A_2)\underline{X}_0 + (B_1 - B_2)\underline{U}_0\right)\hat{d}$$

$$\hat{y} = C\hat{x} + E\hat{u} + \left((C_1 - C_2)\underline{X}_0 + (E_1 - E_2)\underline{U}_0\right)\hat{d}$$

- $\hat{x}$, $\hat{y}$, $\hat{u}$, $\hat{d}$ – small signal variations
- усредњење стања, пертурбација, елиминација DC чланова, одбацивање 2nd order terms

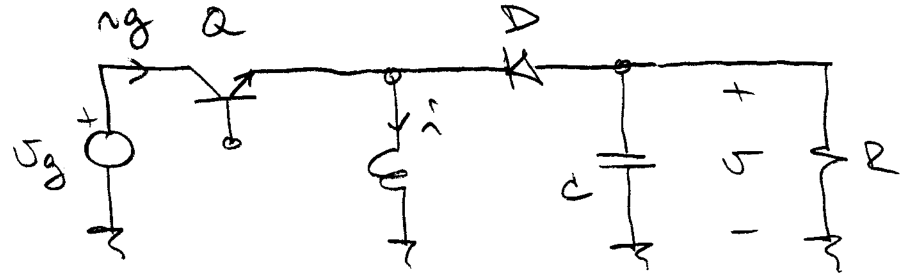
- Изводи се све из међукорака, nonlinear large-signal dynamic model

$$\boxed{\frac{d\underline{x}}{dt} = (DA_1 + D'A_2)\,\underline{x} + (DB_1 + D'B_2)\,\underline{u}}$$

- Вежбање (израда на испиту)

  Користити state-space averaging и извести small-signal AC model за boost конвертор.

- Пример Buck-Boost конвертор са губицима, transistor and diode voltage drops V_T & V_D



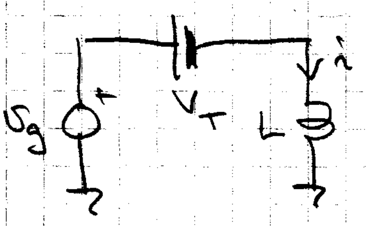
determine both output voltage v and the line current i_g

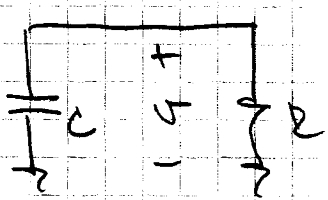
output vector $\underline{y}(t) = \begin{bmatrix} v(t) \\ i_g(t) \end{bmatrix}$

state vector $\underline{x}(t) = \begin{bmatrix} i(t) \\ v(t) \end{bmatrix}$

input vector $u(t) = \begin{bmatrix} v_g(t) \\ V_T \\ V_D \end{bmatrix}$

Q on





$$L\frac{di}{dt} = v_g - V_T \rightarrow \frac{di}{dt} = \frac{v_g}{L} - \frac{V_T}{L}$$

$$C\frac{dv}{dt} = -\frac{v}{R} \rightarrow \frac{dv}{dt} = -\frac{v}{RC}$$

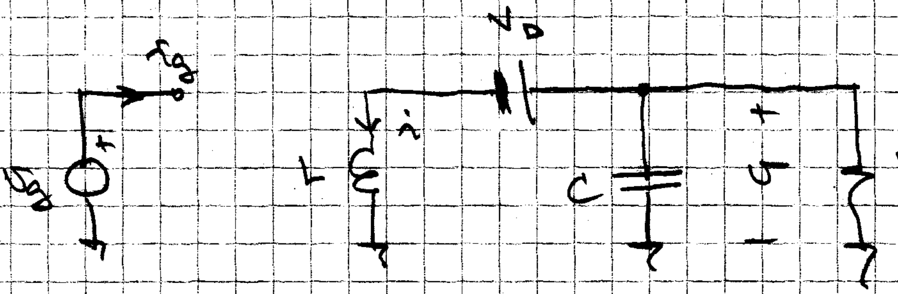
$$i_g = i$$

$$\frac{d\underline{x}}{dt} = \frac{d}{dt}\begin{bmatrix} i \\ v \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}}_{A_1} \underbrace{\begin{bmatrix} i \\ v \end{bmatrix}}_{\underline{x}} +$$

$$\underbrace{\begin{bmatrix} \frac{1}{L} & -\frac{1}{L} & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{B_1} \underbrace{\begin{bmatrix} v_g \\ V_T \\ V_D \end{bmatrix}}_{\underline{u}}$$

$$\underline{y} = \begin{bmatrix} v \\ i_g \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{C_1} \underbrace{\begin{bmatrix} i \\ v \end{bmatrix}}_{\underline{x}} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{E_1} \underbrace{\begin{bmatrix} v_g \\ V_T \\ V_D \end{bmatrix}}_{\underline{u}}$$

Q off



$$L\frac{di}{dt} = v - V_D \qquad \frac{di}{dt} = \frac{v}{L} - \frac{V_D}{L}$$

$$C\frac{dv}{dt} = -\frac{v}{R} - i \qquad \frac{dv}{dt} = -\frac{i}{C} - \frac{v}{RC}$$

$$i_g = 0$$

$$\frac{d}{dt}\begin{bmatrix} i \\ v \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{RC} \end{bmatrix}}_{A_2} \underbrace{\begin{bmatrix} i \\ v \end{bmatrix}}_{\underline{x}} + \underbrace{\begin{bmatrix} 0 & 0 & -\frac{1}{L} \\ 0 & 0 & 0 \end{bmatrix}}_{B_2} \underbrace{\begin{bmatrix} v_g \\ V_T \\ V_D \end{bmatrix}}_{\underline{y}}$$

$$\begin{bmatrix} v \\ i_g \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}}_{C_2} \underbrace{\begin{bmatrix} i \\ v \end{bmatrix}}_{\underline{x}} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{E_2} \underbrace{\begin{bmatrix} v_g \\ V_T \\ V_D \end{bmatrix}}_{\underline{y}}$$

Сада само применимо готове формуле за усредњене моделе

DC Model
$$\underline{0} = A\underline{X}_0 + B\underline{u}_0$$
$$\underline{y}_0 = C\underline{X}_0 + E\underline{u}_0$$

AC Model
$$\frac{d\hat{\underline{x}}}{dt} = A\hat{\underline{x}} + B\hat{\underline{u}} + \left((A_1-A_2)\underline{X}_0 + (B_1-B_2)\underline{u}_0\right)\hat{d}$$
$$\hat{y} = C\hat{\underline{x}} + E\hat{u} + \left((C_1-C_2)\underline{X}_0 + (E_1-E_2)\underline{u}_0\right)\hat{d}$$

$$A = D_0 A_1 + D_0' A_2$$
$$B = D_0 B_1 + D_0' B_2$$
$$C = D_0 C_1 + D_0' C_2$$
$$E = D_0 E_1 + D_0' E_2$$

Директном [illegible]:

$$A = \begin{bmatrix} 0 & \frac{D_0'}{L} \\ -\frac{D_0'}{C} & -\frac{1}{RC} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{D_0}{L} & -\frac{D_0}{L} & -\frac{D_0'}{L} \\ 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 \\ D_0 & 0 \end{bmatrix}$$

$$A_1 - A_2 = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix}$$

$$B_1 - B_2 = \begin{bmatrix} \frac{1}{L} & -\frac{1}{L} & \frac{1}{L} \\ 0 & 0 & 0 \end{bmatrix}$$

$$C_1 - C_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$E_1 - E_2 = \underline{0}$$

$$(A_1 - A_2)\,\underline{x}_0 + (B_1 - B_2)\,\underline{u}_0 =$$

$$= \begin{bmatrix} -\frac{V_0}{L} \\ \frac{I_0}{C} \end{bmatrix} + \begin{bmatrix} \frac{V_g - V_T + V_D}{L} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{V_g - V_0 - V_T + V_D}{L} \\ \frac{I_0}{C} \end{bmatrix}$$

$$(C_1 - C_2)\,\underline{x}_0 + (E_1 - E_2)\,\underline{u}_0 = \begin{bmatrix} 0 \\ I_0 \end{bmatrix}$$

Модели:

<u>dc model</u>

$$\underline{0} = A\,\underline{x}_0 + B\,\underline{u}_0$$

$$\underline{y}_0 = C\,\underline{x}_0 + E\,\underline{u}_0$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{D_0'}{L} \\ -\frac{D_0'}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} I_0 \\ V_0 \end{bmatrix} + \begin{bmatrix} \frac{D_0}{L} & -\frac{D_0}{L} & -\frac{D_0'}{L} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_g \\ V_T \\ V_D \end{bmatrix}$$

$$\underline{y}_0 = \begin{bmatrix} V_0 \\ I_g \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ D_0 & 0 \end{bmatrix} \begin{bmatrix} I_0 \\ V_0 \end{bmatrix} + \underline{0}$$

rearrange

$$0 = \frac{D_o'}{L} V_o + \frac{D_o}{L} V_g - \frac{D_o}{L} V_T - \frac{D_o'}{L} V_D$$

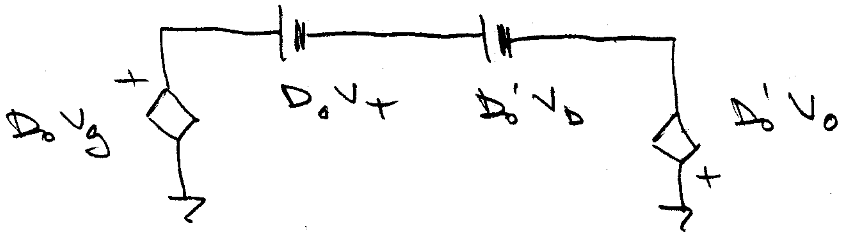
$$0 = D_o' V_o + D_o V_g - D_o V_T - D_o' V_D \qquad (1)$$

$$0 = -\frac{D_o'}{C} I_o - \frac{V_o}{RC}$$

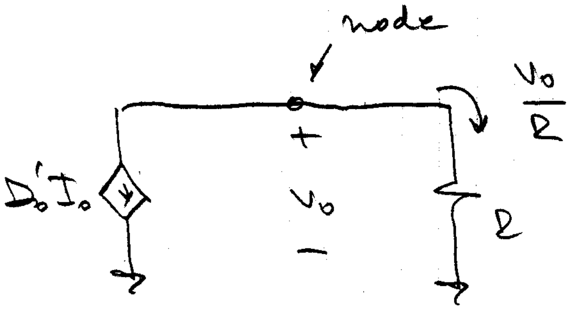
$$0 = D_o' I_o + \frac{V_o}{R} \qquad (2)$$

$$I_g = D_o I_o \qquad (3)$$

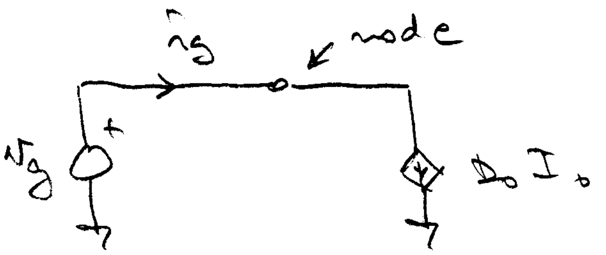
Eq. (1): Loop equation (&3H)



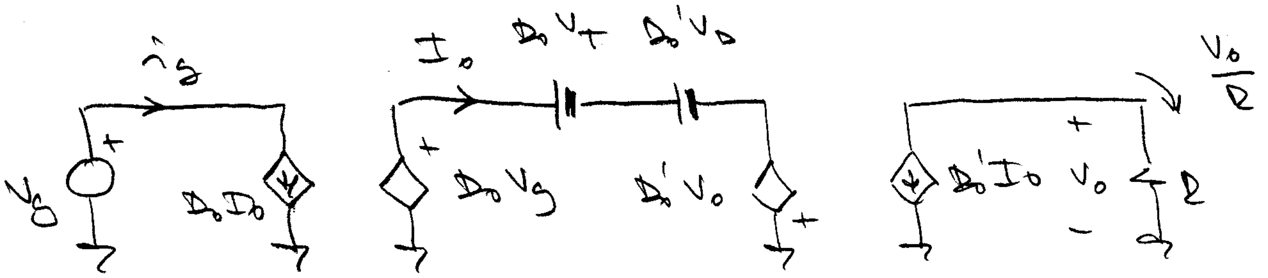
Eq. (2): Node equation (&3C)

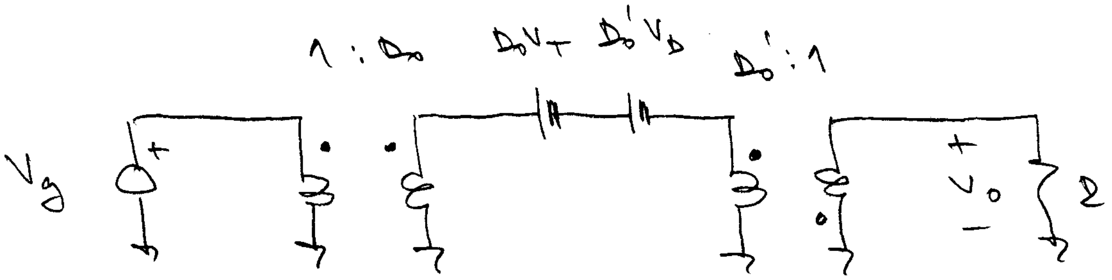


Eq. (3) : Node equation (KCL)



Combine all three circuits together





DC transformers

DC model for the Buck-Boost converter

AC Model

$$\frac{d\hat{x}}{dt} = A\hat{x} + B\hat{u} + ((A_1 - A_2)\underline{x}_o + (B_1 - B_2)\underline{u}_o)\hat{d}$$

$$\frac{d}{dt}\begin{bmatrix}\hat{i}\\ \hat{v}\end{bmatrix}=\begin{bmatrix}0 & \frac{D_0'}{L}\\ -\frac{D_0'}{C} & -\frac{1}{RC}\end{bmatrix}\begin{bmatrix}\hat{i}\\ \hat{v}\end{bmatrix}+$$

$$+\begin{bmatrix}\frac{D_0}{L} & -\frac{D_0}{L} & -\frac{D_0'}{L}\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\hat{v}_g\\ \cancel{\hat{v}_T}\ 0\\ \cancel{\hat{v}_D}\ 0\end{bmatrix}+\begin{bmatrix}\frac{V_g-V_0-V_T+V_D}{L}\\ \frac{I_0}{C}\end{bmatrix}\hat{d}$$

circuit realization

$$L\frac{d\hat{i}}{dt}=D_0'\hat{v}+D_0\hat{v}_g+(V_g-V_0-V_T+V_D)\hat{d} \qquad (4)$$

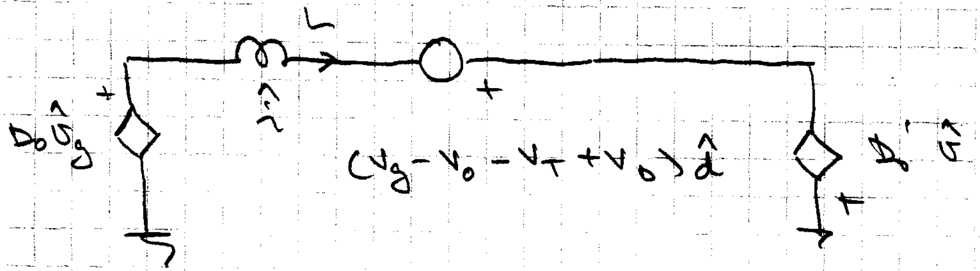
$$C\frac{d\hat{v}}{dt}=-D_0'\hat{i}-\frac{\hat{v}}{R}+I_0\hat{d} \qquad (5)$$

output relations

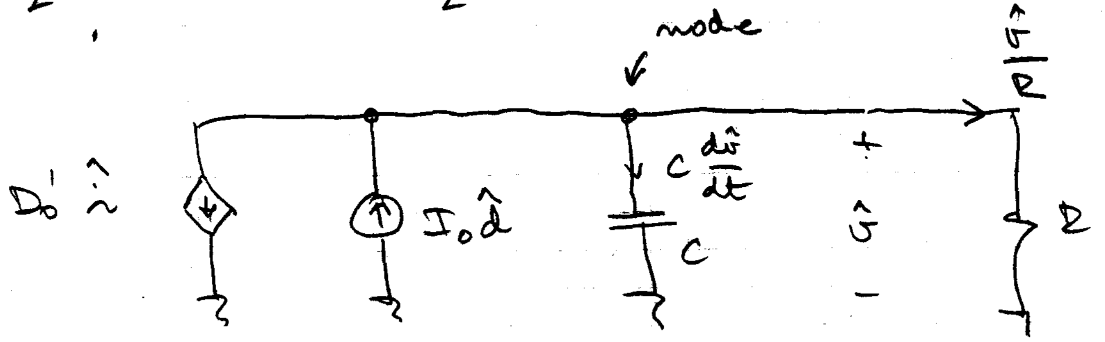
$$\begin{bmatrix}\hat{v}\\ \hat{i}_g\end{bmatrix}=\begin{bmatrix}0 & 1\\ D_0 & 0\end{bmatrix}\begin{bmatrix}\hat{i}\\ \hat{v}\end{bmatrix}+\begin{bmatrix}0\\ I_0\end{bmatrix}\hat{d}$$

$$\hat{i}_g=D_0\hat{i}+I_0\hat{d} \qquad (6)$$

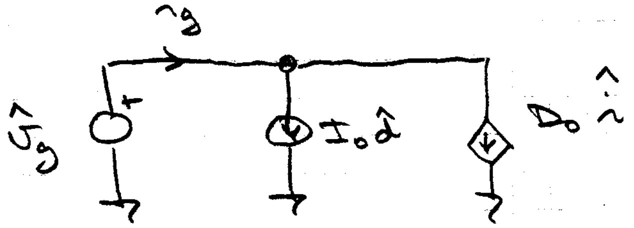
Eq. (4): loop equation



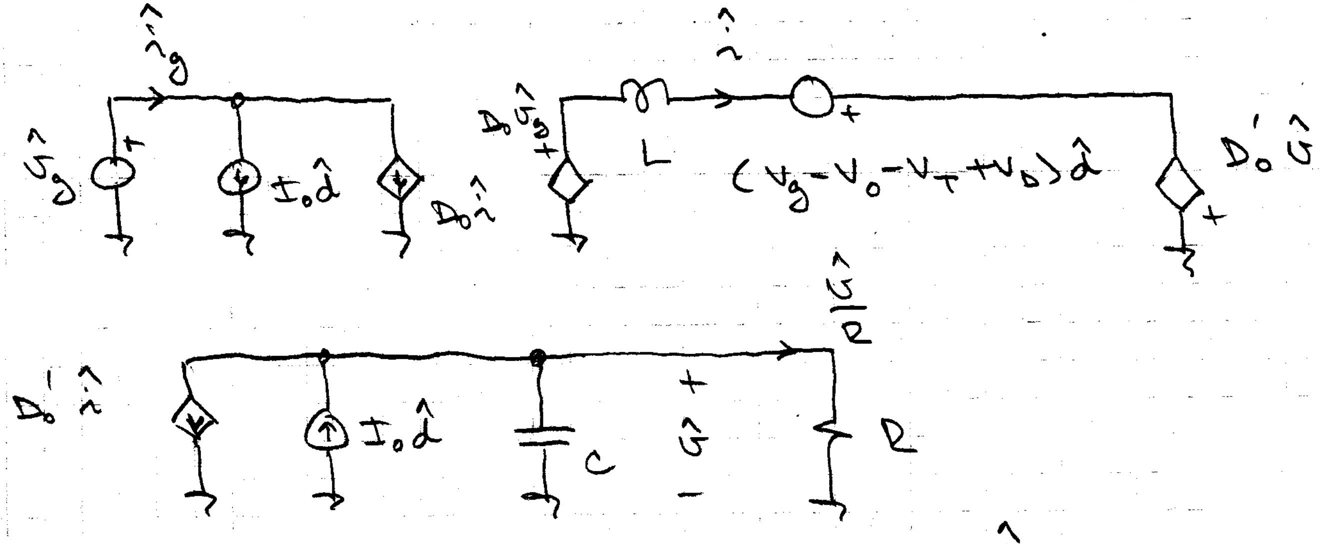
Eq. (5): Node equation

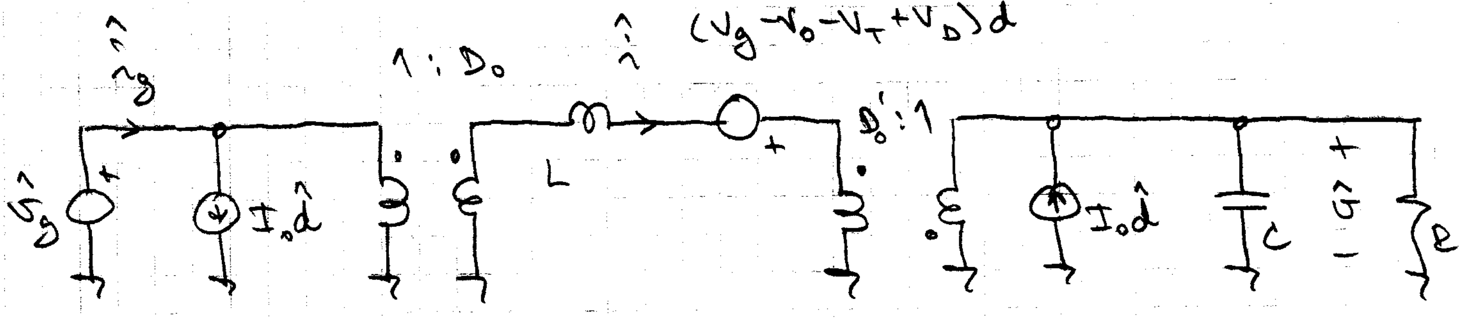


Eq. (6): Node equation



Combination of all three ac circuit yields





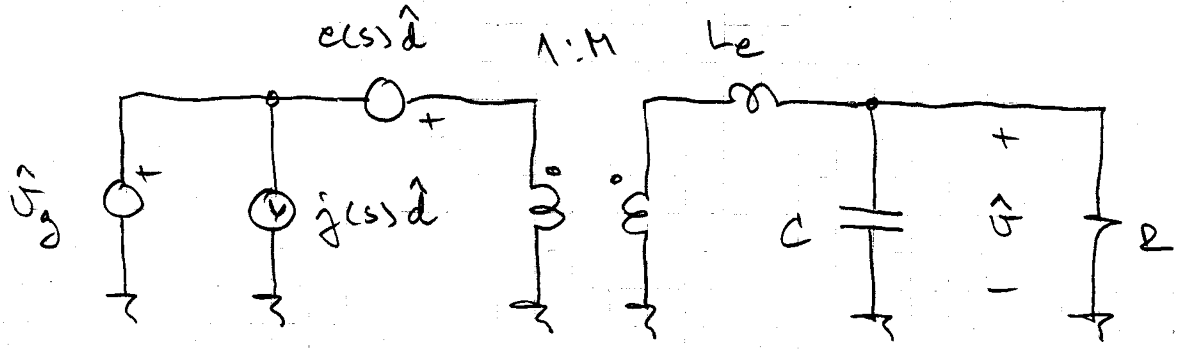
↑ complete ac model

Канонички модел

- општа форма еквивалентног кола за ма који конвертор
- Сви конвертори имају основне функције:

1) Трансформација напонских и струјних нивоа, идеално са $\eta = 100\%$
2) Нискофреквентно филтрирање
3) Контрола излаза варирањем D

Стога је и слично еквивалентно коло



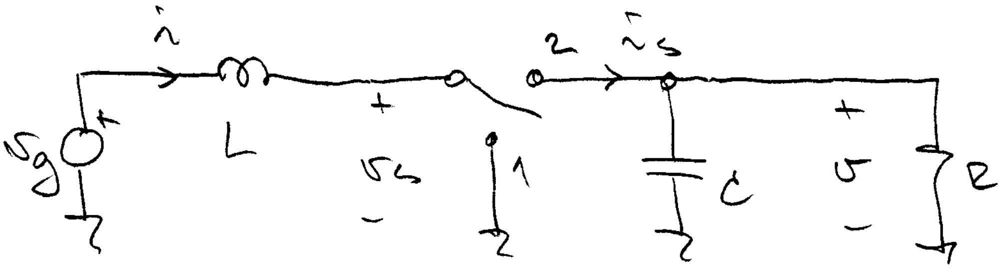
For some common ideal converters

converter	$M(D_0)$	L_e	$e(s)$	$j(s)$
Buck	D_0	L	V_0/D_0^2	V_0/R
Boost	$1/D_0'$	$L/D_0'^2$	$V_0(1-s\frac{L_e}{R})$	$\frac{V_0}{D_0'^2 R}$
Buck-Boost	$-D_0/D_0'$	$L/D_0'^2$	$-\frac{V_0}{D_0^2}(1-s\frac{D_0 L_e}{R})$	$-\frac{V_0}{D_0'^2 R}$

Circuit Averaging

- иста од state-space averaging
- more physical insight
- користи се за конструисање исправљаче и квазирезонантне конверторе

- Пример: Boost converter



Position 1: $v_s = 0$

$i_s = 0$

position 2: $v_s = v$

$i_s = i$

- Averaging:

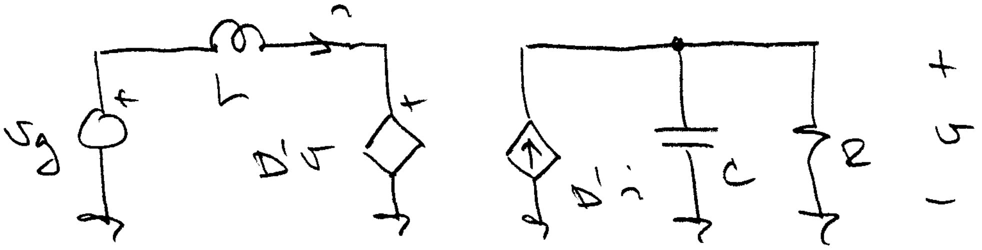
$$\bar{v}_s = \frac{1}{T_s}(0 \cdot DT_s + v \cdot D'T_s)$$

$$\bar{v}_s = D' v$$

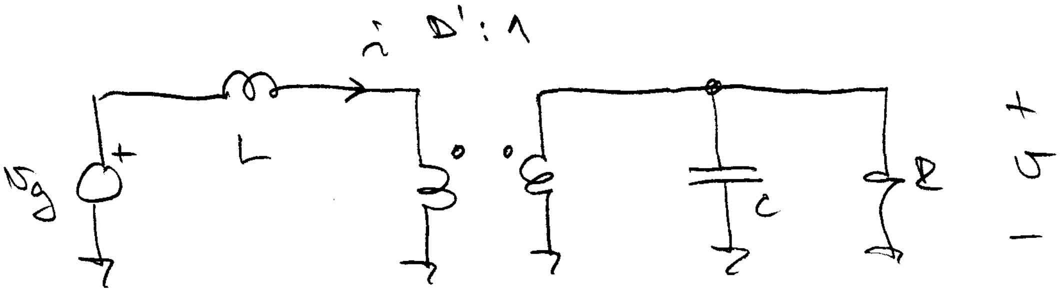
$$\bar{i}_s = \frac{1}{T_s}(0 \cdot DT_s + i\, D'T_s)$$

$$\bar{i}_s = D' i$$

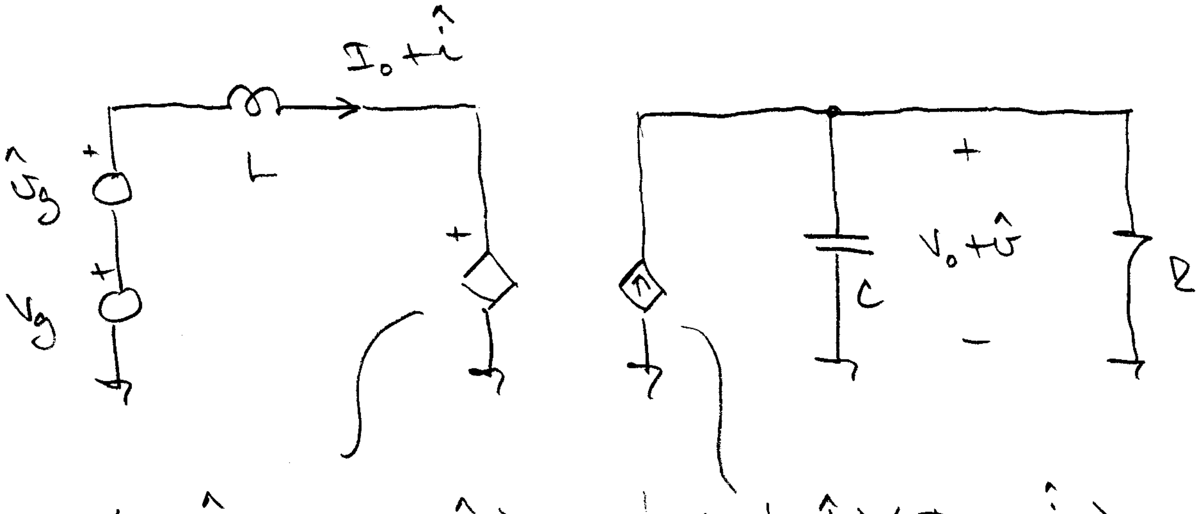
- Averaged circuit



- Примена идеалног трансформатора



- Пертурбација и линеаризација



$(D_0' - \hat{d})(V_0 + \hat{v}) =$

$= D_0' V_0 +$

$+ D_0' \hat{v} - V_0 \hat{d} -$

$- \cancelto{0}{\hat{d}\hat{v}} =$

$= D_0' (V_0 + \hat{v}) -$

$- V_0 \hat{d}$

$(D_0' - \hat{d})(I_0 + \hat{i}) =$

$= D_0' I_0 -$

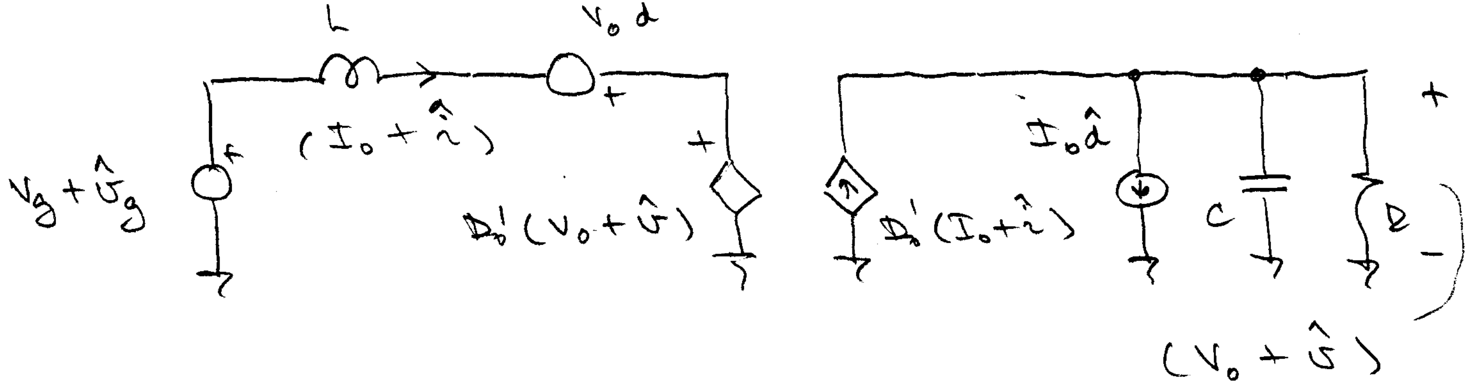
$- \hat{d} I_0 + D_0' \hat{i} -$

$- \cancelto{0}{\hat{d}\hat{i}} =$

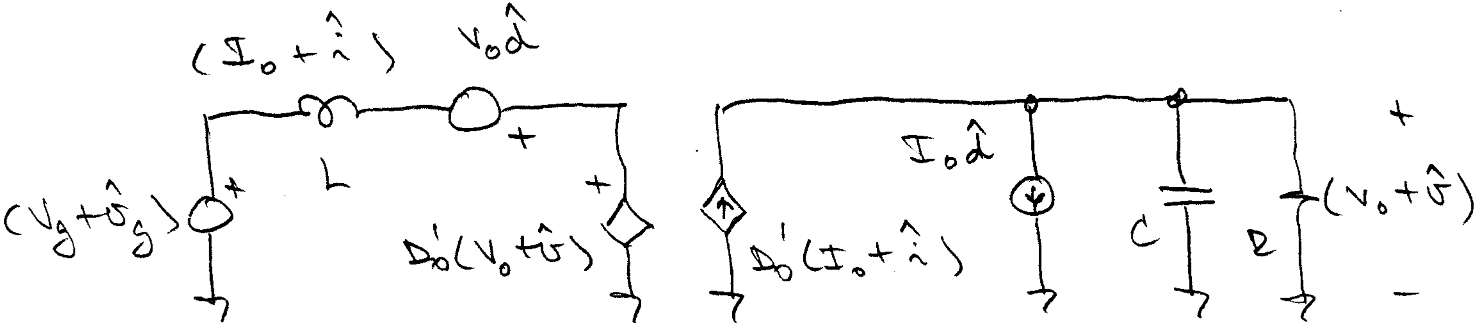
$= D_0' (I_0 + \hat{i}) -$

$- \hat{d} I_0$

- физичка интерпретација



Смештам у трансформатору



(ово је complete averaged & linearized model, canonical form

- Вежба: урадити исто за buck и buck-boost, check на canonical form using circuit averaging

Summary of the circuit averaging procedure

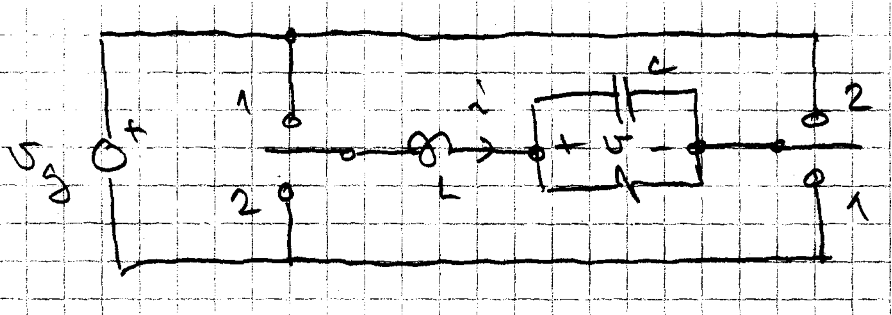
1. Replace switching elements with equivalent current and voltage sources, дефинисани тако да се у колу ништа не догоди.

2. Усредњити све voltages and currents over the switching period

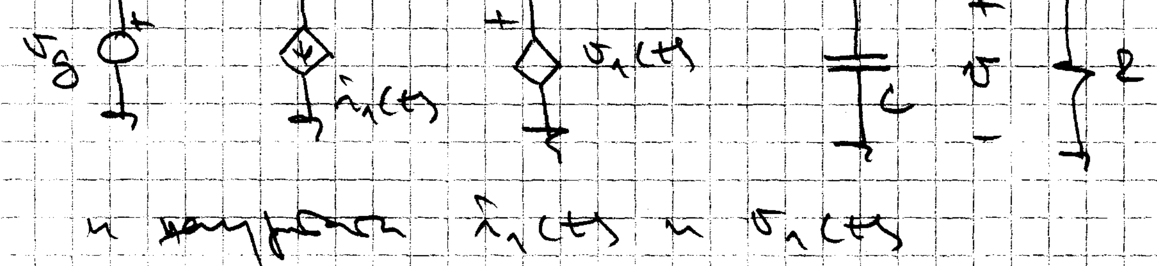
3. Пертурбација и линеаризација, груписање чланова

4. Манипулација да кола добију леп облик

Вежба: Bridge Inverter EXTRA CREDIT



a) Показати да се конвертор може приказати као



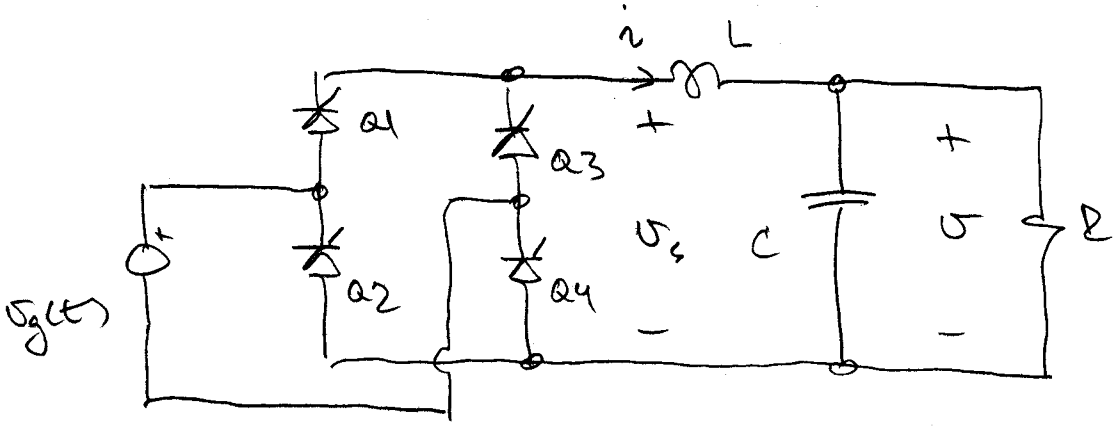
и одредити $i_1(t)$ и $v_1(t)$

б) Извести large-signal circuit-averaged (нелинеарни усредњени) модел овог конвертора

в) Пертурбација и линеаризација, еквивалентни линеаризовани модел

Домаћи задатак:

Small-signal relations in a controlled bridge rectifier



a) small-signal transfer functions

$\frac{\hat{v}}{\hat{\alpha}}(s)$, $\frac{\hat{v}}{\hat{v}_g}(s)$.

б) steady-state relation

$V_0 = f(V_{g0}, A)$

в) equivalent circuit

Type I^c

$$i = I + \hat{i}(t)$$

$$v = V_0 + \hat{v}(t)$$

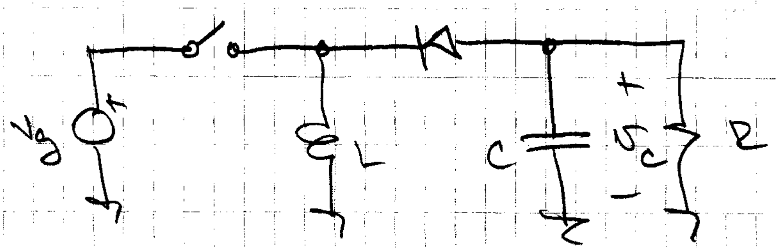
$$v_g = (V_{g0} + \hat{v}_g(t)) \sin \omega t$$

- Switching frequency $\frac{\omega}{2\pi}$ non $\frac{\omega}{4\pi}$

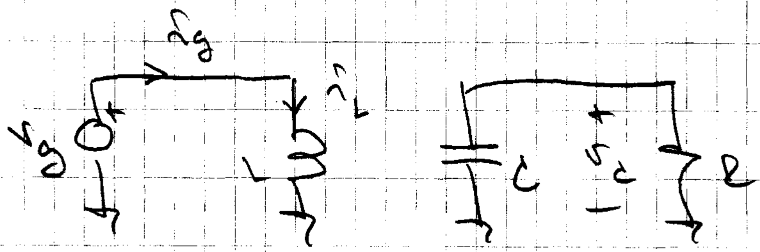
Проширење усредњавања у простору стања на моделовање конвертора у дисконтинуалном моду

- Linear ripple approximation струје калема се не може применити, као код DC струја

- Buck - Boost example



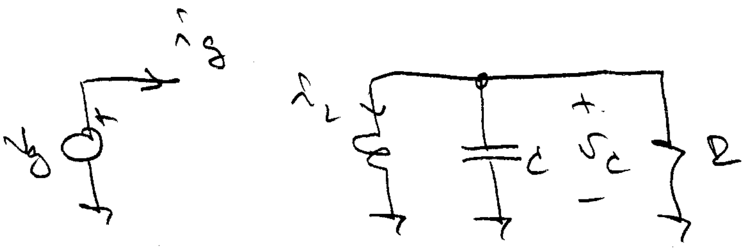
1st Interval, D_1T_s



$$L\frac{di_L}{dt} = v_g \qquad C\frac{dv_c}{dt} = -\frac{v_c}{R}$$

$$i_g = i_L \qquad (1)$$

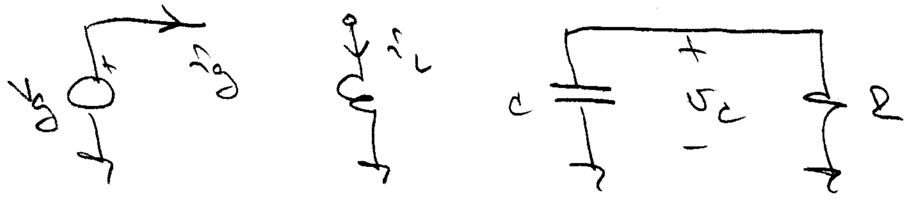
2nd interval D_2T_s



$$L\frac{di_L}{dt}=v_c \qquad C\frac{dv_c}{dt}=-i_L-\frac{v_c}{R} \qquad (2)$$

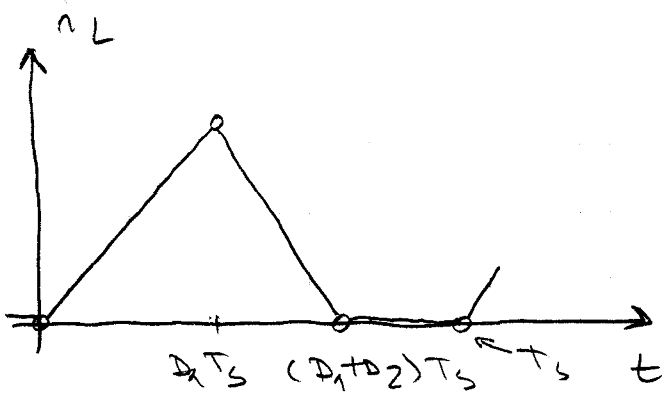
$$i_g=0$$

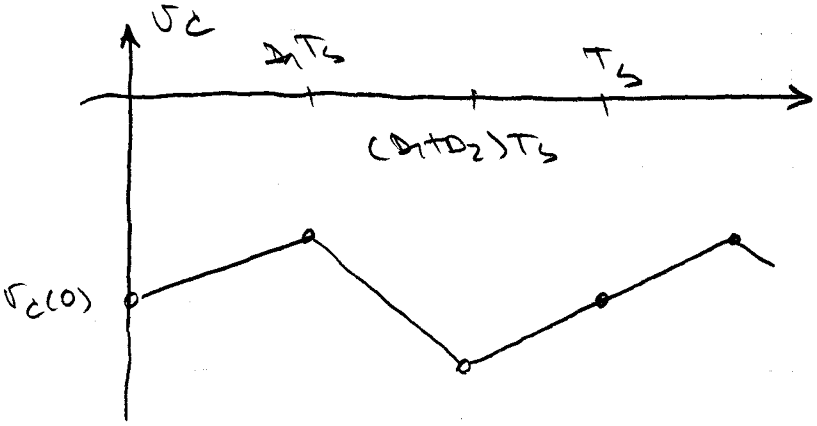
3rd interval D_3T_s



$$L\frac{di_L}{dt}=0 \;,\; i_L=0 \qquad C\frac{dv_c}{dt}=-\frac{v_c}{R} \qquad (3)$$

$$i_g=0$$





1st interval

$$\hat{i}_L(D_1T_s) = \hat{i}_{peak} = \underbrace{\frac{V_g}{L}}_{\text{slope}}\underbrace{D_1T_s}_{\text{time}} \qquad (\hat{i}_L(0)=0) \qquad (4)$$

$$\hat{i}_g = \hat{i}_L$$

$$v_c(D_1T_s) = v_c(0) - \frac{v_c}{RC}D_1T_s$$

2nd interval

$$\hat{i}_L((D_1+D_2)T_s) = \hat{i}_L(D_1T_s) + \frac{v_c}{L}D_2T_s$$

$$\hat{i}_g = 0 \qquad (5)$$

$$v_c((D_1+D_2)T_s) = v_c(D_1T_s) - \underbrace{\left(\frac{\hat{i}_L}{C} + \frac{v_c}{RC}\right)}_{\text{slope}}D_2T_s$$

Напомена, $\hat{i}_L$ није константно током D_2T_s, треба узети средњу вредност

$$v_C((D_1+D_2)T_S) \cong v_C(D_1T_S) -$$

$$- \frac{\bar{i}_D\big|_{D_2T_S} + v_C/R}{C} D_2T_S \cong$$

$$\cong v_C(D_1T_S) - \frac{\frac{1}{2}i_{peak} + \frac{v_C}{R}}{C} D_2T_S \qquad (6)$$

$i_L((D_1+D_2)T_S) = 0$ — 2nd interval calculation

$$0 = i_L(D_1T_S) + \frac{v_C}{L} D_2T_S$$

$$0 = \frac{V_g}{L} D_1T_S + \frac{v_C}{L} D_2T_S$$

$$0 = D_1 V_g + D_2 v_C \qquad (8)$$

$$\frac{v_C}{V_g} = - \frac{D_1}{D_2} \qquad (9)$$

3rd interval

$$\begin{aligned} i_L &= 0 \\ i_g &= 0 \\ v_C(T_S) &= v_C((D_1+D_2)T_S) - \frac{v_C}{RC} D_3T_S \end{aligned} \qquad (10)$$

Dynamic Equation for output (capacitor) voltage

Комбиновањем Eq. (4), (6) и (10) да би се добило $v_C(T_s)$ у функцији $v_C(0)$ даје

$$v_C(T_s) = v_C(0) - \frac{v_C}{RC} T_s - \frac{\frac{1}{2}\hat{i}_{peak}}{C} D_2 T_s \qquad (11)$$

Апроксимација извода:

$$\frac{dv_C}{dt} \cong \frac{\Delta v_C}{\Delta t} = \frac{v_C(T_s) - v_C(0)}{T_s}$$

From (11)

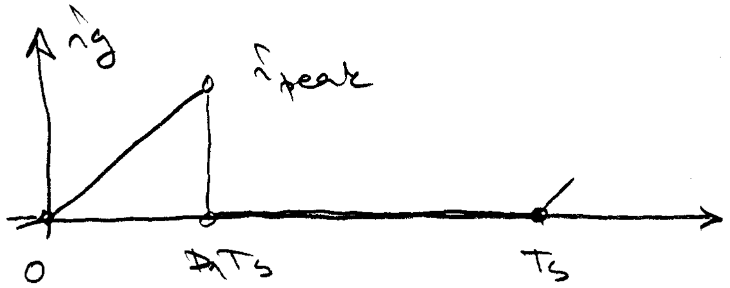
$$\frac{v_C(T_s) - v_C(0)}{T_s} = -\frac{v_C}{RC} - \frac{D_2 \hat{i}_{peak}}{2C} . \qquad (12)$$

$$\hat{i}_{peak} = \frac{V_g}{L} D_1 T_s$$

$$C\frac{dv_C}{dt} = -\frac{v_C}{R} - \frac{D_1 D_2 V_g T_s}{2L} \qquad (13)$$

Equation for the average input current $\bar{i}_g$:

(посебно само у континуалном моделу, input & output properties)



$$\bar{i}_g = \frac{1}{T_s}\int_0^{T_s} i_g(\tau)\,d\tau = \frac{1}{T_s}\left(\frac{1}{2} i_{peak}\right) D_1 T_s =$$

$$= \frac{D_1^2 V_g T_s}{2L} \qquad (14)$$

Имамо :

(15) $D_1 V_g + D_2 v_C = 0$ из Eq. (8)
expression for D_2

(16) $C\dfrac{dv_C}{dt} = -\dfrac{v_C}{R} - \dfrac{D_1 D_2 V_g T_s}{2L}$ из Eq. (13)
charge balance

(17) $\bar{i}_g = \dfrac{D_1^2 V_g T_s}{2L}$ из Eq. (14)
average input current

Ово је нелинеарни систем пошто D_1, D_2, V_g и v_c стоје међусобно измножени у разним комбинацијама

Note: нема expression за $\frac{di_L}{dt}$. $\frac{di_L}{dt} = 0$, пошто $i_L(0) = i_L(T_s) = 0$

Резултат: Преносна карактеристика конвертора има само један пол, зависан од C

Small - Signal AC Equations

- perturb & linearize

$$D_1(t) = D_{10} + \hat{d}_1(t)$$

$$D_2(t) = D_{20} + \hat{d}_2(t)$$

$$v_g(t) = V_{g0} + \hat{v}_g(t)$$

$$v_c(t) = V_{c0} + \hat{v}_c(t)$$

$$\bar{i}_g(t) = I_{g0} + \hat{i}_g(t)$$

Eq. (15)

$$(D_{10} + \hat{d}_1)(V_{g0} + \hat{v}_g) + (D_{20} + \hat{d}_2)(V_{c0} + \hat{v}_c) = 0$$

$$0 = \underbrace{(D_{10} V_{g0} + D_{20} V_{c0})}_{\text{DC terms}} +$$

$$+ \underbrace{D_{10} \hat{v}_g + V_{g0} \hat{d}_1 + D_{20} \hat{v}_c + V_{c0} \hat{d}_2}_{\text{linear ac}} +$$

$$+ \underbrace{\hat{d}_1 \hat{v}_g + \hat{d}_2 \hat{v}_c}_{\text{nonlinear}}$$

Eq. (16) becomes

$$C \frac{d}{dt} \left(\cancelto{0}{V_{c0}} + \hat{v}_c \right) = - \frac{V_{c0} + \hat{v}_c}{R} - \frac{(D_{10} + \hat{d}_1)(D_{20} + \hat{d}_2)(V_{g0} + \hat{v}_g) T_s}{2L}$$

$$C \frac{d\hat{v}_c}{dt} = \underbrace{- \frac{V_{c0}}{R} - \frac{D_{10} D_{20} V_{g0} T_s}{2L}}_{\text{dc}} -$$

$$\underbrace{- \frac{D_{10} D_{20} \hat{v}_g T_s}{2L} - \frac{D_{10} \hat{d}_2 V_{g0} T_s}{2L} - \frac{\hat{v}_c}{R}}_{\text{linear ac}} -$$

$$\underbrace{- \frac{\hat{d}_1 D_{20} V_{g0} T_s}{2L}}_{\text{linear ac}} - \text{nonlinear terms}$$

DC terms

$$0 = D_{10} V_{g0} + D_{20} V_{c0}$$ expression for D_{20}

$$0 = -\frac{V_{c0}}{R} - \frac{D_{10} D_{20} V_{g0} T_s}{2L}$$ charge balance

$$I_{g0} = \frac{D_{10}^2 V_{g0} T_s}{2L}$$ input current

3 equations, 3 unknowns (D_{20}, V_{c0}, I_{g0})

Solution:

$$D_{20} = \sqrt{K}$$ where $K = \frac{2L}{RT_s}$

$$V_{c0} = -\frac{V_{g0} D_{10}}{\sqrt{K}}$$

$$I_{g0} = \frac{D_{10}^2}{K} \frac{V_{g0}}{R}$$

Small-Signal AC Terms

$$0 = D_{10}\hat{v}_g + V_{go}\hat{d}_1 + D_{20}\hat{v}_c + V_{co}\hat{d}_2$$ (exp. for $\hat{d}_2$)

$$C\frac{d\hat{v}_c}{dt} = -\frac{\hat{v}_c}{R} - \frac{T_s}{2L}\left(D_{10}D_{20}\hat{v}_g + D_{10}V_{go}\hat{d}_2 + D_{20}V_{go}\hat{d}_1\right)$$ (capacitor charge)

$$\hat{i}_g = \frac{D_{10}^2 T_s}{2L}\hat{v}_g + \frac{D_{10}V_{go}T_s}{L}\hat{d}_1 =$$

$$= \frac{D_{10}^2}{RK}\hat{v}_g + \frac{2D_{10}V_{go}}{RK}\hat{d}_1$$ (input current)

$$\hat{d}_2 = -\frac{D_{10}\hat{v}_g + V_{go}\hat{d}_1 + D_{20}\hat{v}_c}{V_{co}}$$

→ yields: commutation
$\hat{d}_2$ ↔ [illegible] $\hat{j}$-the

$$C\frac{d\hat{v}_c}{dt} = -\frac{\hat{v}_c}{R} - \frac{T_s}{2L}\left(\hat{v}_g\left(D_{10}D_{20} - D_{10}^2\frac{V_{go}}{V_{co}}\right) + \right.$$

$$\left. + \hat{d}_1\left(D_{20}V_{go} - D_{10}\frac{V_{go}^2}{V_{co}}\right) - \hat{v}_c D_{10}D_{20}\frac{V_{go}}{V_{co}}\right)$$

$$C\frac{d\hat{v}_c}{dt} = -\hat{v}_c\left(\frac{1}{R} + \frac{1}{R}\right) + \hat{v}_g\frac{2D_{10}}{R\sqrt{K}} + \hat{d}_1\frac{2V_{go}}{R\sqrt{K}}$$

↑

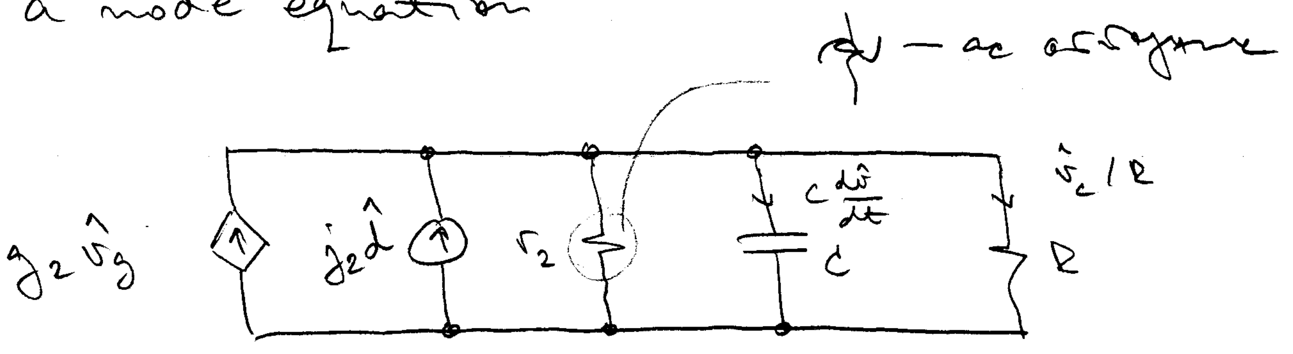
[illegible]
damps

Equivalent circuit model

Output relation

$$C\frac{d\hat{v}_c}{dt} = -\hat{v}_c\left(\frac{1}{R}+\frac{1}{R}\right) + \hat{v}_g\,\frac{2D_{10}}{R\sqrt{K}} + \hat{d}_1\,\frac{2V_{g0}}{R\sqrt{K}}$$

a mode equation

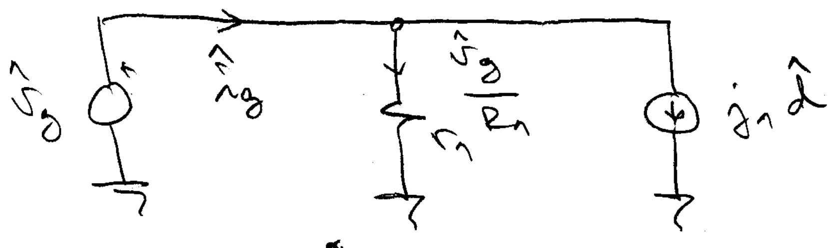


$$g_2 = \frac{2D_{10}}{R\sqrt{K}}, \quad j_2 = \frac{2V_{g0}}{R\sqrt{K}}, \quad r_2 = R$$

[illegible] = incremental ac resistance

Input relation

$$\hat{i}_g = \frac{D_{10}^2}{RK}\hat{v}_g + \frac{2D_{10}V_{g0}}{RK}\hat{d}_1$$



also ac resistor

$$r_1 = R\frac{K}{D_{10}^2}$$

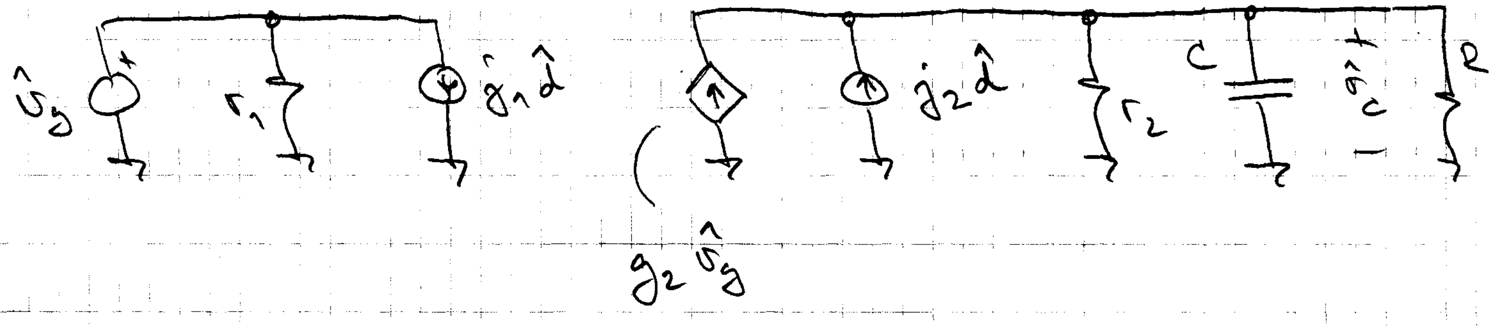
$$g_1 = \frac{V_{g0}}{R}\,\frac{2D_{10}}{K}$$

r_1 & r_2 are effective ac resistances only;

they do not enter into the dc model, nor they

represent power loss

Complete ac small-signal model for
buck-boost in discontinuous mode



r_1 & r_2 are ac resistors

Summary of the Analysis Technique

1 Must determine $v(T_s)$ in terms of $v(0)$, then approximate
$$\frac{dv}{dt} = \frac{v(T_s) - v(0)}{T_s}$$

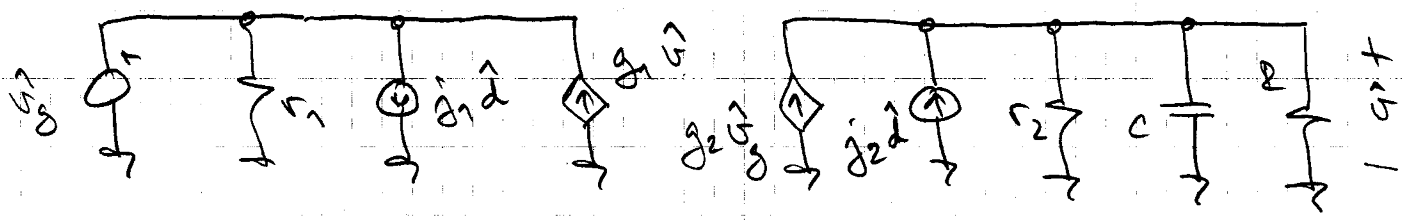
2 Cannot approximate $i(t)$ as nearly constant, since inductor current ripple is not small. Instead, use average value of i.

3 An additional equation is needed to determine the length of the second interval D_2T_s: The second interval ends when $i((D_1+D_2)T_s) = 0$.

Result: Nonlinear Diff. Eq. which describe $\frac{dv}{dt}$. No equation for $\frac{di}{dt}$, since $i(T_s) = i(0) = 0 \Rightarrow \frac{di}{dt} = 0$, and the inductor does not contribute a pole to the converter dynamics.

- One can construct small-signal ac model by perturbation and linearization, as usual

Canonical Circuit Model - DCM

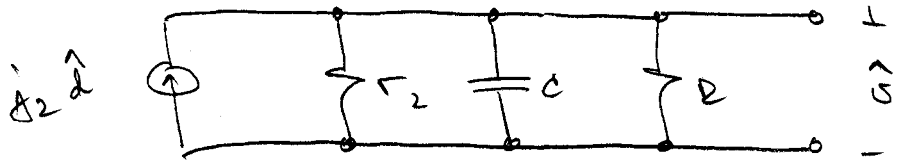


Converter	r_1	j_1	g_1	g_2	j_2	r_2
buck	$\frac{1-M}{M^2}R$	$\frac{2V_0}{R}\sqrt{\frac{1-M^2}{K}}$	$\frac{M^2}{1-M}\ \frac{1}{R}$	$\frac{M(2-M)}{(1-M)R}$	$\frac{2V_0}{RM}\sqrt{\frac{1-M}{K}}$	$(1-M)R$
boost	$\frac{M-1}{M^2}R$	$\frac{2V_0}{R}\sqrt{\frac{M}{K(M-1)}}$	$\frac{M}{M-1}\ \frac{1}{R}$	$\frac{M(2M-1)}{(M-1)R}$	$\frac{2V_0}{R\sqrt{KM(M-1)}}$	$\frac{M-1}{M}R$
buck- -boost	$\frac{R}{M^2}$	$\frac{2\lvert V_0\rvert}{R\sqrt{K}}$	0	$\frac{2M}{R}$	$\frac{2\lvert V_0\rvert}{R\sqrt{K}M}$	R

$$M = \frac{V_0}{V_g} = \begin{cases} \dfrac{2}{1+\sqrt{1+4K/D_1^2}} & \text{buck} \\[2ex] \dfrac{1+\sqrt{1+4D_1^2/K}}{2} & \text{boost} \\[2ex] -\dfrac{D_1}{\sqrt{K}} & \text{buck-boost} \end{cases}$$

Small-signal transfer functions are easily found from the model:

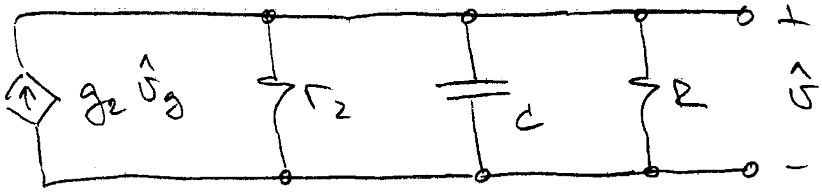
а) $\frac{\hat{v}}{\hat{d}}$: $\hat{v}_g = 0$



$$\hat{v} = j_2 \hat{d} \left(r_2 \parallel \frac{1}{sC} \parallel R \right)$$

$$\frac{\hat{v}}{\hat{d}} = G_{do} \frac{1}{1 + \frac{s}{\omega_p}}$$ – само један пол

б) $\frac{\hat{v}}{\hat{v}_g}$: set $\hat{d} = 0$



$$\hat{v} = g_2 \hat{v}_g \left(r_2 \parallel \frac{1}{sC} \parallel R \right)$$

$$\frac{\hat{v}}{\hat{v}_g} = \frac{G_{go}}{1 + \frac{s}{\omega_p}}$$

6) Z_{out} : $\hat{v}_g = 0$, $\hat{d} = 0$

$$Z_{out} = r_2 \parallel \frac{1}{sC} \parallel R$$

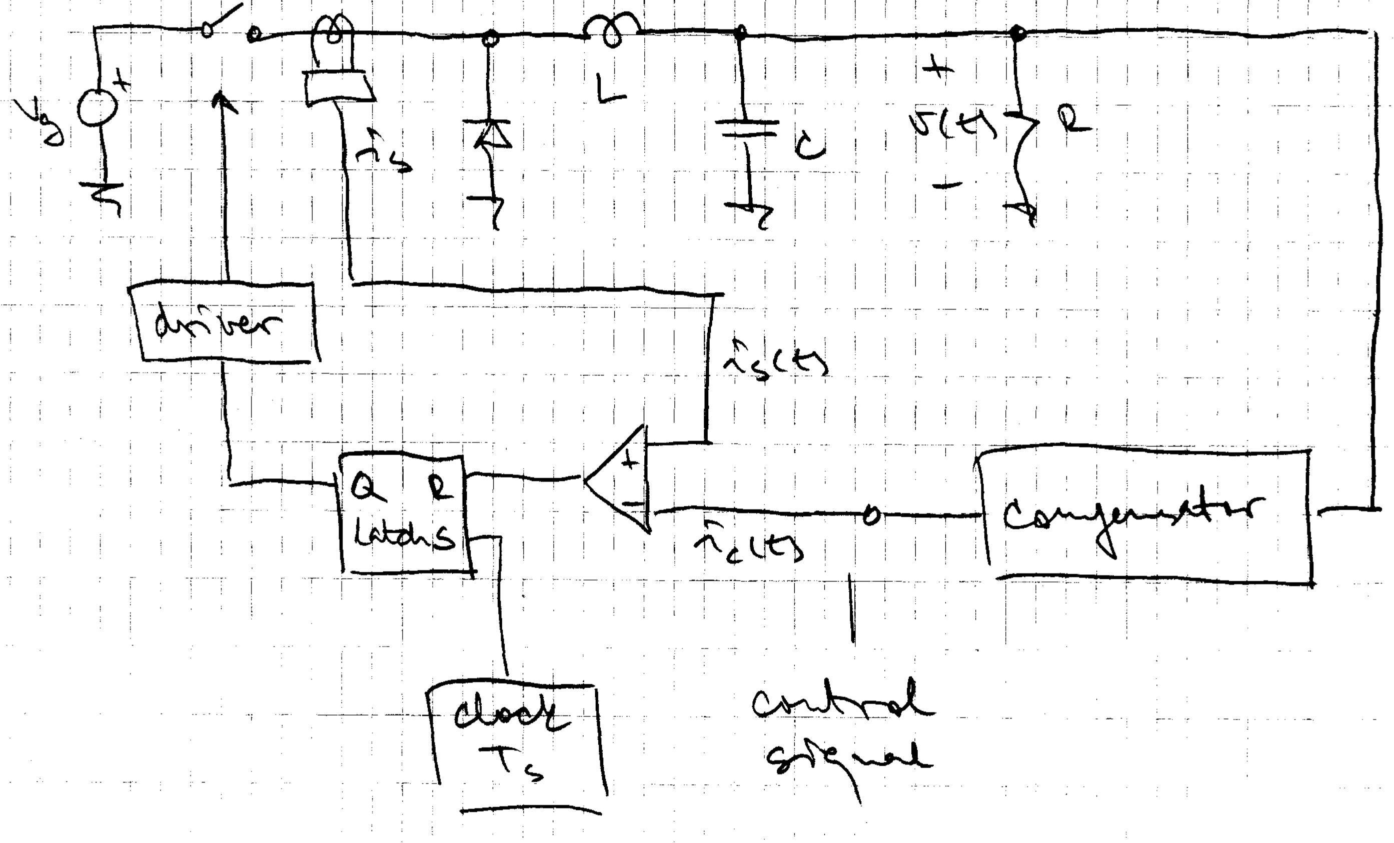
*

Технике струјног програмирања

- до сада - duty ratio programmed mode
  $v(t)$ је контролисано са $D(t)$

- у задње време (10 година) "current-programmed" mode, излаз је контролисан избором врхне вредности струје транзистора (peak transistor current) peak ($i_s(t) \cong i_c(t)$).

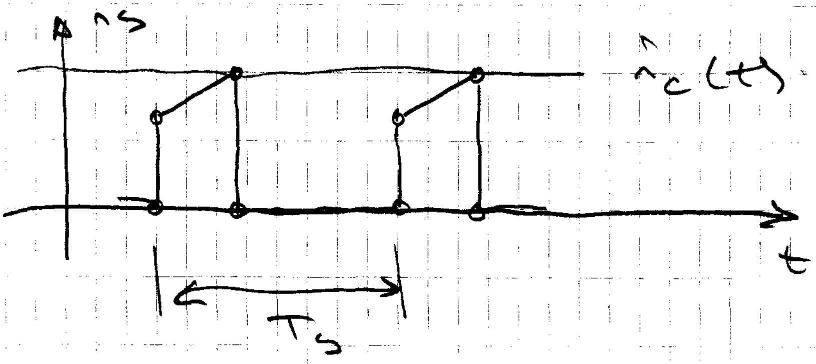
- Пример: buck converter



- где постоје регулације, локална је резистивна

- D(t) није директно контролисано

- мало јачи контролер



Предности / мане технике струјног програмирања

1° dynamics is potentially simpler

$\frac{\hat{v}}{\hat{i}_c}$ има један пол мање од $\frac{\hat{v}}{\hat{d}}$.

Реално постоји и други пол, али на знатно вишим учестаностима од PWM [illegible]

2° Преносфункција [illegible] [illegible] сама по себи, [illegible] се тачно како треба

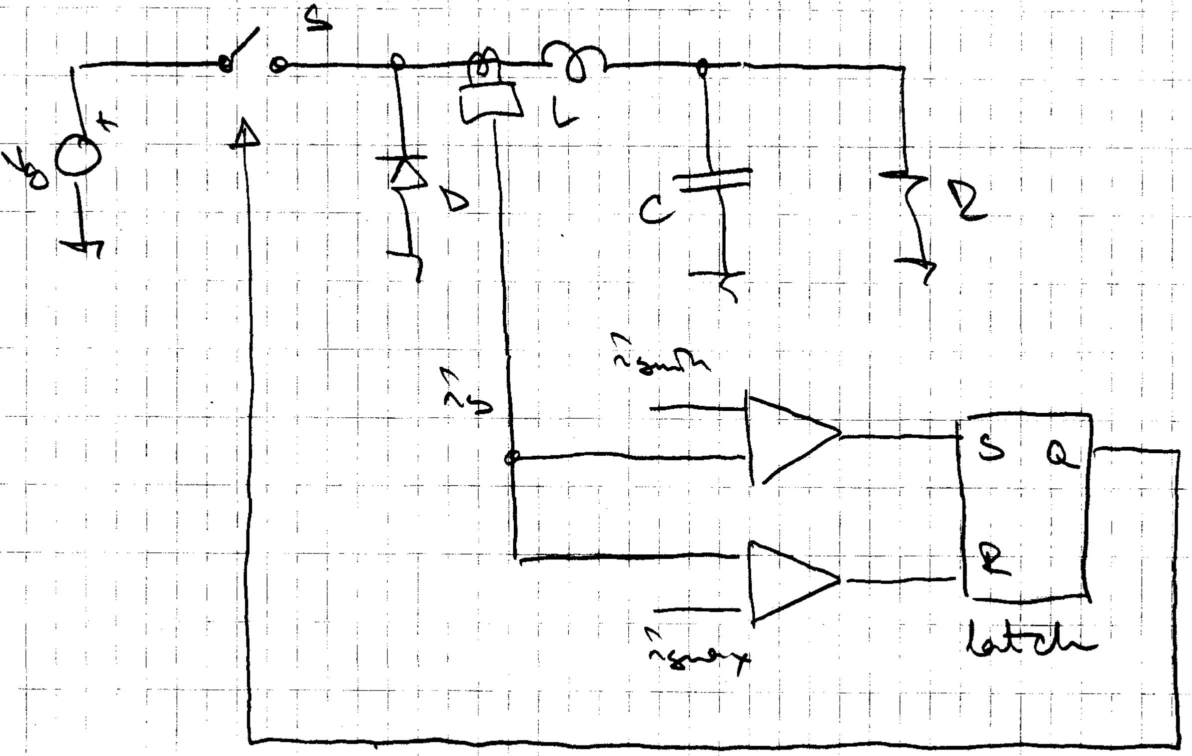
3° Много лакше подешавање, open loop може директно, ако сви [illegible] исто то може и closed loop.

4. Код push-pull-а и код full-bridge-а нема проблема са засићењем језгра

Мане:

1. За steady-state D > 0.5 струјна петља је нестабилна. Отклања се додатном "artificial ramp" на i_s. Биће објашњено

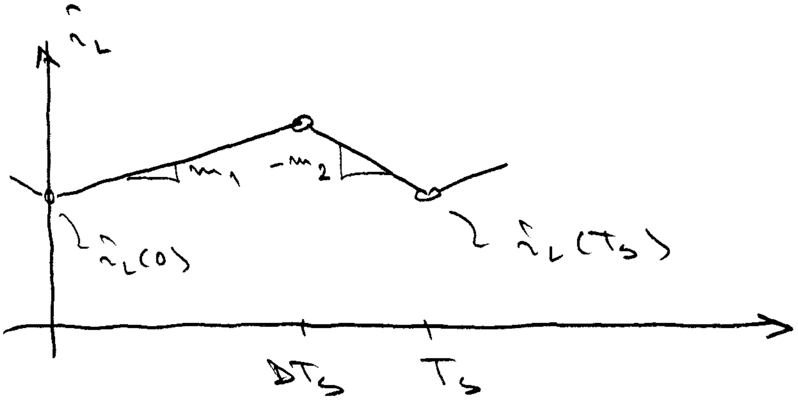
Хистерезисна контрола:



- нема проблема стабилности
- f_s није константно
- компликовано
- Лако не [illegible] и не [illegible]
- компликовано задавање i_c → $i_c + \Delta i = i_{max}$
  ↳ $i_c - \Delta i = i_{min}$
  ↳ када не пређе 0!

③

Discrete-Time Analysis of Basic Current-Mode Converter



$m_1 = \frac{d\hat{i}_L}{dt}$: 1st interval

$-m_2 = \frac{d\hat{i}_L}{dt}$: 2nd interval

	m_1	m_2
buck	$\frac{V_g - V}{L}$	V/L
boost	V_g/L	$(V - V_g)/L$
buck-boost	V_g/L	$-V/L$

$$\hat{i}_L(DT_s) = \hat{i}_c = \hat{i}_L(0) + m_1 DT_s$$

$$D = \frac{\hat{i}_c - \hat{i}_L(0)}{m_1 T_s}$$

2nd interval

$$\hat{i}_L(T_s) = \hat{i}_L(DT_s) - D'T_s m_2 =$$

$$= \hat{i}_L(0) + m_1 DT_s - m_2 D'T_s \qquad (*)$$

steady-state

$$\hat{i}_L(0) = \hat{i}_L(T_s)$$

$$0 = m_1 D - m_2 D' \qquad (**)$$

Consider small perturbation in $\hat{i}_L(0)$:

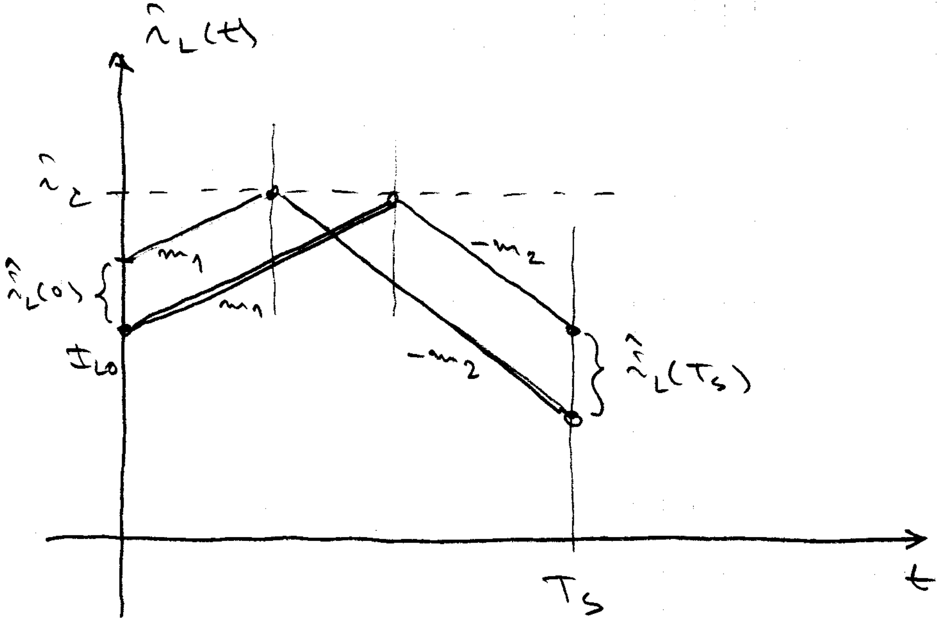
$$\hat{i}_L(0) = I_{L0} + \hat{\hat{i}}_L(0)$$

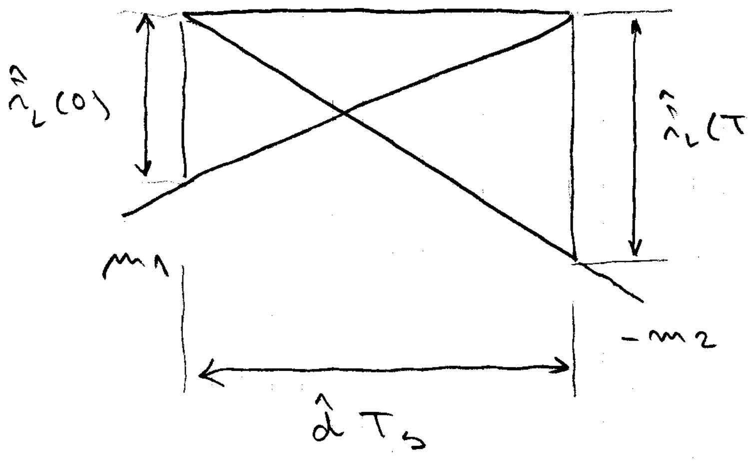
I_{L0} је steady-state $\hat{i}_L(0)$ које задовољава једначину (*)

Претпоставка $|\hat{\hat{i}}_L(0)| << |I_{L0}|$, хоћу да видим да ли се и расте или опада

нестабилно стабилно

Занима ме $\hat{\hat{i}}_L(nT_s)$





$\hat{d} < 0$

$\hat{\hat{i}}_L(0) > 0$

$\hat{\hat{i}}_L(T_s) < 0$

$$-\hat{\hat{i}}_L(0) = m_1 \hat{d}\, T_s$$

$$-\hat{\hat{i}}_L(T_s) = -m_2 \hat{d}\, T_s$$

eliminate $\hat{d}\, T_s$

$$-\frac{\hat{\hat{i}}_L(0)}{m_1} = \frac{\hat{\hat{i}}_L(T_s)}{m_2}$$

$$\hat{i}_L(T_s) = \hat{i}_L(0)\left(-\frac{m_2}{m_1}\right)$$

$$\frac{m_2}{m_1} = \frac{D}{D'} \qquad (**)$$

$$\hat{i}_L(T_s) = \hat{i}_L(0)\left(-\frac{D}{D'}\right)$$

$$\boxed{\hat{i}_L(nT_s) = \hat{i}_L(0)\left(-\frac{D}{D'}\right)^n}$$

за стабилност

$$\frac{D}{D'} < 1 \qquad \frac{D}{1-D} < 1$$

$$D < 1 - D \qquad 2D < 1$$

$$\boxed{D < \frac{1}{2}}$$ - стабилно

- Два примера из Бобове књиге

Example Operation of boost converter with $V_g = 20$, $V = 50$

note $\frac{V}{V_g} = \frac{1}{D'} \Rightarrow D' = \frac{2}{5} \Rightarrow D = \frac{3}{5} > \frac{1}{2}$

so the current programmed mode should be unstable.

$$\left(-\frac{D}{D'}\right) = \left(-\frac{3/5}{2/5}\right) = -1.5$$

n	$\hat{i}(nT_s) = \hat{i}(0)\left(-\frac{D}{D'}\right)^n$
0	$\hat{i}(0)$
1	$-1.5\ \hat{i}(0)$
2	$+2.25\ \hat{i}(0)$
3	$-3.375\ \hat{i}(0)$

etc.

- growing oscillation

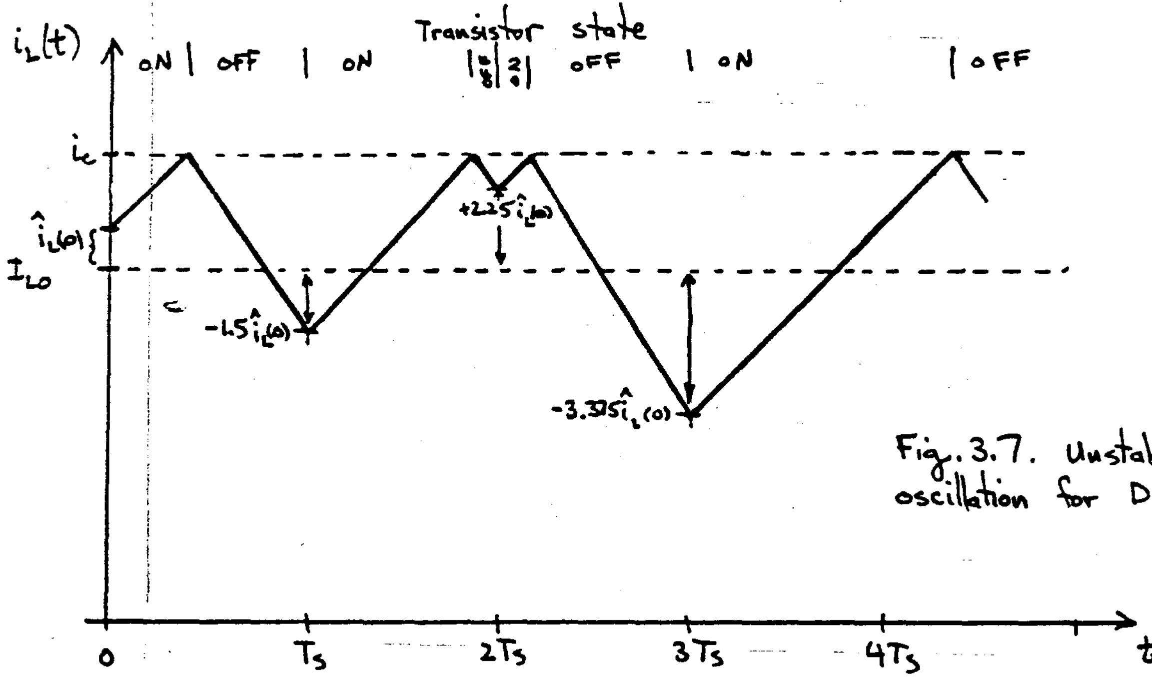


Fig. 3.7. Unstable oscillation for D=0.6

It can be seen from Fig. 3.7 that the oscillations grow in amplitude, and the current-mode controller does not operate correctly. However, once the oscillations become large, they no longer grow without bound. Instead, the inherent nonlinearity (saturation) of the system limits the maximum amplitude of the oscillations.

For $V_g = 20$, $V = 30$:

then $D = \frac{1}{3}$, $-\frac{D}{D'} = -\frac{1}{2}$

n	$\hat{i}(nT_s) = \hat{i}(0)\left(-\frac{D}{D'}\right)^n$
0	$\hat{i}(0)$
1	$-\frac{1}{2}\hat{i}(0)$
2	$\frac{1}{4}\hat{i}(0)$
3	$-\frac{1}{8}\hat{i}(0)$

etc.

disturbance becomes small after a few intervals

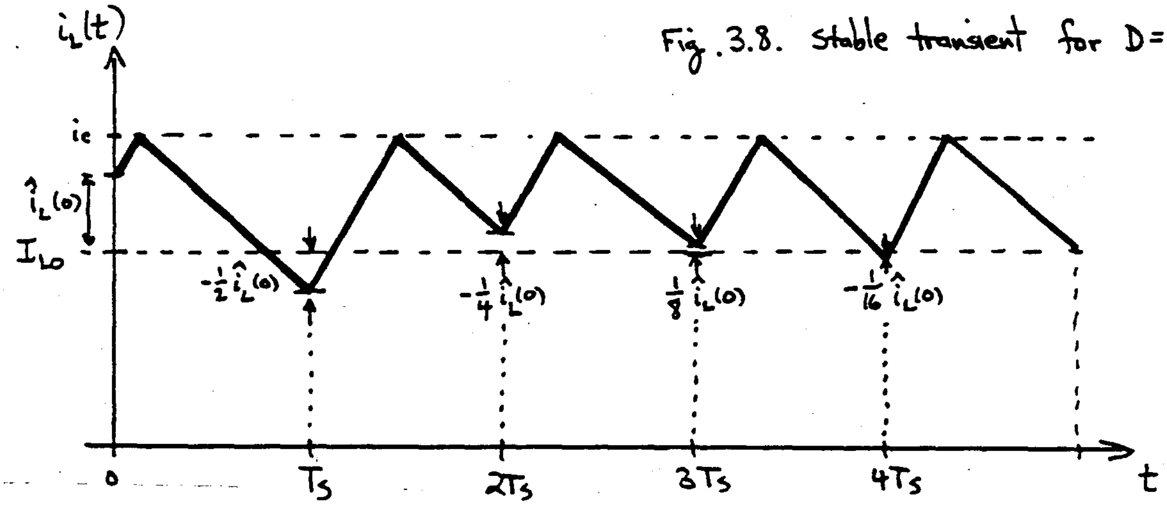
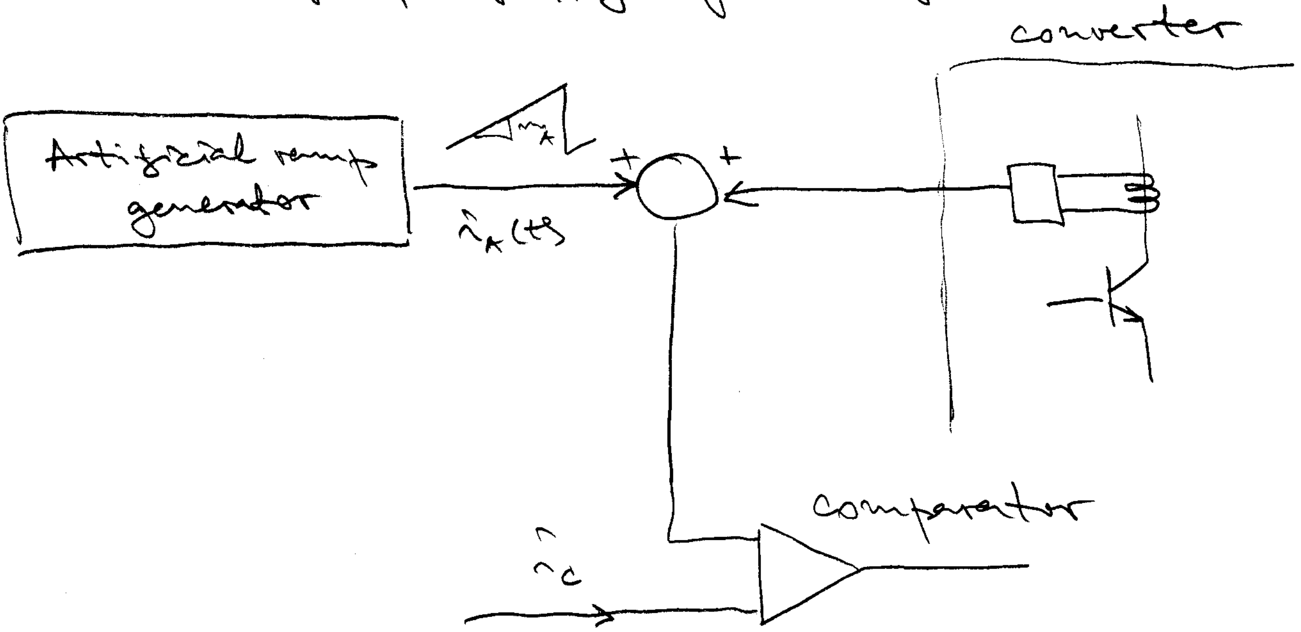


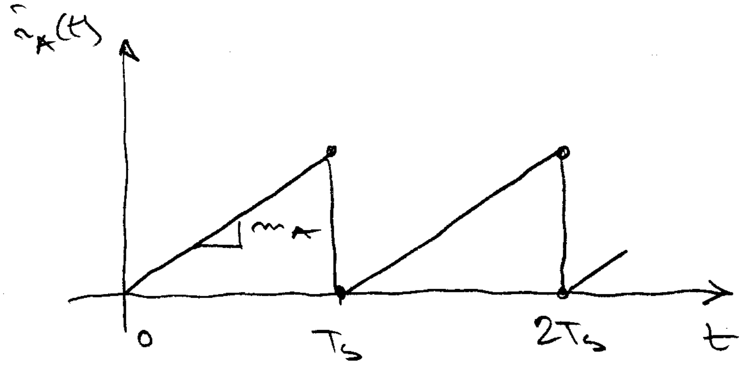
Fig. 3.8. Stable transient for $D = \frac{1}{3}$

Нестабилност за $D > \frac{1}{2}$ не зависи од converter topology.

Artificial Ramp

commonly-used solution, додаје се artificial ramp на мерену струју транзистора





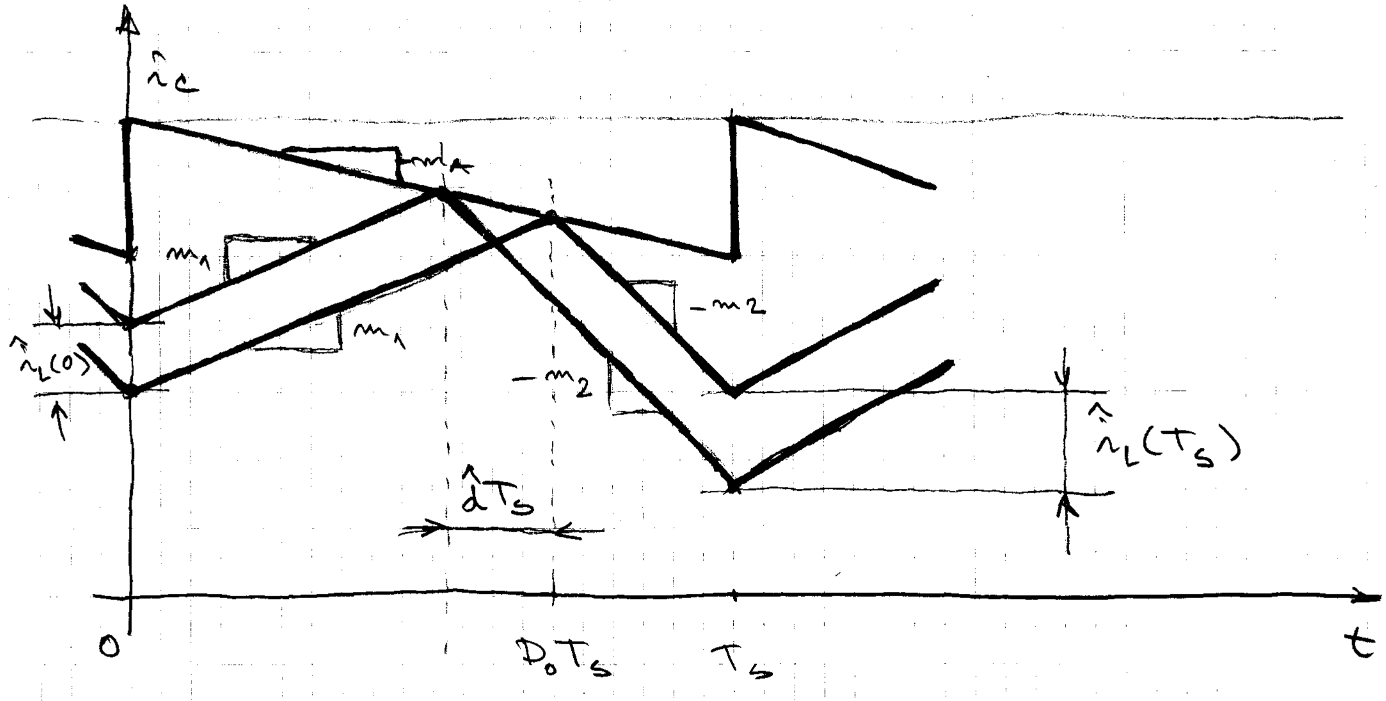
Транзистор се пали када

$$\hat{i}_A(t) + \hat{i}_L(t) = \hat{i}_c$$

односно

$$\hat{i}_L(t) = \hat{i}_c - \hat{i}_A(t)$$

важи само у тренутку када се транзистор пали



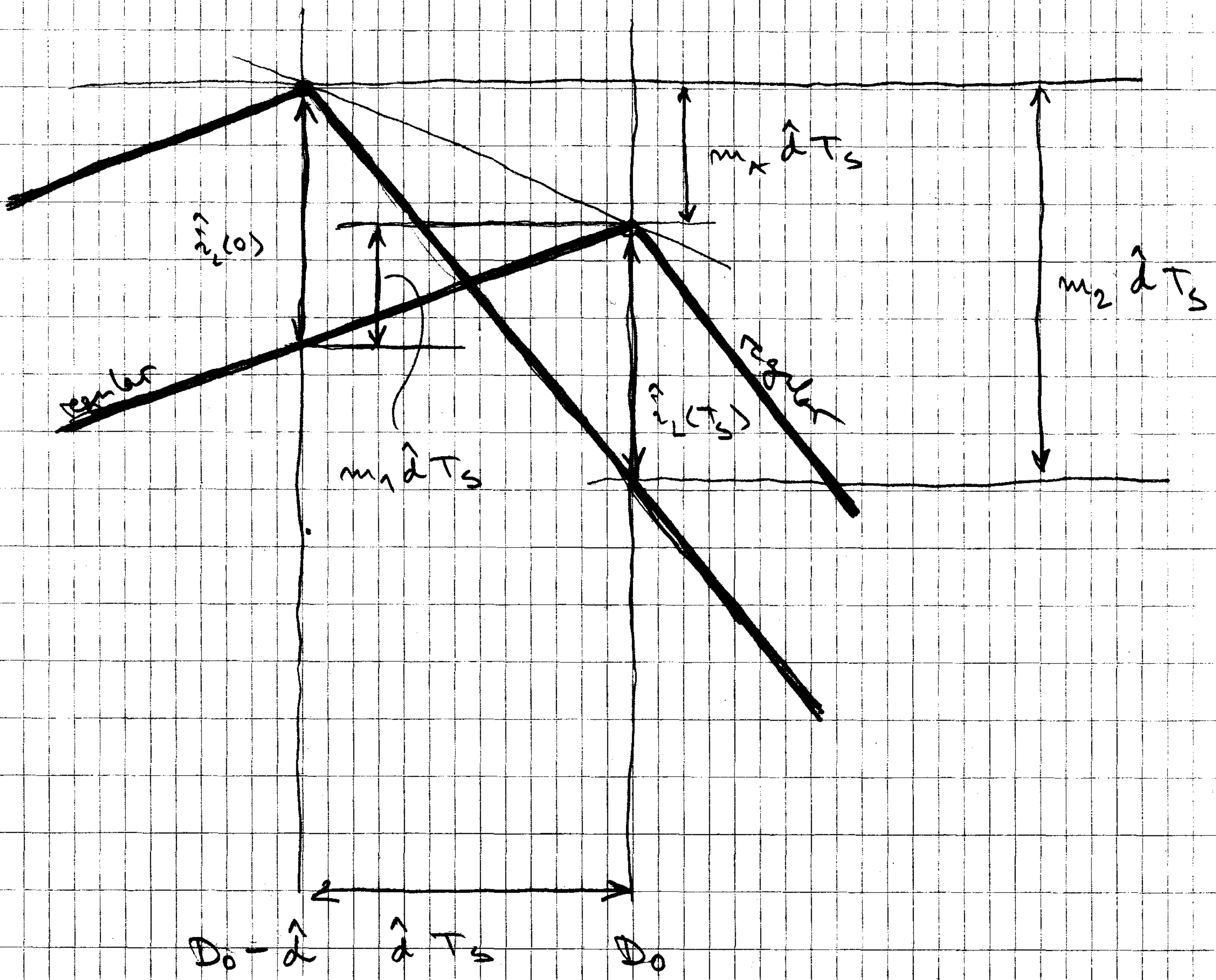
$$-\hat{i}_L(0) = \hat{d}T_s m_1 - \hat{d}T_s(-m_A) =$$

$$= \hat{d}T_s(m_1 + m_A)$$

$$-\hat{i}_L(T_s) = -m_2\hat{d}T_s - \hat{d}T_s(-m_A) =$$

$$= \hat{d}T_s(m_A - m_2)$$

погледај! →



$$\hat{i}_L(0) = m_A \hat{d} T_s + m_1 \hat{d} T_s = (m_1 + m_A) \hat{d} T_s$$

$$\hat{i}_L(T_s) = - m_2 \hat{d} T_s + m_A \hat{d} T_s = (m_A - m_2) \hat{d} T_s$$

$$\frac{\hat{i}_L(T_s)}{\hat{i}_L(0)} = \frac{m_A - m_2}{m_1 + m_A}$$

$$\frac{\hat{i}_L(T_s)}{\hat{i}_L(0)} = \left(-\frac{m_2 - m_A}{m_1 + m_A}\right)$$

$$\boxed{\hat{i}_L(nT_s) = \left(-\frac{m_2 - m_A}{m_1 + m_A}\right)^n \hat{i}_L(0)}$$

$m_A = 0$ – идеално разлагајући случај

$\hat{i}_L \to \infty$ ако $\left|\frac{m_2 - m_A}{m_1 + m_A}\right| > 1$

$\hat{i}_L \to 0$ ако $\left|\frac{m_2 - m_A}{m_1 + m_A}\right| < 1$

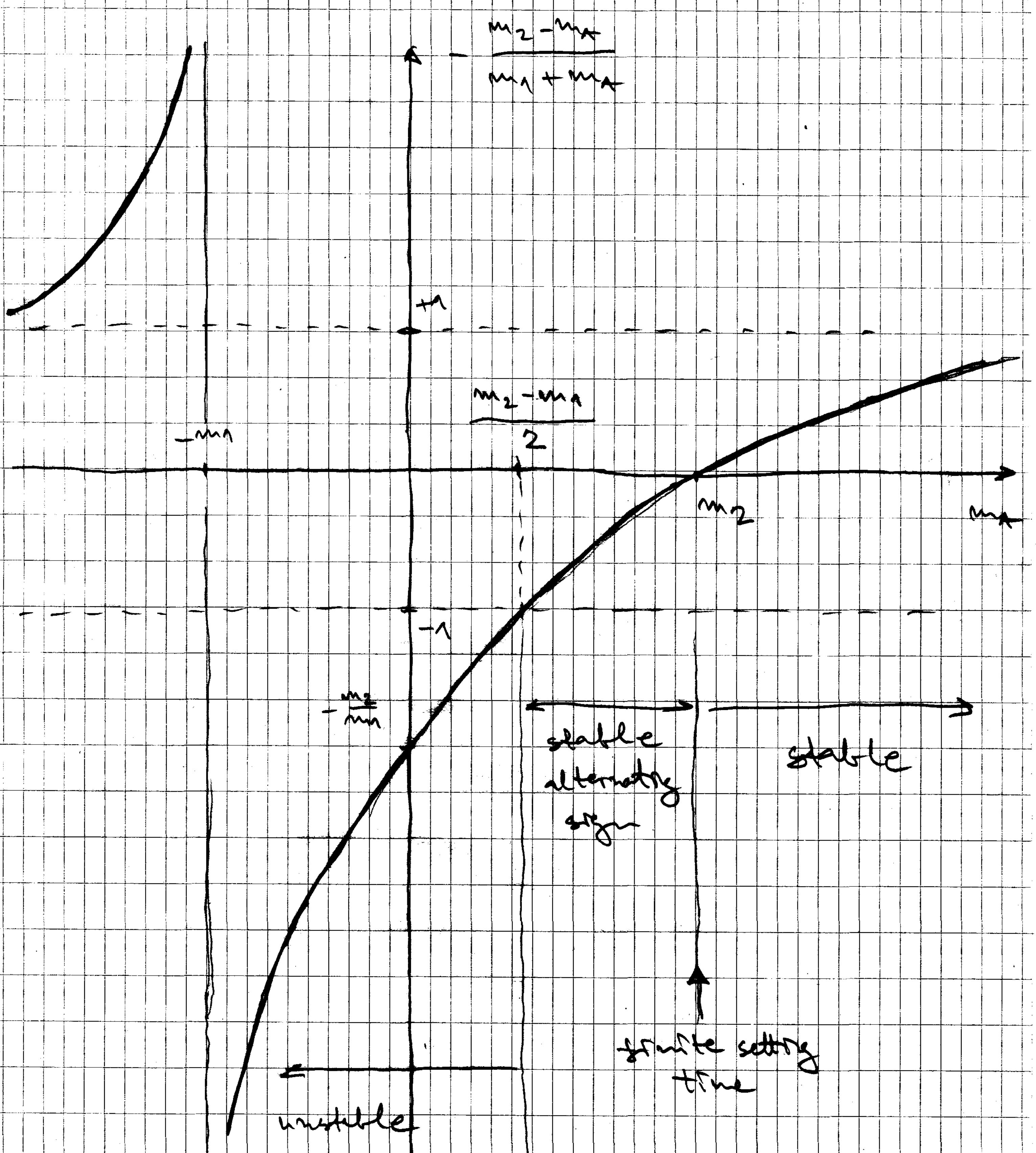
Нула: $-\frac{m_2 - m_A}{m_1 + m_A} \quad (m_A)$

случајеви

$D_0 > \frac{1}{2} \qquad m_2 > m_1$

$D_0 < \frac{1}{2} \qquad m_1 > m_2$.

$D_0 > \frac{1}{2}$, $m_2 > m_1$



$m_1 > m_2$ – stable

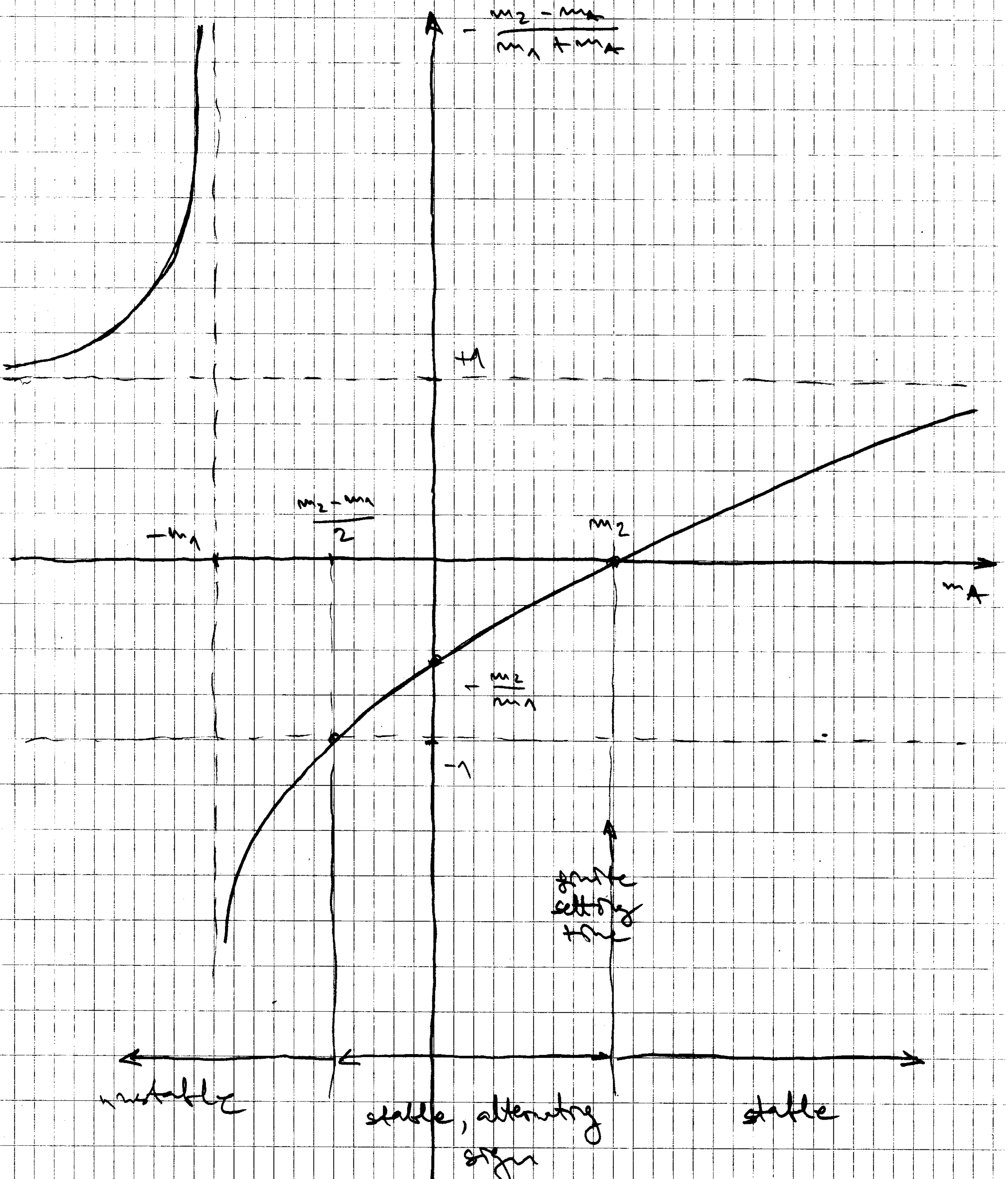
$\frac{m_2 - m_1}{2} < m_1 < m_2$ – stable, alternating sign

$m_1 < \frac{m_2 - m_1}{2}$ – unstable

mai lente m_1 mai mare $\hat{\imath}_c$ mai lente ca $\bar{\hat{\imath}}_L$, mai $\hat{\imath}_c$ mare

$m_1 = m_2$ – finite setting time

$D_0 < \frac{1}{2}$, $m_2 < m_1$



$m_1 > m_2$ – stable

$m_2 > m_1 > \frac{m_2 - m_1}{2}$ – stable, alternating sign

$m_1 = m_2$ – finite setting time

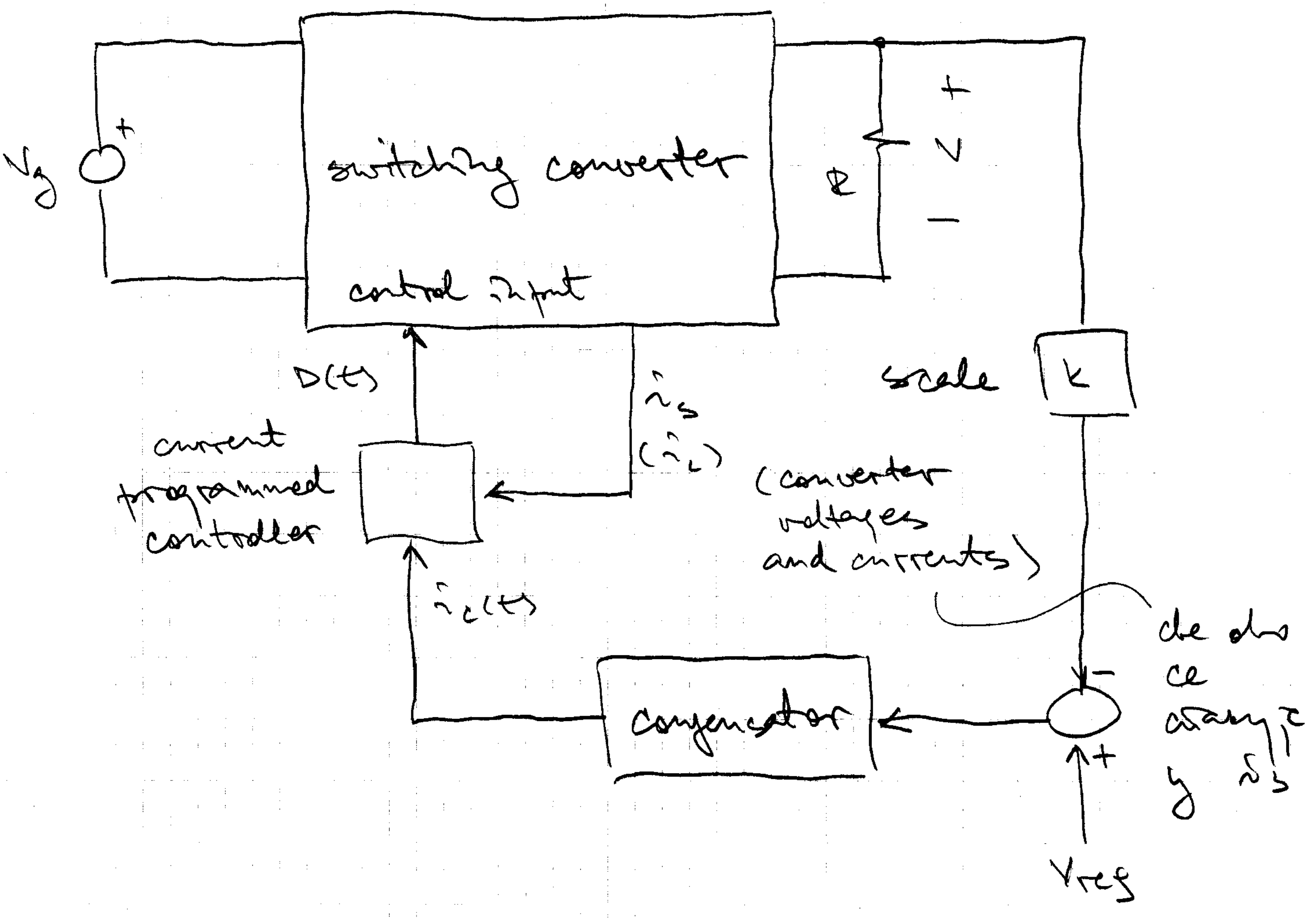
$m_1 < \frac{m_2 - m_1}{2}$ – unstable

finite setting time?

m_2 се мења са Do, нпр. са променом параметара. Не може се направити finite setting time кад параметар доста варира

Регулација претварача са струјним програмирањем

- дупла петља регулације



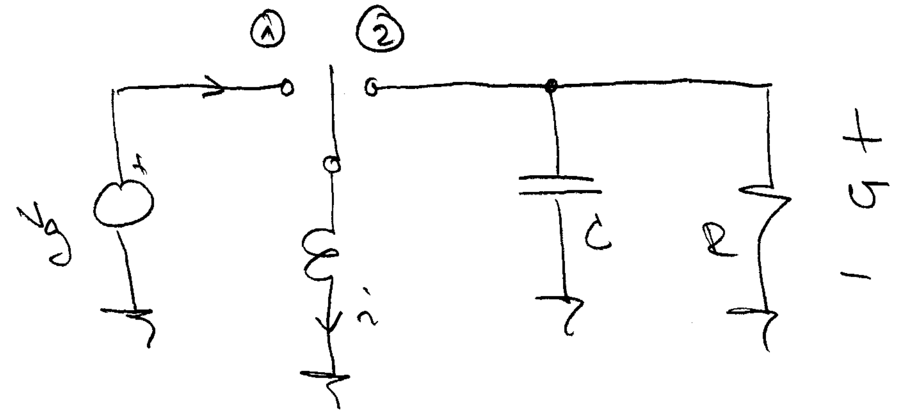
- equivalent circuit, small signal model ,

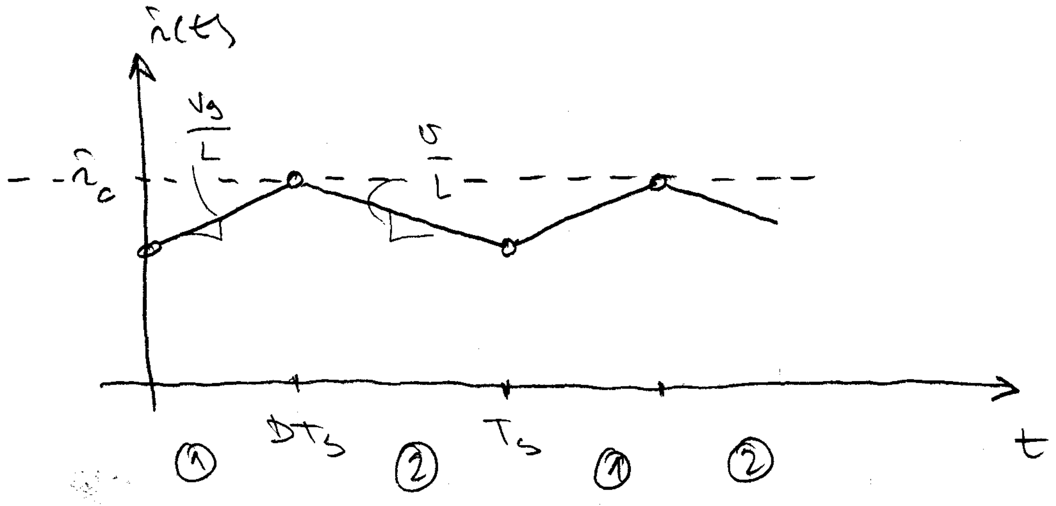
$\cancel{\hat{d}} \curvearrowright \hat{i}_c$

$i_L \cong i_C$ – ово <u>углавном</u> ради – сјајно

- реално понашање ради на високим учестаностима
- опет state-space averaging

Simplified Equivalent Circuit Modelling, Buck-Boost Example





State-Space Equations

$$L \frac{d\hat{i}}{dt} = D_0' \hat{v} + D_0 \hat{v}_g + (V_g - V_0)\hat{d}$$

$$C \frac{d\hat{v}}{dt} = -D_0' \hat{i} - \frac{\hat{v}}{R} + I_0 \hat{d}$$

$$\hat{i}_g = D_0 \hat{i} + I_0 \hat{d}$$

Laplace transform

$$sL\,\hat{i}(s) = D_0' \hat{v}(s) + D_0 \hat{v}_g(s) + (V_g - V_0)\hat{d}(s)$$

$$sC\,\hat{v}(s) = -D_0' \hat{i}(s) - \frac{\hat{v}(s)}{R} + I_0 \hat{d}(s)$$

$$\hat{i}_g(s) = D_0 \hat{i}(s) + I_0 \hat{d}(s)$$

converter is stable, ripple is small, artificial ramp is not too large

$$\hat{i}_c(s) \cong \hat{i}(s)$$

$$sL\,\hat{i}_c(s) \cong D_0' \hat{v}(s) + D_0 \hat{v}_g(s) + (V_g - V_0)\hat{d}(s)$$

$$\hat{d} = \frac{sL\,\hat{i}_c(s) - D_0' \hat{v}(s) - D_0 \hat{v}_g(s)}{V_g - V_0}$$

- циљ: елиминисати $\hat{d}(s)$ и заменити са $\hat{i}_c(s)$

$$sC\,\hat{v}(s) = -D_0'\,\hat{i}_c(s) - \frac{\hat{v}(s)}{R} + I_0\,\frac{sL\,\hat{i}_c(s) - D_0'\,\hat{v}(s) - D_0\,\hat{v}_g(s)}{V_g - V_0}$$

$$\hat{i}_g(s) = D_0\,\hat{i}_c(s) + I_0\,\frac{sL\,\hat{i}_c(s) - D_0'\,\hat{v}(s) - D_0\,\hat{v}_g(s)}{V_g - V_0}$$

- сређено и груписано изгледа

$$sC\,\hat{v}(s) = \left(\frac{sLI_0}{V_g - V_0} - D_0'\right)\hat{i}_c(s) -$$

$$- \left(\frac{D_0' I_0}{V_g - V_0} + \frac{1}{R}\right)\hat{v}(s) - \left(\frac{D_0 I_0}{V_g - V_0}\right)\hat{v}_g(s)$$

$$\hat{i}_g(s) = \left(D_0 + \frac{sLI_0}{V_g - V_0}\right)\hat{i}_c(s) -$$

$$- \left(\frac{D_0' I_0}{V_g - V_0}\right)\hat{v} - \left(\frac{D_0 I_0}{V_g - V_0}\right)\hat{v}_g(s)$$

убацимо steady-state equations

$$V_0 = -\frac{D_0}{D_0'}V_g \;;\quad I_0 = -\frac{V_0}{D_0' R} \rightarrow \frac{D_0 V_g}{D_0'^2 R}$$

$$\frac{I_0}{V_g - V_0} = \frac{D_0 V_g}{D_0'^2 R \left(V_g + \frac{D_0}{D_0'} V_g\right)} = \frac{D_0}{D_0' R}$$

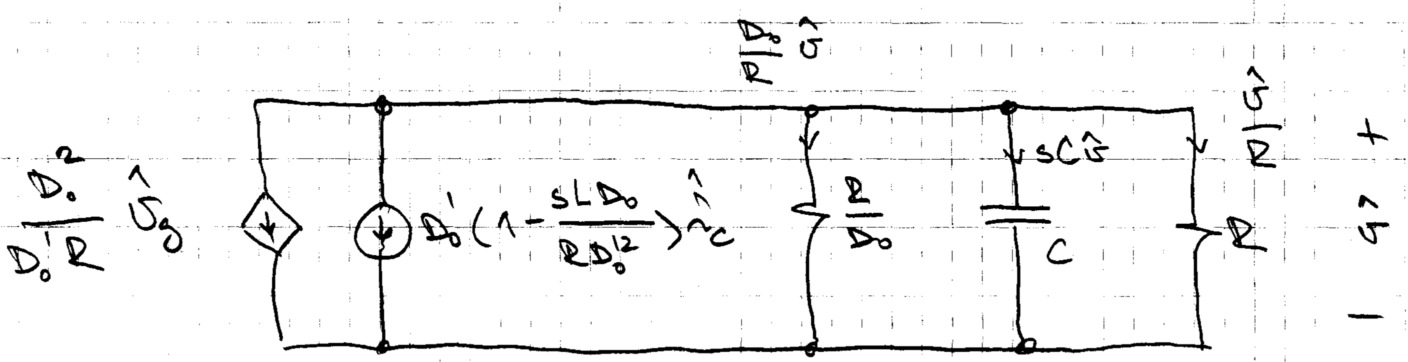
па је:

$$sC\hat{v} = \left(\frac{sLD_0}{D_0' R} - D_0'\right)\hat{i}_c - \left(\frac{D_0}{R} + \frac{1}{R}\right)\hat{v} - \\ - \left(\frac{D_0^2}{D_0' R}\right)\hat{v}_g$$

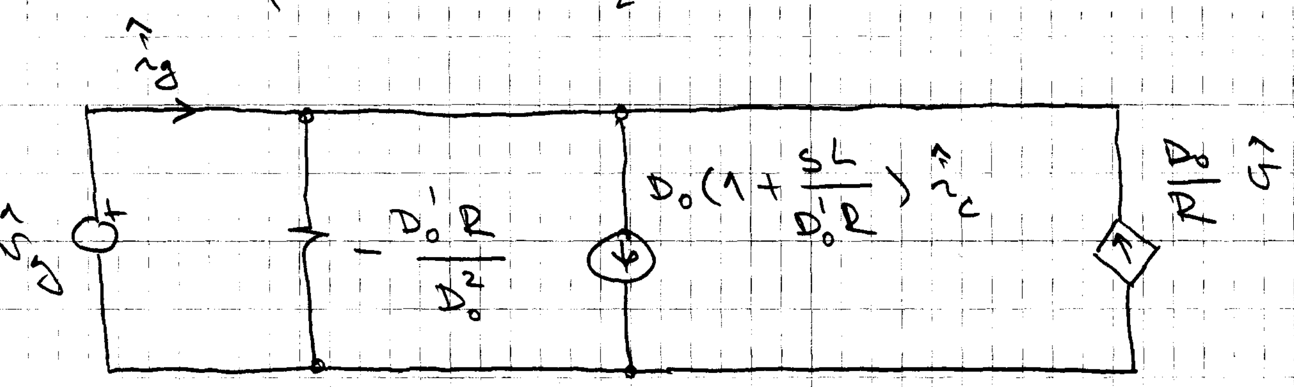
$$\hat{i}_g = \left(\frac{sLD_0}{RD_0'} + D_0\right)\hat{i}_c - \left(\frac{D_0}{R}\right)\hat{v} - \left(\frac{D_0^2}{D_0' R}\right)\hat{v}_g$$

↑ ово су основне једначине, следи екв. коло

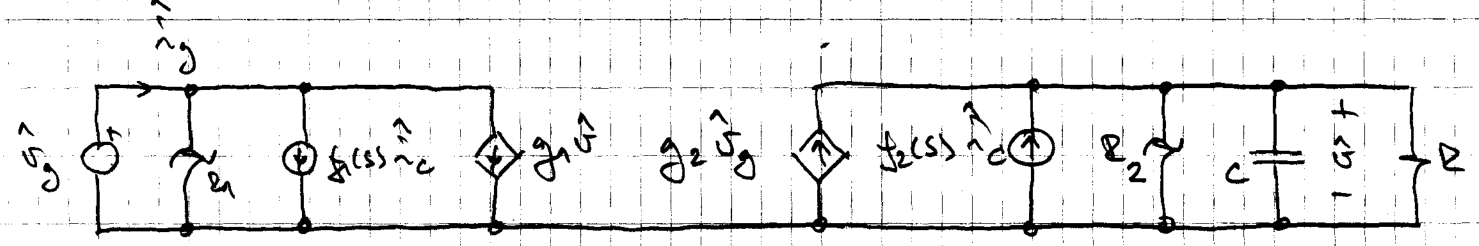
- једначина излазног кондензатора



- the input current equation:



Слагањем:



$$R_1 = -\frac{D_0'}{D_0}R, \quad f_1(s) = D_0\left(1+s\frac{L}{D_0'R}\right), \quad g_1 = -\frac{D_0}{R}$$

$$R_2 = \frac{R}{D_0}, \quad f_2(s) = -D_0'\left(1-s\frac{D_0L}{D_0'^2R}\right), \quad g_2 = -\frac{D_0^2}{D_0'R}$$

Из овог кола се лаке преносне функције, излазна импеданса, . . .

Summary: Control to output transfer function

$$\frac{\hat{v}}{\hat{i}_c} = f_2 \; R_2 \| R \| \frac{1}{sC} \qquad (\hat{v}_g = 0)$$

$$\boxed{\frac{\hat{v}}{\hat{i}_c} = -R \, \frac{1-D_0}{1+D_0} \, \frac{1 - s\,\frac{D_0 L}{D_0'^2 R}}{1 + s\,\frac{RC}{1+D_0}}}$$

Line-to-output transfer function

$$\frac{\hat{v}}{\hat{v}_g} = g_2 \; R_2 \| R \| \frac{1}{sC} \qquad (\hat{i}_c = 0)$$

$$\boxed{\frac{\hat{v}}{\hat{v}_g} = -\frac{D_0^2}{1-D_0^2} \, \frac{1}{1 + s\,\frac{RC}{1+D_0}}}$$

output impedance

$$Z_{out} = R_2 \| R \| \frac{1}{sC} = \frac{R}{1+D_0} \, \frac{1}{1 + s\,\frac{RC}{1+D_0}}$$

$\hat{i}_c = 0$, $\hat{v}_g = 0$

Пример 2: buck converter

$$sL\,\hat{i}(s) = \hat{d}\,V_g + D_0\,\hat{v}_g - \hat{v}$$

$$sC\,\hat{v}(s) = \hat{i} - \hat{v}/R$$

$$\hat{i}_g(s) = \hat{d}\,I_0 + D_0\,\hat{i}$$

↑ из state-space averaging-a

апроксимација

$$\hat{i}(s) \cong \hat{i}_c(s)$$

$$sL\,\hat{i}_c(s) = \hat{d}\,V_g + D_0\,\hat{v}_g - \hat{v}$$

$$\hat{d}(s) \cong \frac{sL\hat{i}_c - D_0\hat{v}_g + \hat{v}}{V_g}$$

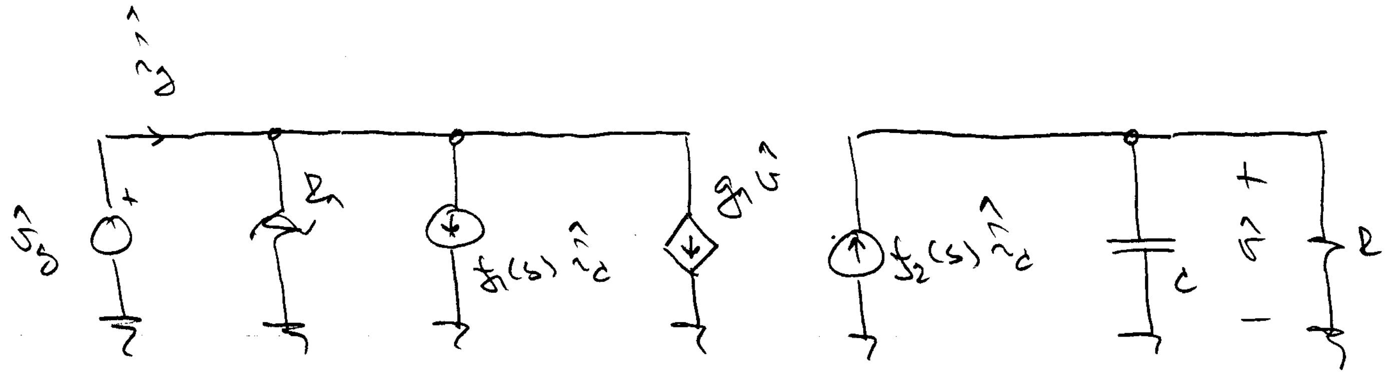
заменом $\hat{d}(s)$ и узмем dc identity

$$I_0/V_g = D_0/R$$

$$sC\,\hat{v} = \hat{i}_c - \frac{\hat{v}}{R}$$

$$\hat{i}_g = \hat{i}_c\,D_0\left(1 + s\frac{L}{R}\right) - \frac{D_0^2}{R}\hat{v}_g + \frac{D_0}{R}\hat{v}$$

first-order small-signal equations



нема зависност од $\hat{v}_g$

$$R_1 = -\frac{R}{D_0^2}\ , \quad f_1(s) = D_0\left(1+s\frac{L}{R}\right)\ , \quad g_1 = \frac{D_0}{R}$$

$$f_2(s) = 1$$

control-to-output transfer function:

$$\frac{\hat{v}}{\hat{i}_c} = f_2(s)\, R \parallel \frac{1}{sC} = \frac{R}{1+sRC}$$

line-to-output transfer function:

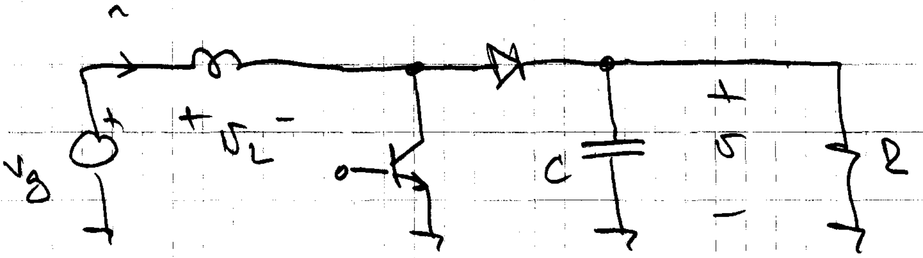
$$\frac{\hat{v}}{\hat{v}_g} = 0 \qquad \text{(good "line rejection")}$$

output impedance

$$Z_{out} = R \parallel \frac{1}{sC} = \frac{R}{1+sRC}$$

(исто [illegible] [illegible])

Вежба: CPM boost converter

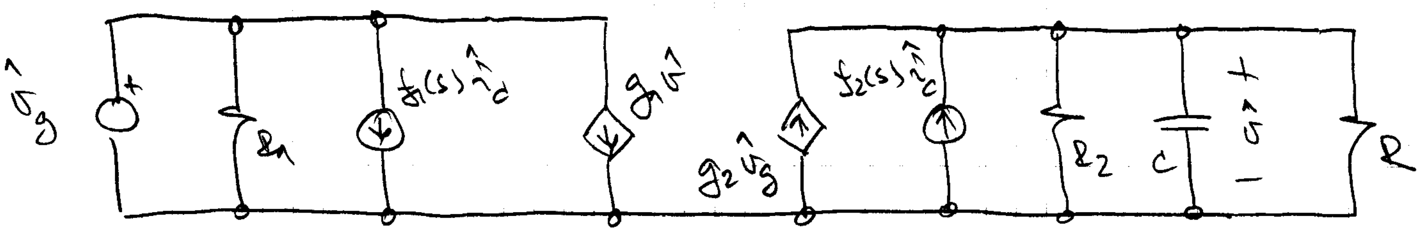


$$L\frac{d\hat{i}}{dt} = \hat{v}_g - D_0'\hat{v} + \hat{d}V_0$$

$$C\frac{d\hat{v}}{dt} = -\frac{\hat{v}}{R} + D_0'\hat{i} - \hat{d}I_0$$

даље сами . . .

A Canonical Model for the Current Programmed Mode



simple first-order model

converter	R_1	$f_1(s)$	g_1	g_2	$f_2(s)$	R_2
buck	$-\frac{R}{D_0^2}$	$D_0(1+s\frac{L}{R})$	$\frac{D_0}{R}$	0	1	∞
boost	∞	1	0	$\frac{1}{D_0' R}$	$D_0'(1-\frac{sL}{D_0'^2 R})$	R
buck-boost	$-\frac{D_0' R}{D_0^2}$	$D_0(1+s\frac{L}{D_0' R})$	$-\frac{D_0}{R}$	$-\frac{D_0^2}{D_0' R}$	$-D_0'(1-\frac{sD_0L}{D_0'^2 R})$	$\frac{R}{D_0}$

РЕЗОНАНТНИ КОНВЕРТОРИ

- идеја: смањити прекидачке губитке
  применом soft-switching-а

- ипак имају веће губитке од square-wave конвертора на нижим учестаностима, мање η

- на одређеној учестаности прекидања η резонантних конвертора постаје веће од η одговарајућих square-wave конвертора, кад почну да доминирају прекидачки губици.

- Где тачно – не зна се, реда 100 kHz

- Смањени "прекидачки" губици по цену повећаних кондукционих губитака

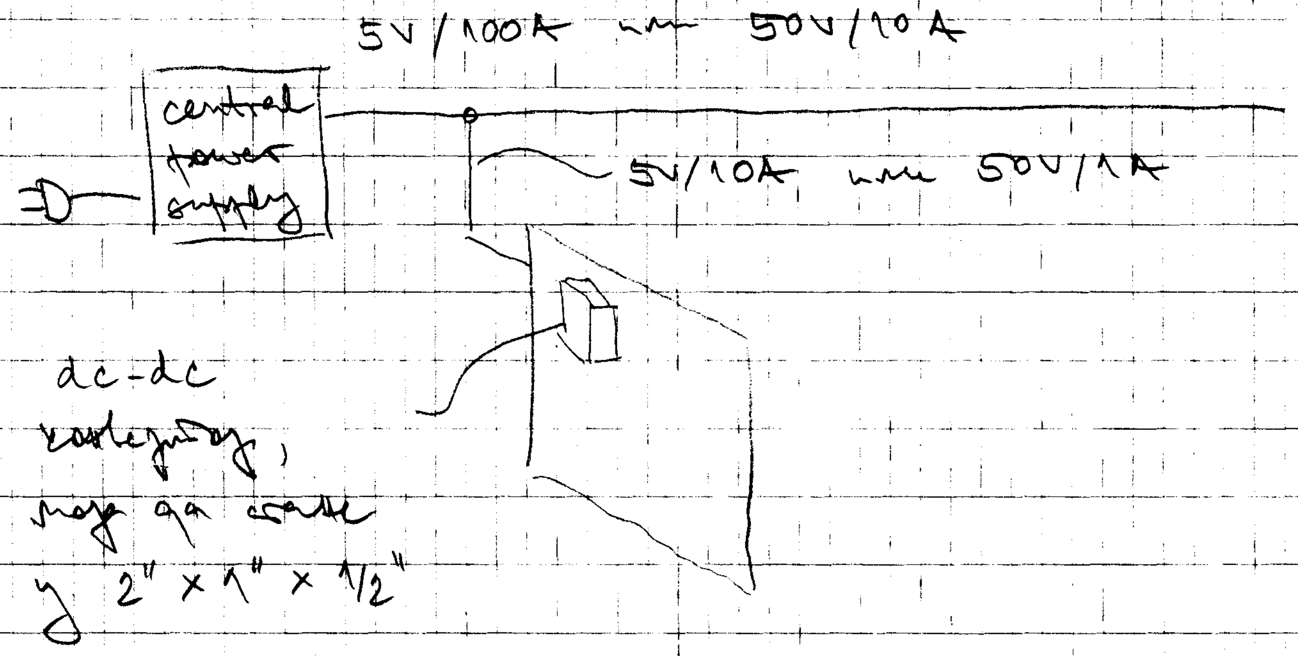
- "прекидачи" – прави прекидачи, паразитне индуктивност и паразитне капацитивност

- Поређење увек проблематично

- Подела:
  1) square-wave конвертори у дисконт. режиму
  2) кола за гашење тиристора – квазирезонантни

- ПРИМЕНА:

1) distributed power processing



- Густина снаге 50W/in³ — комплетно (све са хладњаком, филтром . . .)

2) Aerospace & automotive апликације, потребна мала запремина, тежина и велика "густина снаге"

3) Кола са IGBT. IGBT — ефикасна, моћна, споро гасе због current tail-a, потребан zero current (voltage?) turn-off

Bob: 3.5kW са IGBT — резонантно

4) Генерално су погодни када се процесира мала снага – квазирезонантни

5) код кола са трансформатором (огромна снага) – мора увек

6) никада кад се без примене трансформатора захтева велико η, а димензије нису од значаја

ЗАКЉУЧАК: примена је отворено питање без дефинитивног одговора, од примене до примене треба ићи и дискутовати

Bob: " you should use resonant converters when you have real good reason to do so "

ПРЕДНОСТИ И МАНЕ

ПРЕДНОСТИ:

1) мање димензије и маса конвертора
2) мање СМЕТЊЕ
3) уклањају се "из чујног" подручју
4) мањи губици на високим учестаностима

МАНЕ:

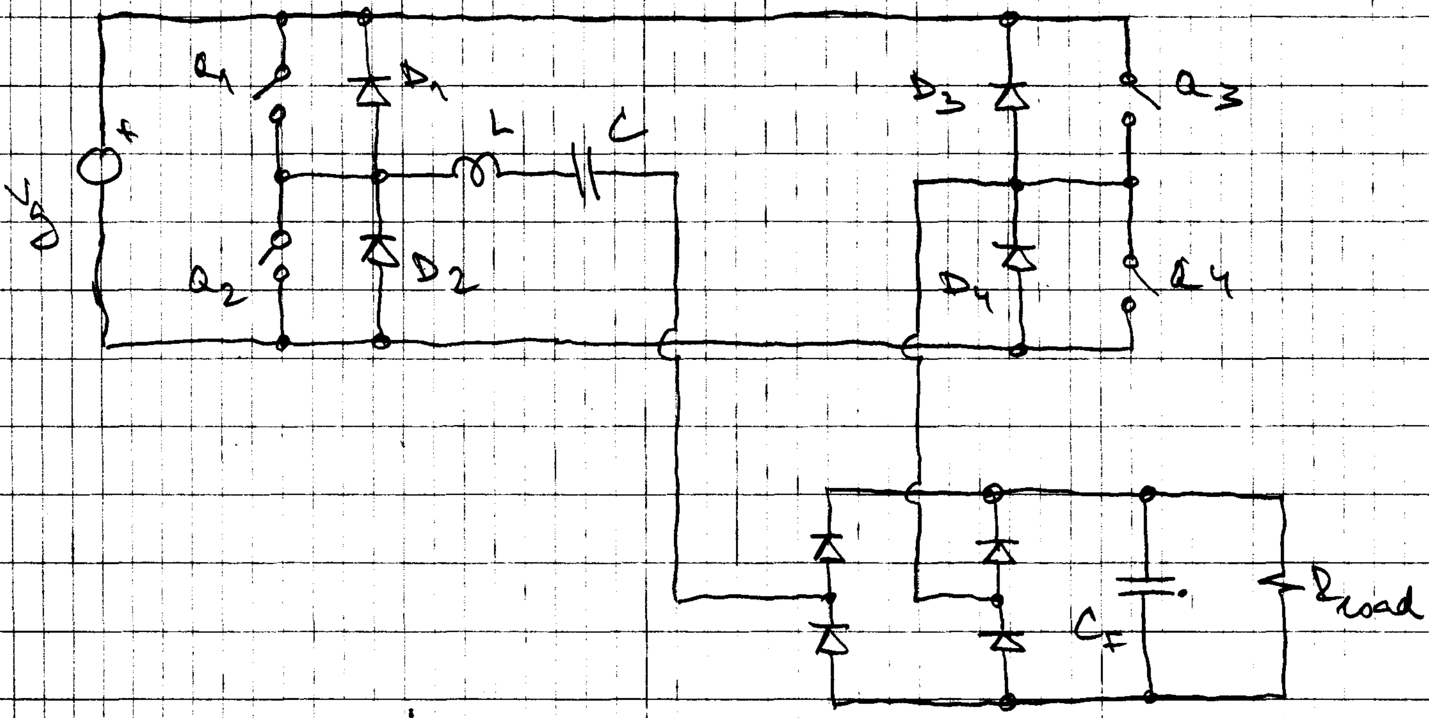
1) повећани кондукциони губици
2) ЗНАТНО сложеније пројектовање
3) ЗНАТНО компликованије управљање

КЛАСИФИКАЦИЈА

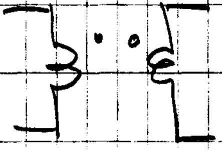
- има безброј класификација, једна од њих:

1) (Totally) Resonant Converters
(Потпуно Резонантни конвертори)

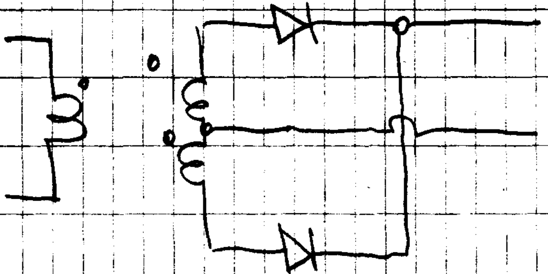
1.а. Серијски резонантни конвертор



+ модификације

а)  за галванско раздвајање, [illegible] [illegible] од звучника !

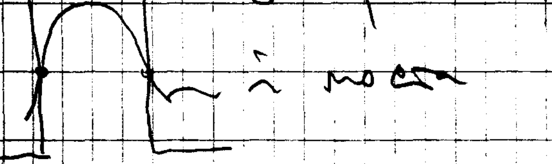
б) двоструки исправљач за мање губитке при ниским напонима



в) half-bridge топологија, за мање снаге

г) елиминација C_o код half-bridge-а, резонантни кондензатор блокира dc компоненту

идеја:



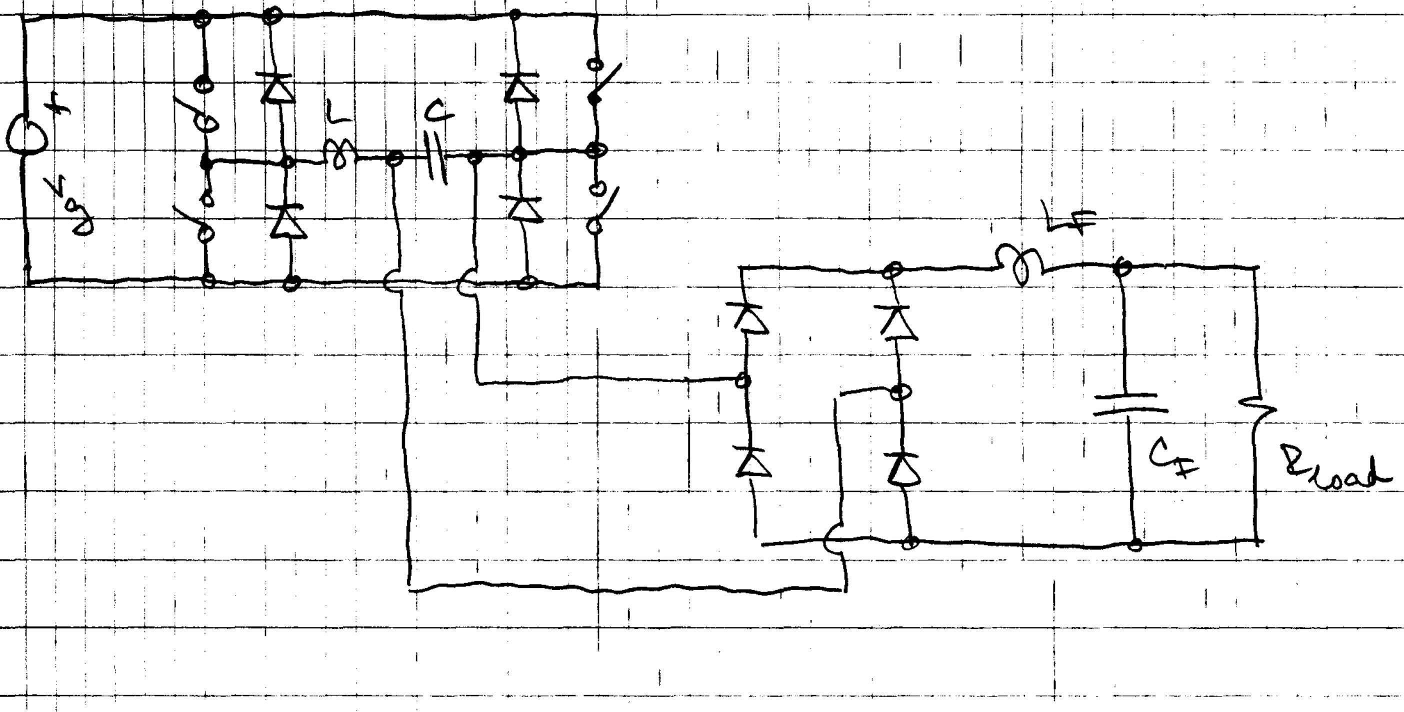
- дефазирање грана само даје soft switching од индуктивног дела – може се додати да оба буду soft

- излазне диоде switch at zero current, њихово искључивање не доприноси значајно губицима

- расипне индуктивности трансформатора не утичу - на ред са L резонантно

- поменуте идеје су основ и за све остале резонантне конверторе

1.8. Паралелни резонантни конвертор



- у много чему аналогна са SRC, али заснована на истом принципу

2) Квазирезонантни конвертори

- zero voltage switching
- zero current switching
- multiresonant
- nonlinear resonant

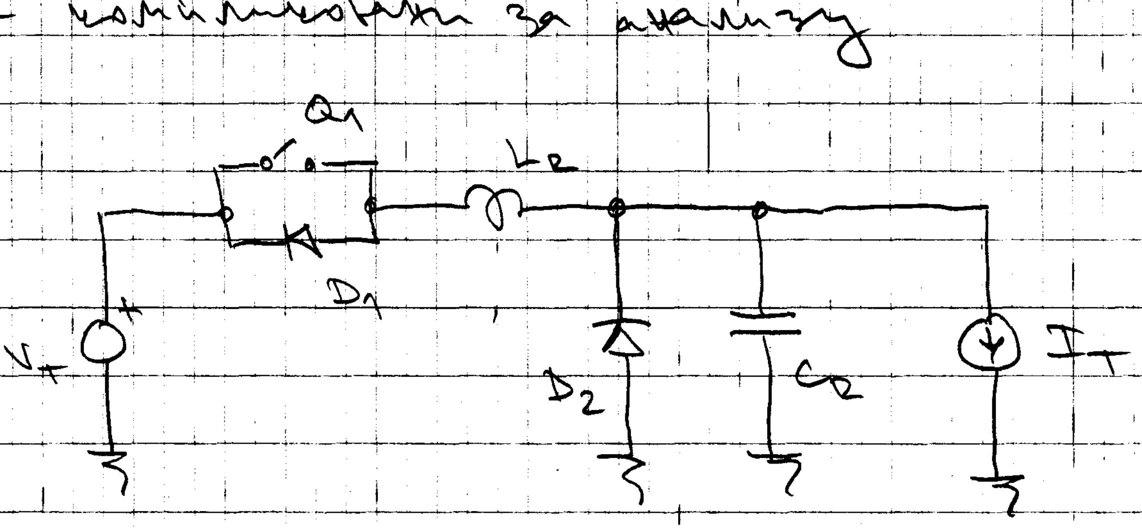
- екситација кола за пражњење индуктора

- има их безброј врста

- генерално већи component stress и кондукциони губици

- добро за мале снаге и где баш мора (трансформатори)

- компликовани за анализу



- безброј модификација

МЕТОДИ АНАЛИЗЕ РЕЗОНАНТНИХ КОНВЕРТОРА

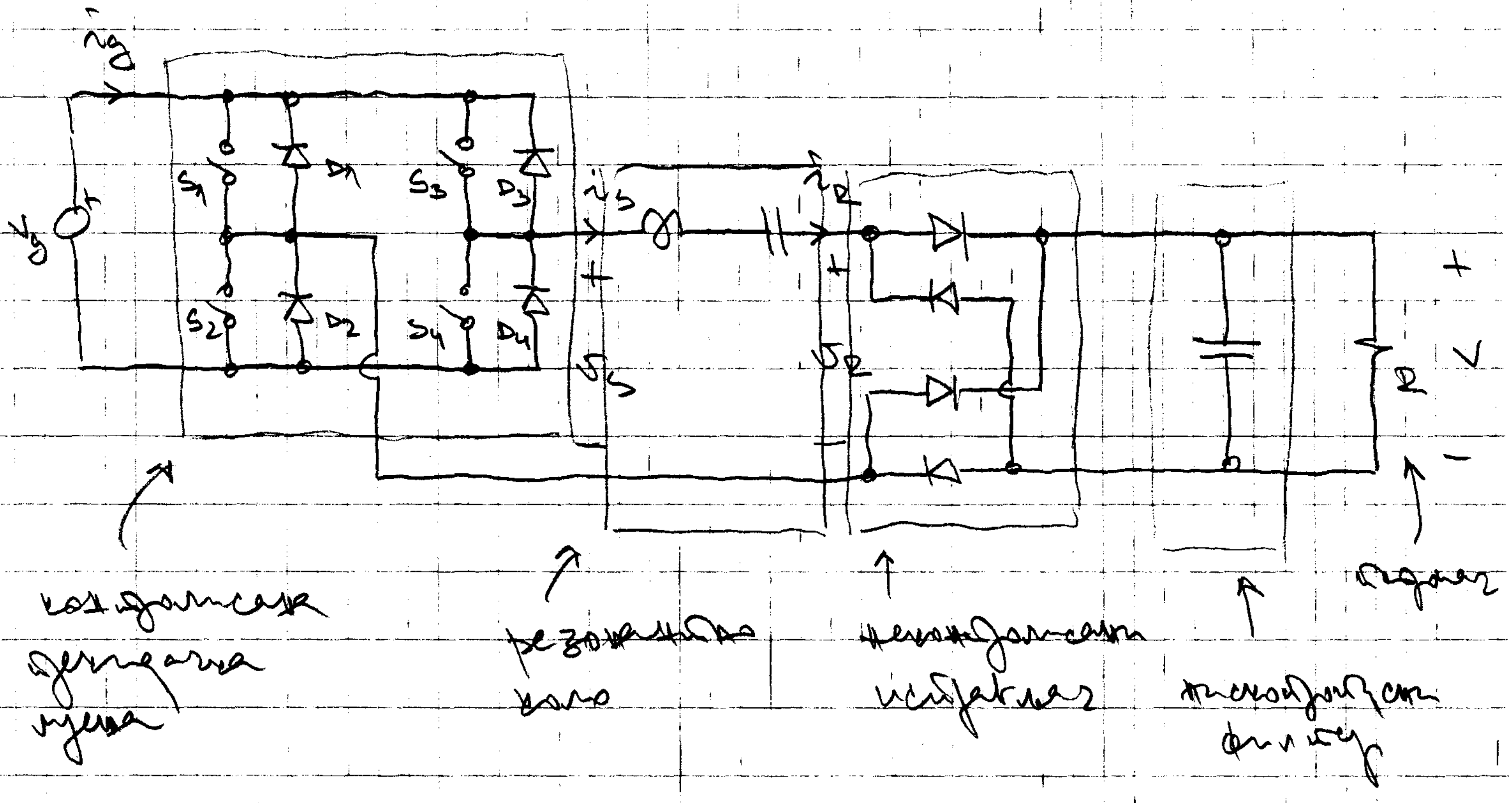
- linear ripple approximation не може да се примени

1) Синусоидална Апроксимација

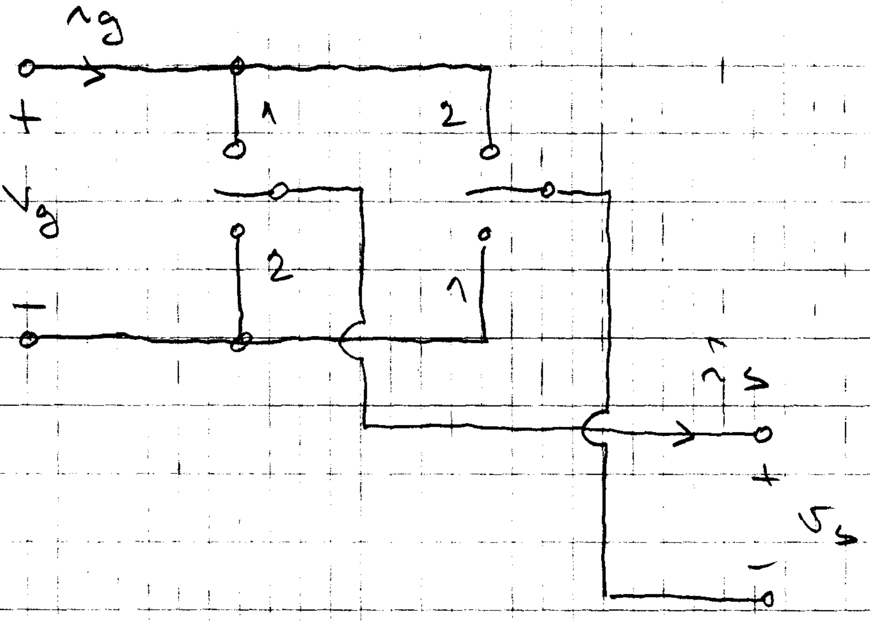
2) Анализа у фазној равни

СИНУСОИДАЛНА АПРОКСИМАЦИЈА

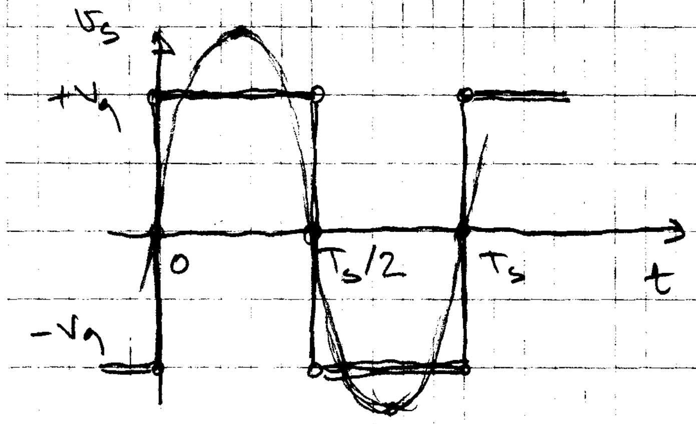
- мање-више одоке теорија
- дуго била једина
- корисна за брзу анализу и пројекту
- Серијски резонантни конвертор



1) Контролисана прекидачка мрежа



- Нека се управља са $D = 0.5$, потпуно контролисана стања



стање ①: $v_s = V_g$ $\hat{i}_s = \hat{i}_g$

стање ②: $v_s = -V_g$ $\hat{i}_s = -\hat{i}_g$

$$v_s(t) = V_g \frac{4}{\pi} \sum_{n=1,3,5..}^{+\infty} \frac{1}{n} \sin(n\omega_s t)$$

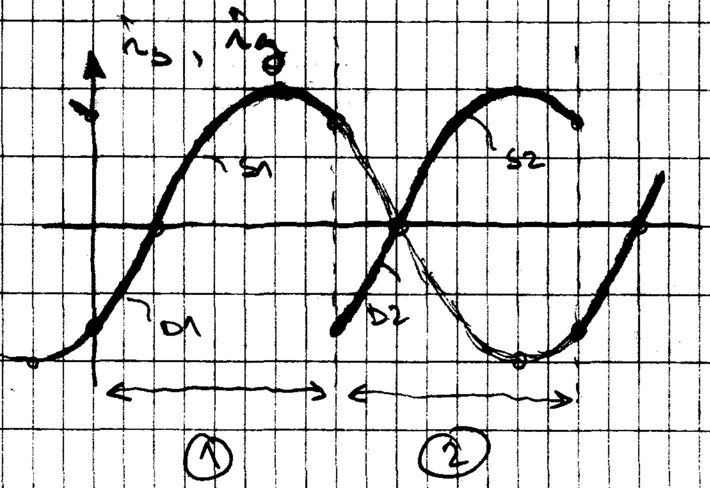
$$\omega_s = \frac{2\pi}{T_s} = 2\pi f_s$$

$$\boxed{v_{s1}(t) = \frac{4}{\pi} V_g \sin\omega_s t}$$

- филтерска својства резонантног кола, велик Q-фактор, $v_s(t) \cong v_{s1}(t)$ - sin app

- сада је побуда простопериодична, претпоставка

$$i_s(t) \cong I_{s1} \sin(\omega_s t - \varphi_s)$$



Прелаз проводења D1-S1 - zero voltage and current turn-on

S1-D2 - hard turn off

current lagging, operation above resonance

у другом је zero-voltage turn off,
current leading, operation below resonance

– може додати, о њој ће бити реч касније

– i_g је периодично са $\frac{T_s}{2}$

$$\bar{i}_g = \frac{1}{\frac{T_s}{2}} \int_0^{T_s/2} i_g(t)\,dt =$$

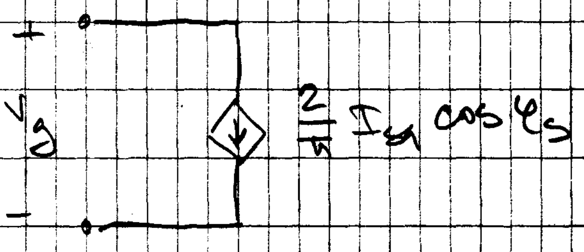
$$= \frac{2}{T_s} \int_0^{T_s/2} I_{s1} \sin(\omega_s t - \varphi_s)\,dt =$$

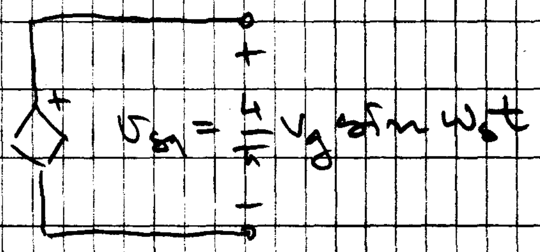
$$= \frac{2}{\pi} I_{s1} \cos \varphi_s$$

$$\boxed{\bar{i}_g = \frac{2}{\pi} I_{s1} \cos \varphi_s}$$

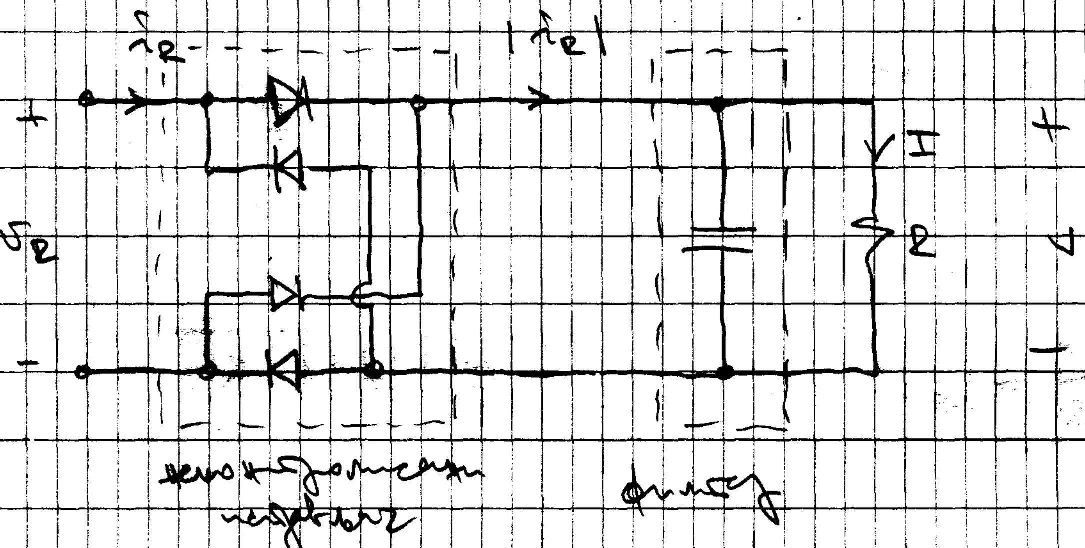
$$\boxed{v_s \cong \frac{4}{\pi} V_g \sin \omega_s t}$$

еквивалентно коло:

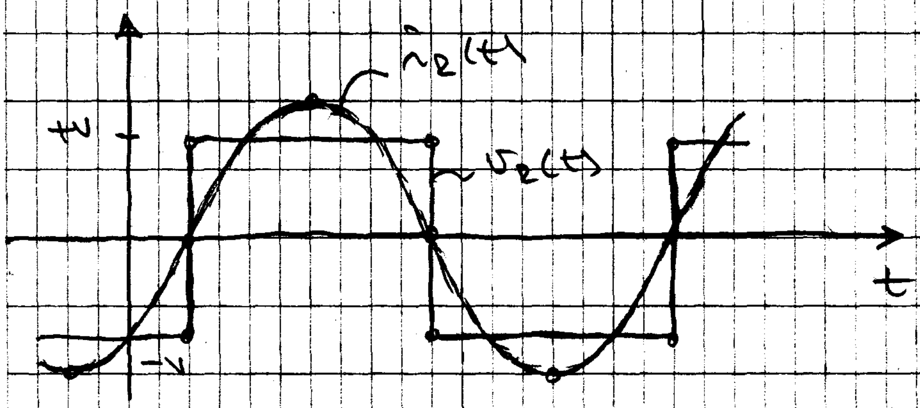




2) исправљач са капацитивним филтром



$i_R(t) = I_{R1} \sin(\omega_s t - \varphi_R)$; $v \cong const$



$$v_R(t) = \frac{4}{\pi} V \sum_{n=1,3,5\ldots}^{\infty} \frac{1}{n}\sin(n\omega_s t - \varphi_R)$$

– синусоидална апроксимација

$$v_R(t) \cong v_{R1}(t) = \frac{4}{\pi} V \sin(\omega_s t - \varphi_R)$$

$$i_R(t) = I_{R1} \sin(\omega_s t - \varphi_R)$$

~ емулација отпорности

$$\frac{v_R}{i_R} = \frac{\frac{4}{\pi}V}{I_{R1}}$$

$I_{R1}(I) = ?$

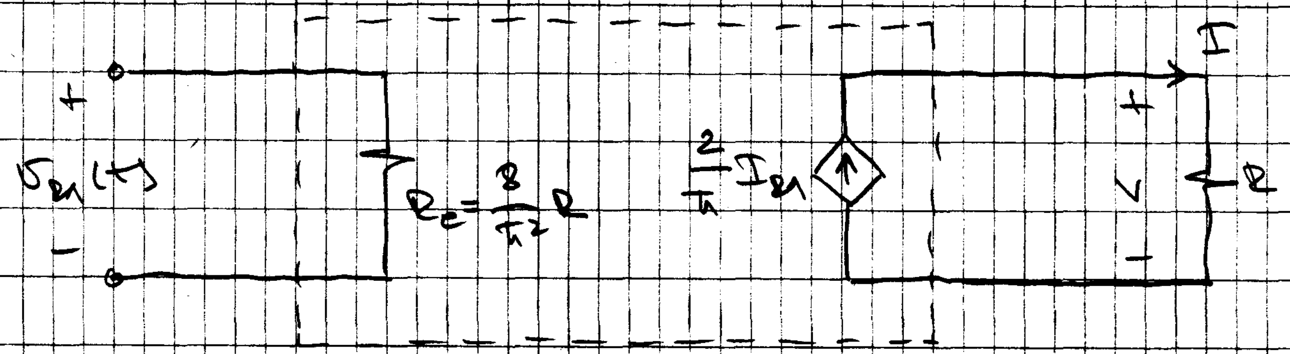
$$I = \overline{|i_R|} = \frac{2}{\pi} I_{R1} \rightarrow I_{R1} = \frac{\pi}{2} I$$

$$\frac{v_R}{i_R} = R_e = \frac{\frac{4}{\pi}V}{\frac{\pi}{2}I} = \frac{8}{\pi^2}\frac{V}{I} = \frac{8}{\pi^2}R$$

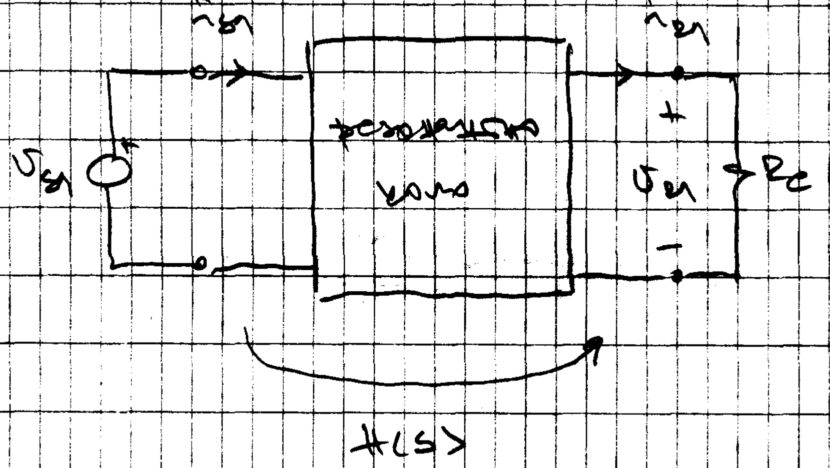
$$\boxed{\frac{v_R}{i_R} = R_e = \frac{8}{\pi^2}R}$$

$$\boxed{I = \frac{2}{\pi} I_{R1}}$$

еквивалентно коло исправљача са филтром



3) Резонантно коло



– стандардно линеарно коло

$$\boxed{\frac{V_{R1}(s)}{V_{s1}(s)} = H(s)}$$ – дефиниција $H(s)$

$$\frac{\text{амплитуда } v_{R1}}{\text{амплитуда } v_{s1}} = \left| H(s) \right| \Big|_{s = j\omega_s}$$

- овде је суштина апроксимације: ако полови H(s) имају висок Q - фактор - све је океј

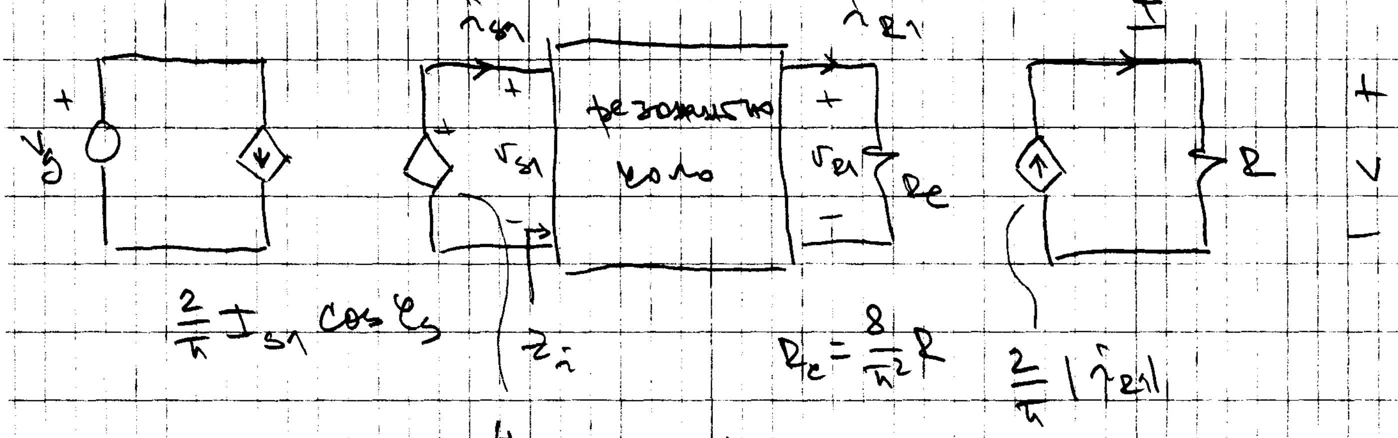
$$I_R(s) = \frac{V_{R1}(s)}{R_e} = \frac{H(s)}{R_e} V_{s1}(s)$$

$$I_{R1} = \frac{|H(j\omega_s)|}{R_e} \cdot V_{s1}$$ амплитуда

амплитуда (peak magnitude)

Преносни однос (conversion ratio)

V / V_g



$$v_{s1} = \frac{4}{\pi} V_g \sin \omega_s t$$

$$\frac{V}{V_g} = R \cdot \frac{2}{\pi} \cdot \frac{1}{R_e} \cdot |H(j\omega_s)| \cdot \frac{4}{\pi}$$

$$\frac{V}{I} \cdot \frac{I}{|\hat{i}_{R1}|} \cdot \frac{|\hat{i}_{R1}|}{|v_{R1}|} \cdot \frac{|v_{R1}|}{|v_{s1}|} \cdot \frac{|v_{s1}|}{V_g}$$

$$\frac{V}{V_g} = \cancel{8} \cdot \frac{\cancel{2}}{\cancel{\pi}} \cdot \frac{\cancel{\pi^2}}{\cancel{8R}} \cdot \left| H(j\omega_s) \right| \cdot \frac{\cancel{4}}{\cancel{\pi}}$$

$$\boxed{\frac{V}{V_g} = \left| H(j\omega_s) \right|}$$

- Ефикасност

$$P_{in} = V_g I_g = V_g \frac{2}{\pi} I_{s1} \cos\varphi_s$$

$$I_{s1}(s) = \frac{V_{s1}(s)}{Z_i(s)} = Y_i(s) V_{s1}(s)$$

$$I_{s1} \cos\varphi_s = Re(I_{s1}(j\omega_s))$$

$$Re(I_{s1}) = V_{s1} Re(Y_i(j\omega_s))$$

$$Re(I_{s1}) = \frac{4}{\pi} V_g Re(Y_i(j\omega_s))$$

$$\boxed{P_{in} = \frac{8}{\pi^2} V_g^2 Re(Y_i(j\omega_s))}$$

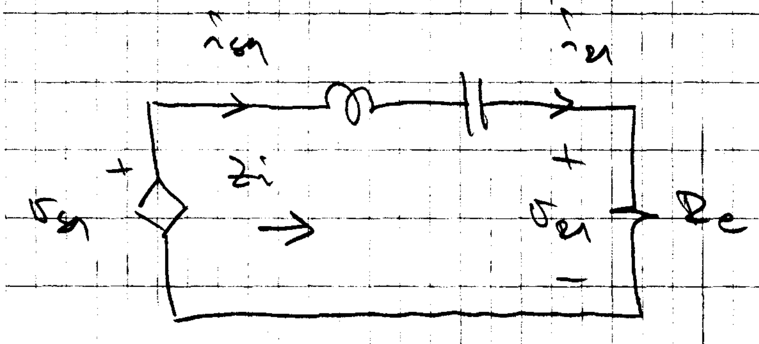
$$P_{out} = I V = \frac{V_{e1}^2}{2 R_e}$$

$$V_{e1}^2 = V_{s1}^2 \left| H(j\omega_s) \right|^2$$

$$P_{out} = \left| H(j\omega_s) \right|^2 \frac{V_{s1}^2}{2 R_e} = \left| H(j\omega_s) \right|^2 \frac{n^2 V_g^2}{8 R}$$

$$\boxed{\eta = \frac{P_{out}}{P_{in}} = \frac{\left| H(j\omega_s) \right|^2}{R_e \, Re(Y_i(j\omega_s))}}$$

Серијски резонантни конвертор



$$H(s) = \frac{R_e}{Z_i(s)} = \frac{R_e}{R_e + sL + \frac{1}{sC}}$$

$$H(s) = \frac{\frac{s}{Q_e \omega_0}}{1 + \frac{s}{Q_e \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}}$$

$$R_0 = \sqrt{\frac{L}{C}}$$

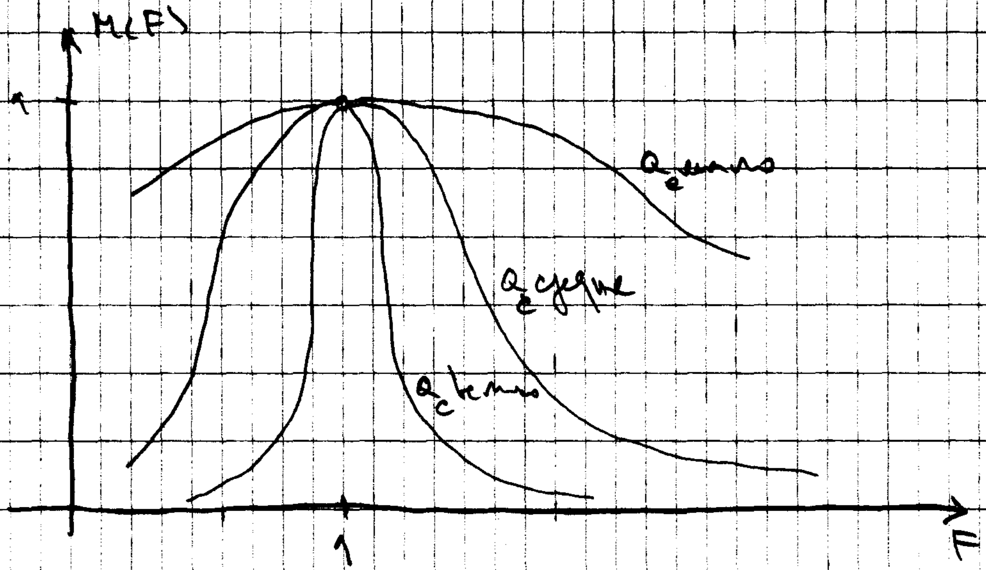
$$Q_e = \frac{R_0}{R_e}$$

DC conversion ratio $M = V / V_g$

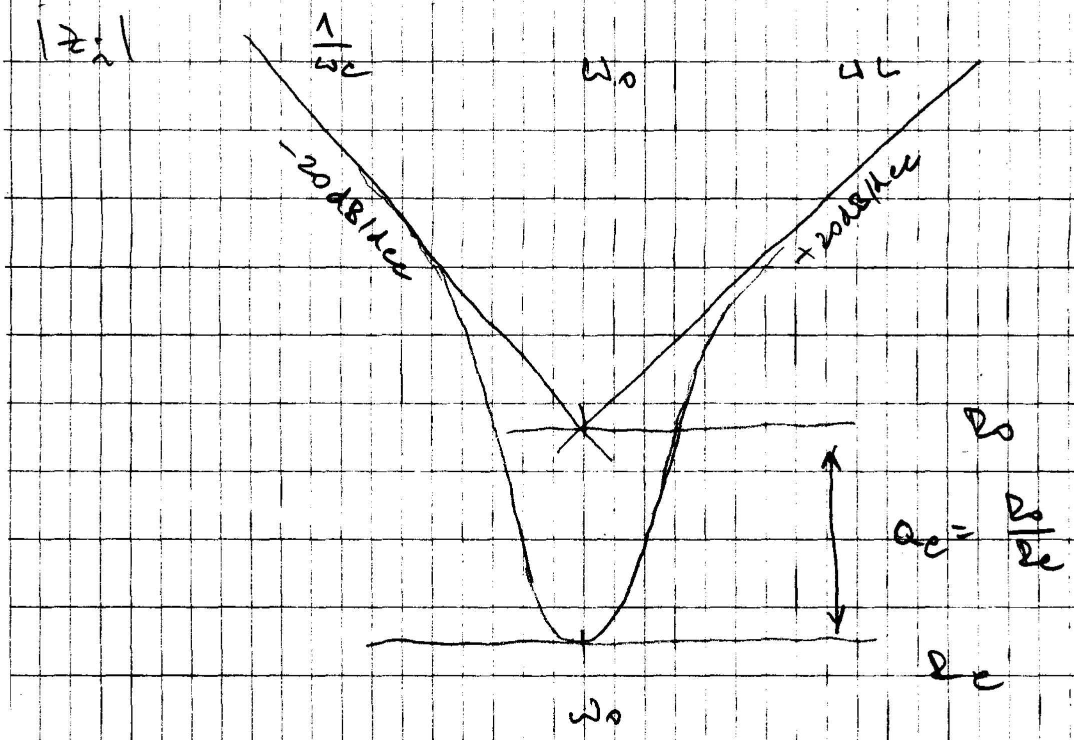
$$\boxed{M = |H(j\omega_s)| = \frac{1}{\sqrt{1 + Q_e^2\left(\frac{1}{F} - F\right)^2}}}$$

$\boxed{F = \frac{f_s}{f_0}}$ – нормализована учестаност прекидања

– контрола – варира се f_s, тиме и F



- цртање Бодеових карактеристика за Z_i и H



$Z_i = sL + \frac{1}{sC} + R_e$

$\omega = \omega_0$:

$\omega_0 L = \frac{1}{\omega_0 C} = R_0$

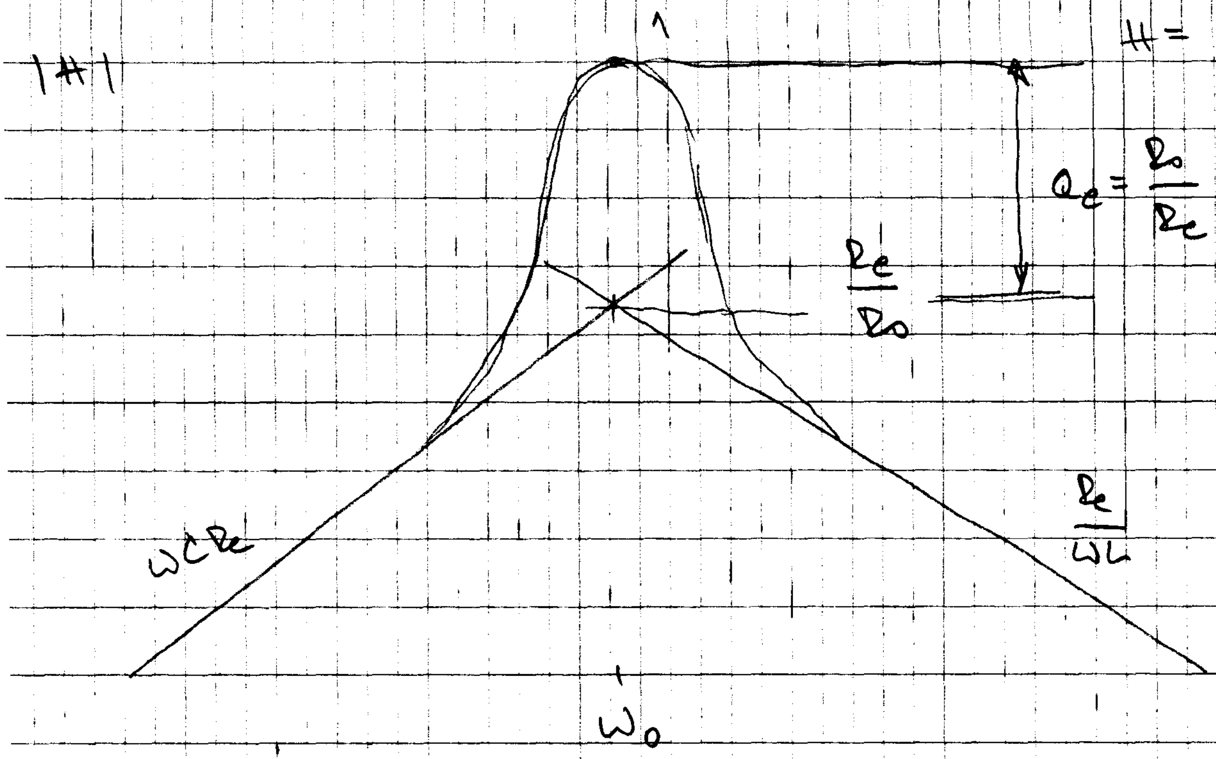
$\omega_0 = \frac{1}{\sqrt{LC}}$

$R_0 = \sqrt{\frac{L}{C}}$

$Z(\omega_0) = R_e$!

за функцију хармоника $\tilde{i}_s$

$H = \frac{R_e}{Z_i}$



- када је синусоидална апроксимација добра?

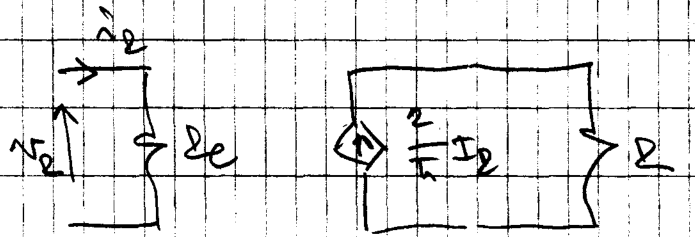
1) Q релативно високо (преко 1)

2) f_s је у близини f_0

Анализа Ефикасности

- модели су недисипативни

- исправљач



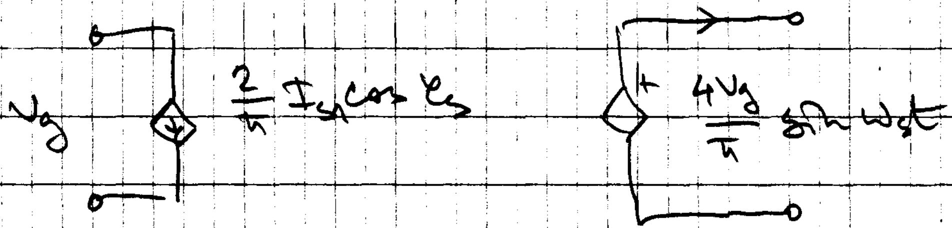
$R_e = \frac{8}{\pi^2} R$

$P_{in} = \frac{1}{2} I_2^2 R_e = \frac{1}{2} I_2^2 \frac{8}{\pi^2} R = \frac{4}{\pi^2} R I_2^2$

$P_{out} = R \left(\frac{2}{\pi} I_2 \right)^2 = \frac{4}{\pi^2} R I_2^2$

- инвертор

$i_{s1} = I_{s1} \cos(\omega_s t - \varphi_s)$



$$P_{in} = V_g \frac{2}{\pi} I_{s1} \cos\varphi_s$$

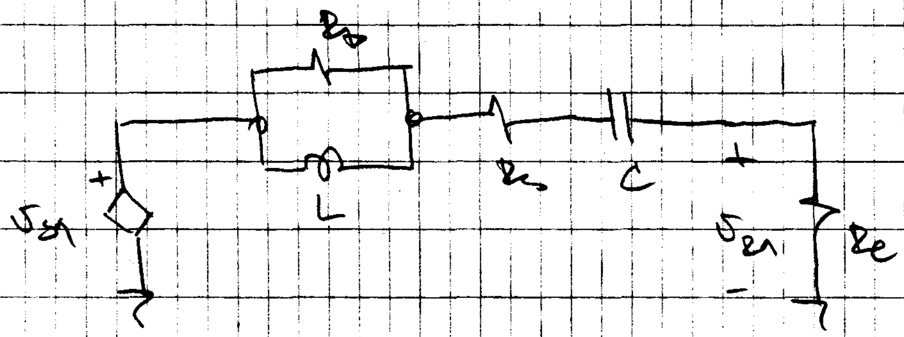
$$P_{out} = \frac{1}{2} \frac{4}{\pi} V_g I_{s1} \cos\varphi_s = \frac{2}{\pi} V_g I_{s1} \cos\varphi_s$$

$$P_{in} = P_{out}$$

- Може се применити за анализу ефикасности

R_p - губици у језгру

R_s - губици у намотајима



Уз доста рачунања

$$H(s) = \frac{\frac{s}{\omega_e}\left(1 + \frac{s}{\omega_p}\right)}{1 + \frac{s}{Q_e \omega_0} + \left(\frac{s}{\omega_0}\right)^2}$$

$$\omega_0 = \frac{1}{\sqrt{LC \frac{R_p + R_s + R_e}{R_p}}}$$

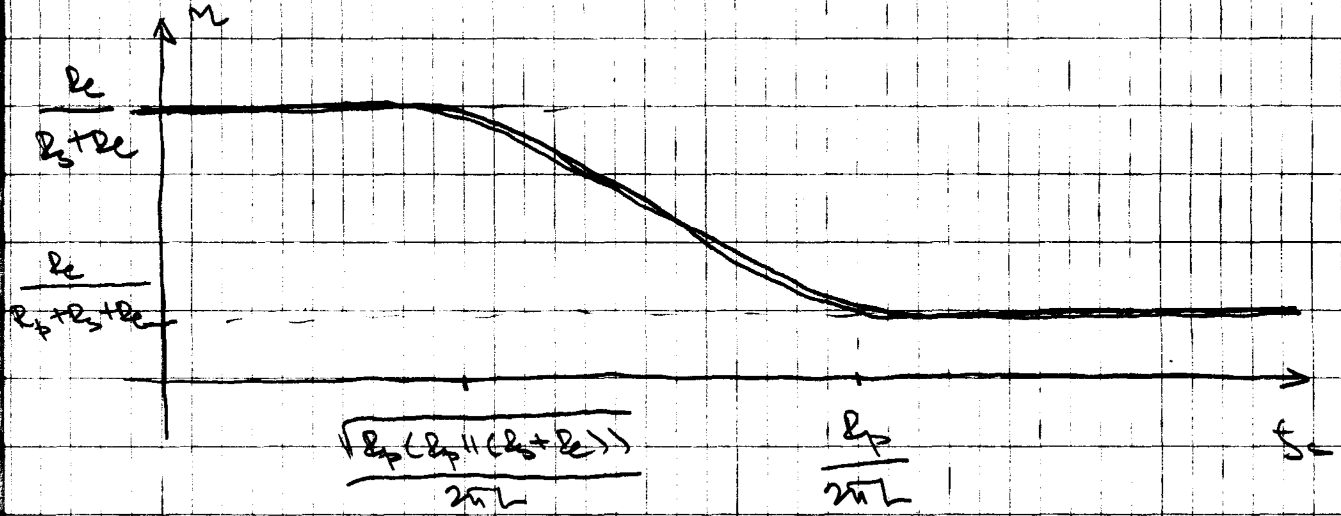
$$\omega_e = \frac{1}{CR_e} \qquad \omega_p = \frac{R_p}{L}$$

$$\frac{1}{Q_e} = \frac{R_0}{R_p} + \frac{R_e+R_s}{R_0}$$

$$R_0 = \sqrt{\frac{L}{C}}\sqrt{1+\frac{R_s+R_e}{R_p}}$$

– уз рачунање

$$\eta = \frac{R_e}{R_s+R_e}\,\frac{1+\left(\frac{L}{R_p}\right)^2\omega_s^2}{1+\frac{L^2}{R_p(R_p\|(R_s+R_e))}\omega_s^2}$$



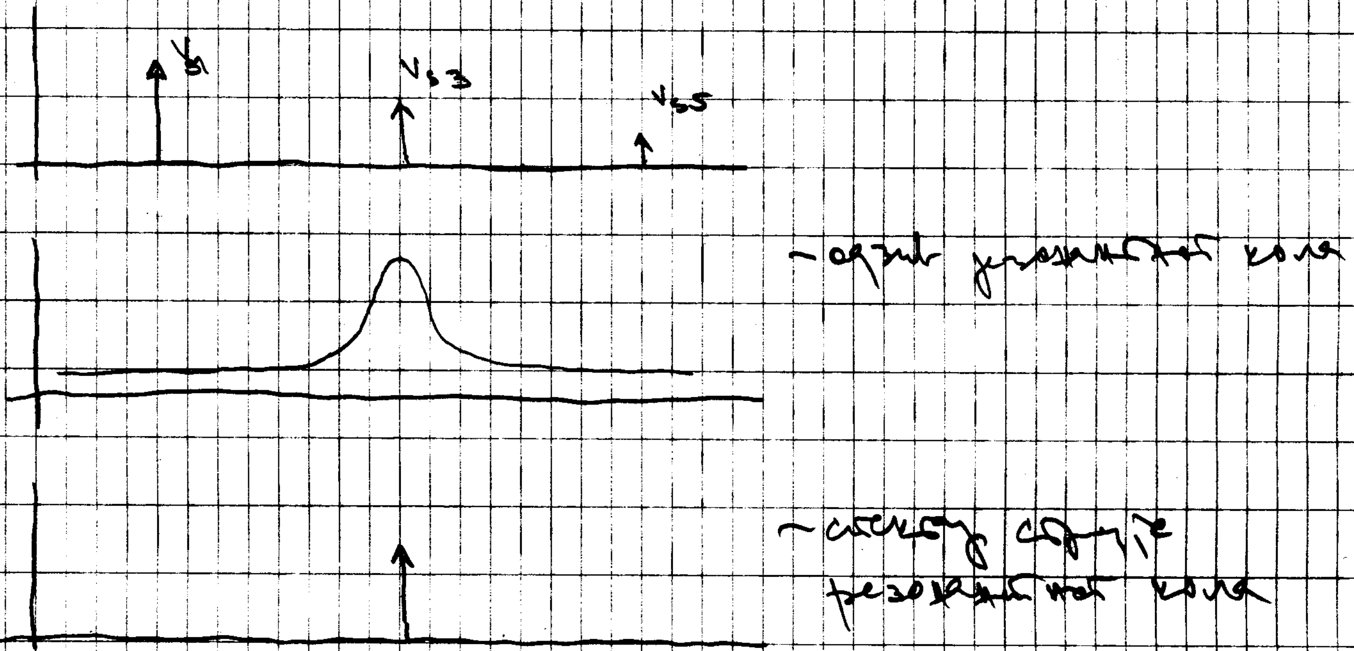
како би се избегла деградација услед губитака у језгру

$$f_s \ll \frac{1}{2\pi L}\sqrt{R_p(R_p\|(R_s+R_e))}$$

Субхармонијски режим рада серијског резонантног конвертора

- $f_0 \cong n f_s$ n – непаран број

спектар v_s

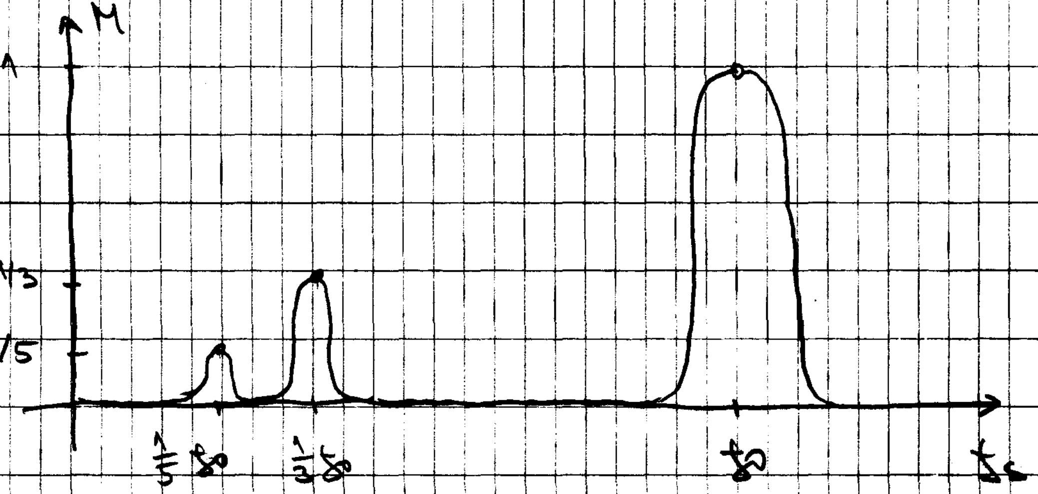


- добро када је $f_0 \cong n f_s$ и Q_e велико
- модел резонантног кола и исправљача исти
- модел инвертора се мења, амплитуда телеграфског напона се дели са n
- резултат

$$M = \frac{V}{V_g} = \frac{|H(jn\omega_s)|}{n}$$

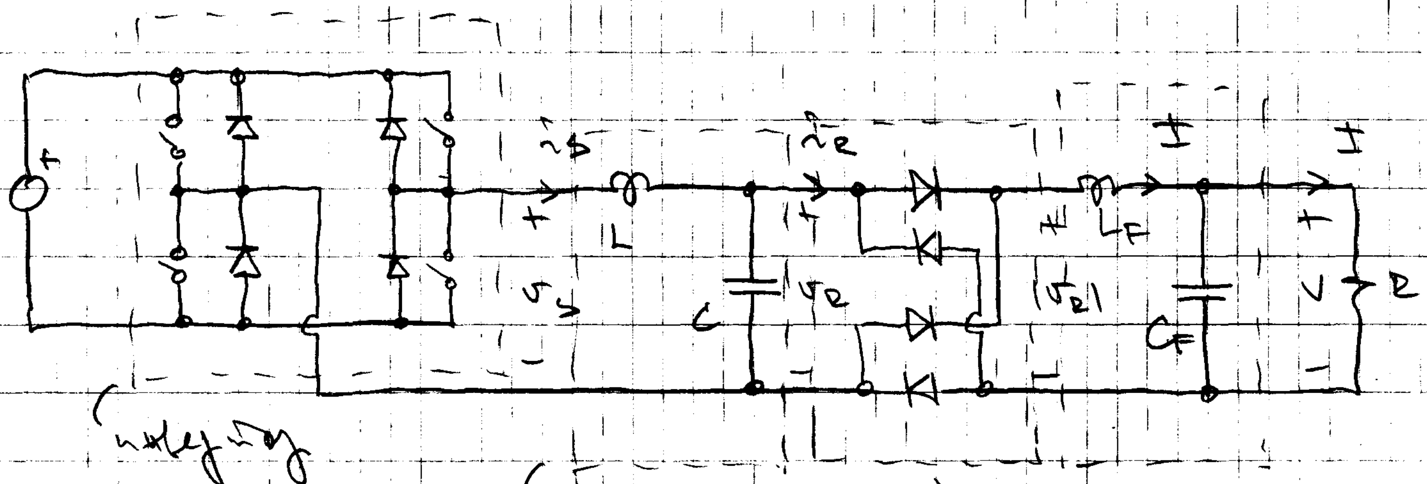
добро за $(n-1) f_s < f_0 < (n+1) f_s$

– типична преносна карактеристика



– добро за велики Q и близину $n f_s$

Паралелни резонантни конвертор



(инвертор

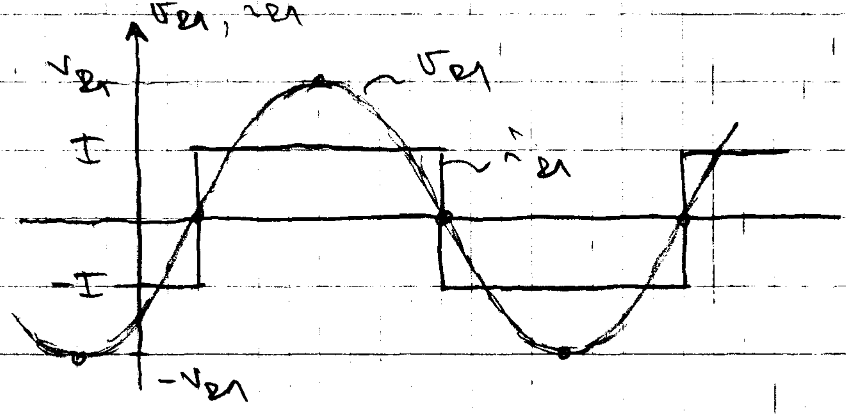
(resonant tank (осцилаторно коло)

исправљач

филтер

- инвертор ради исто као и код SRC

- исправљач другачији, current-loaded



$$v_{R1} = V_{R1}\sin(\omega_s t - \varphi_R)$$

$$i_{R1} = \frac{4I}{\pi}\sin(\omega_s t + \varphi_R)$$

(јасно како

$$\frac{v_{R1}}{i_{R1}} = \frac{\pi}{4}\frac{V_{R1}}{I}$$

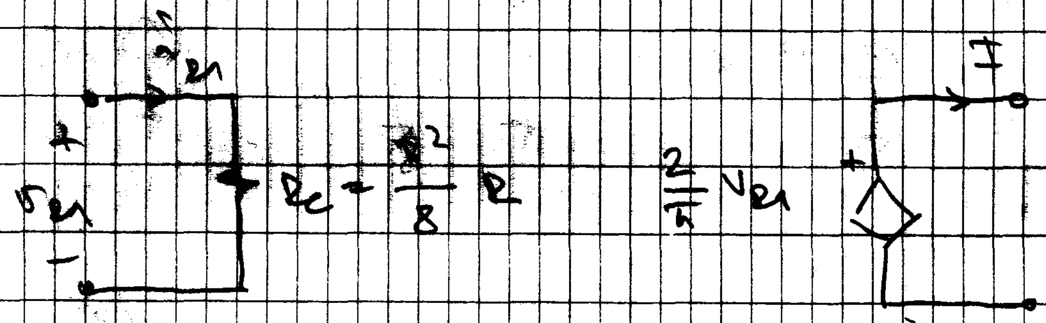
- volt-second balance на L_F

$$\overline{|v_{R1}|} = \frac{2}{\pi} V_R = V \rightarrow V_R = \frac{\pi}{2} V$$

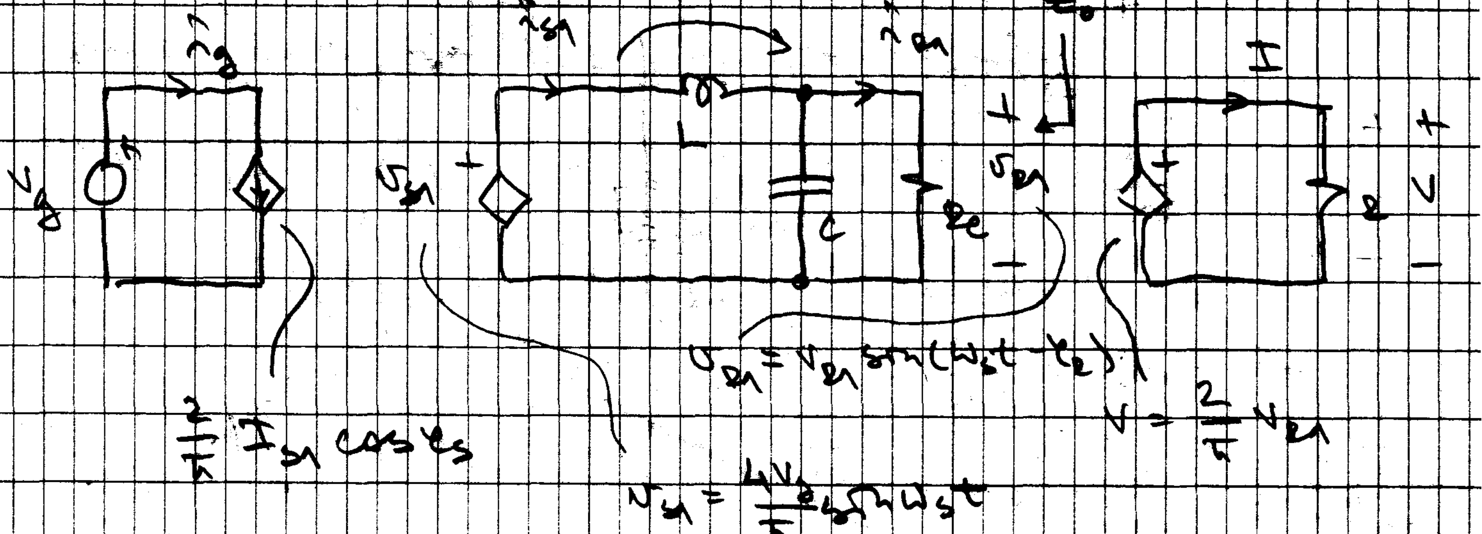
$$R_e = \frac{\pi}{4} \cdot \frac{\frac{\pi}{2} V}{I} = \frac{\pi^2}{8} R$$

$$\boxed{R_e = \frac{\pi^2}{8} R}$$

- модел исправљача, current-loaded



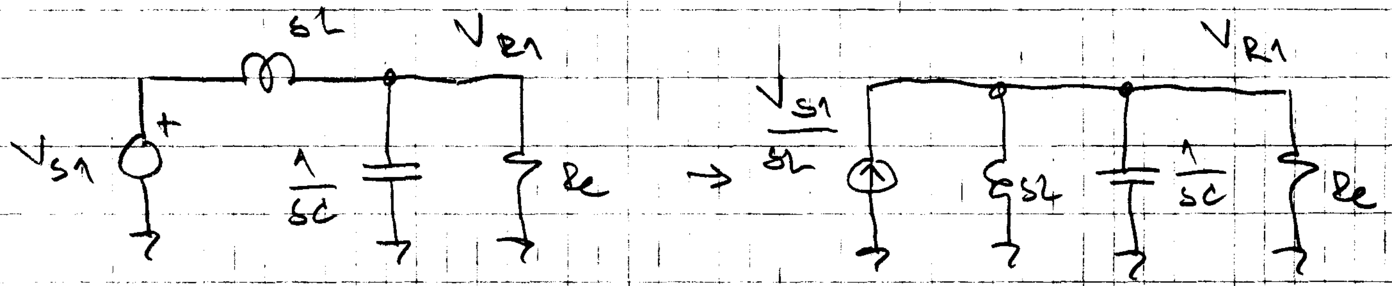
- еквивалентно коло PRC



$$M = \frac{V}{V_g} = \frac{V}{V_{R1}} \cdot \frac{V_{R1}}{V_{S1}} \cdot \frac{V_{S1}}{V_g} =$$

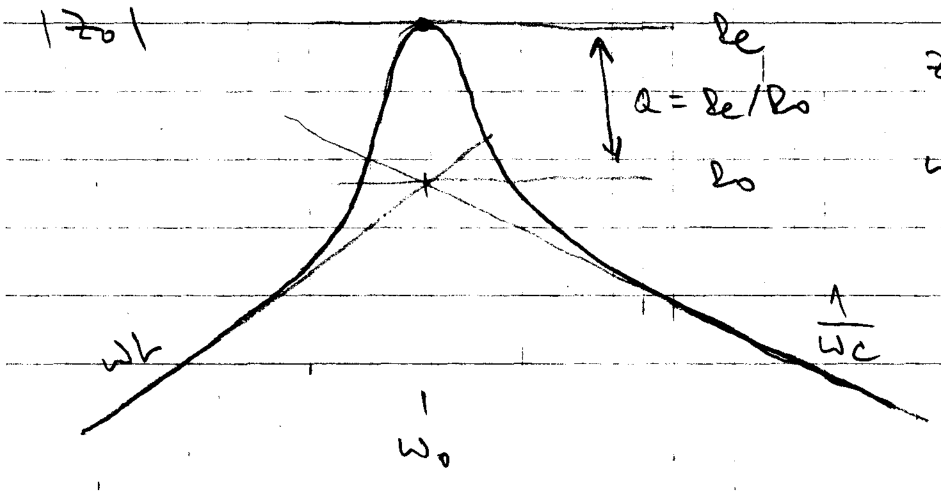
$$= \frac{2}{\pi} \cdot |H(j\omega_s)| \cdot \frac{4}{\pi}$$

$$\boxed{M = \frac{8}{\pi^2} |H(j\omega_s)|}$$



$$\frac{V_{R1}}{V_{S1}} = H(s) = \frac{Z_0}{sL}, \quad Z_0 = sL \,\|\, \frac{1}{sC} \,\|\, R_e$$

Bode plots, za δ(ω) [illegible]



$$Z_0 = sL \,\|\, \frac{1}{sC} \,\|\, R_e$$

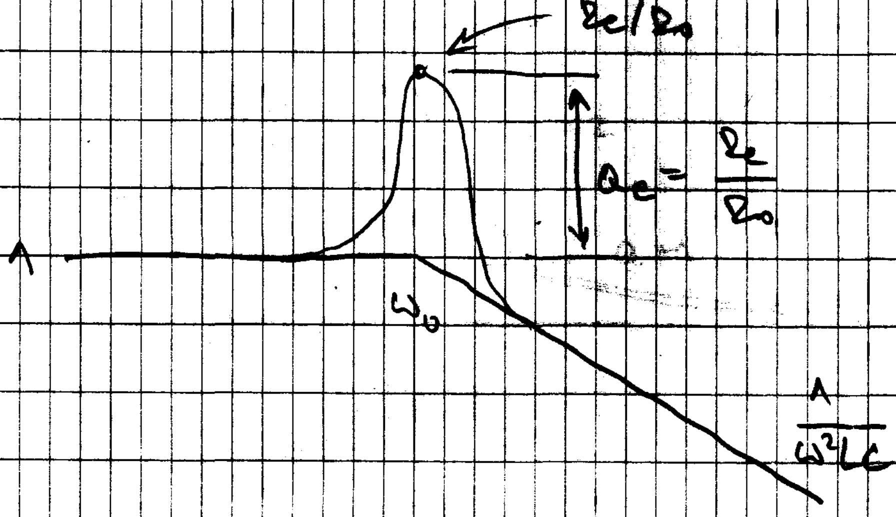
$\omega = \omega_0$:

$$\frac{1}{\omega_0 C} = \omega_0 L = R_0$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad R_0 = \sqrt{\frac{L}{C}}$$

$$Z_0(j\omega_0) = R_e$$

$$H(j\omega) = \frac{Z_0(j\omega)}{j\omega L}$$



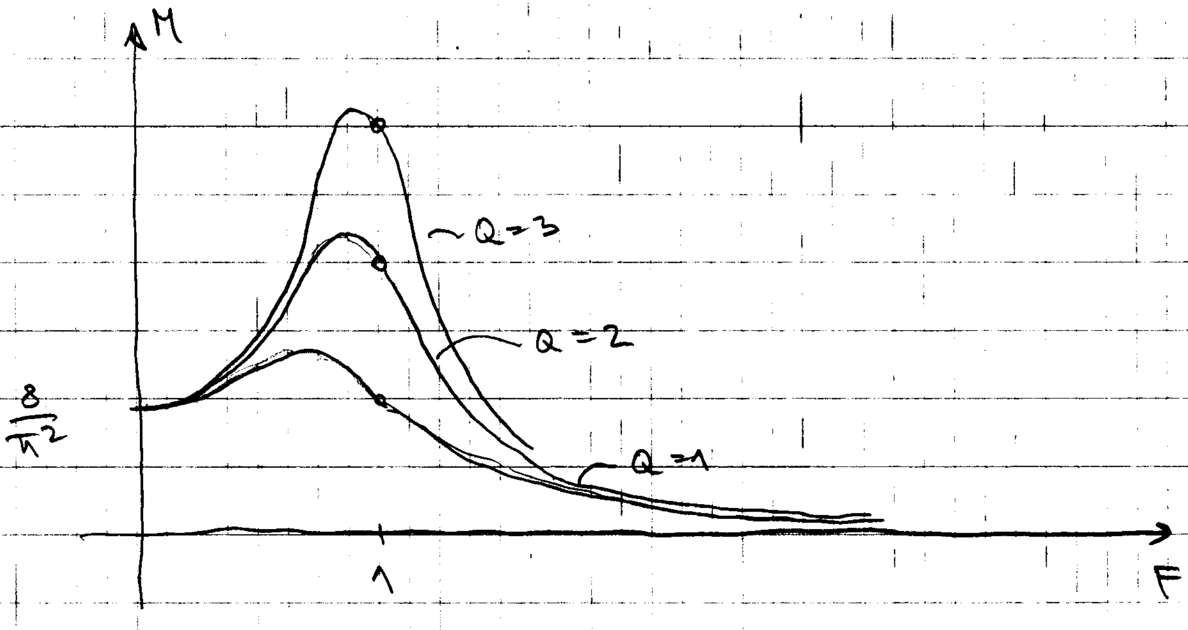
- овим, апроксимација је добра у околини ω_0

$$M = \frac{8}{\pi^2}\left|\frac{1}{1+\frac{s}{Q_e\omega_0}+\left(\frac{s}{\omega_0}\right)^2}\right|_{s=j\omega_s}$$

$$\boxed{M = \frac{8}{\pi^2}\frac{1}{\sqrt{(1-F^2)^2+\left(\frac{F}{Q_e}\right)^2}}}$$ ~ као "boost"

С овим контролом варирамо f_s, т.ј. F

$$F = \frac{f_s}{f_0}$$ — нормализована учестаност



Услови комутације прекидача

– за оба конвертора

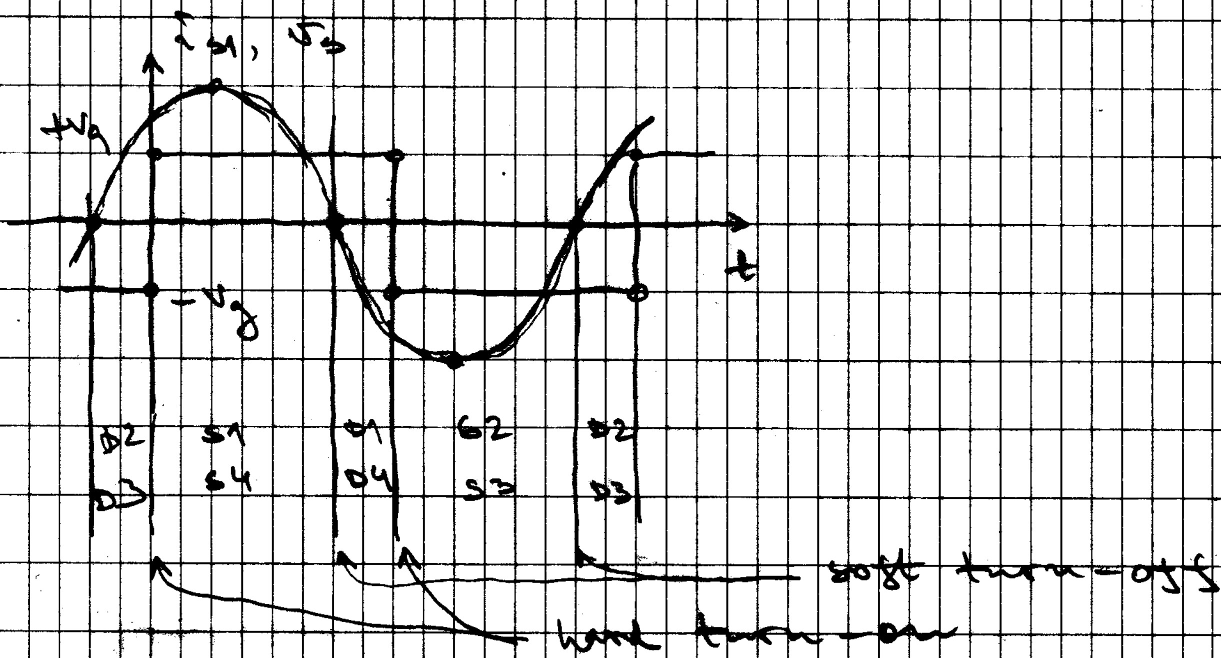
1) испод резонантне учестаности струја предњачи (С доминантно)

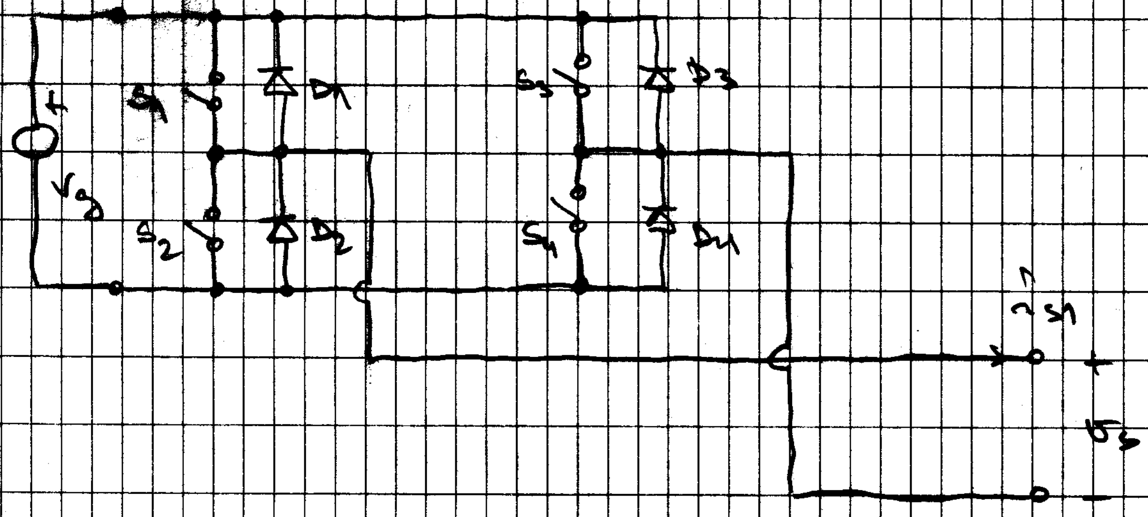
2) изнад резонантне учестаности струја касни, L

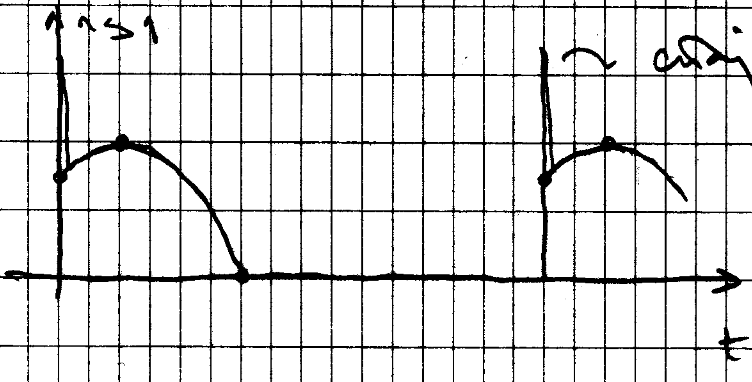
above/below resonance

lead/lagg

1) $f_s < f_0$, подрезонантни случај

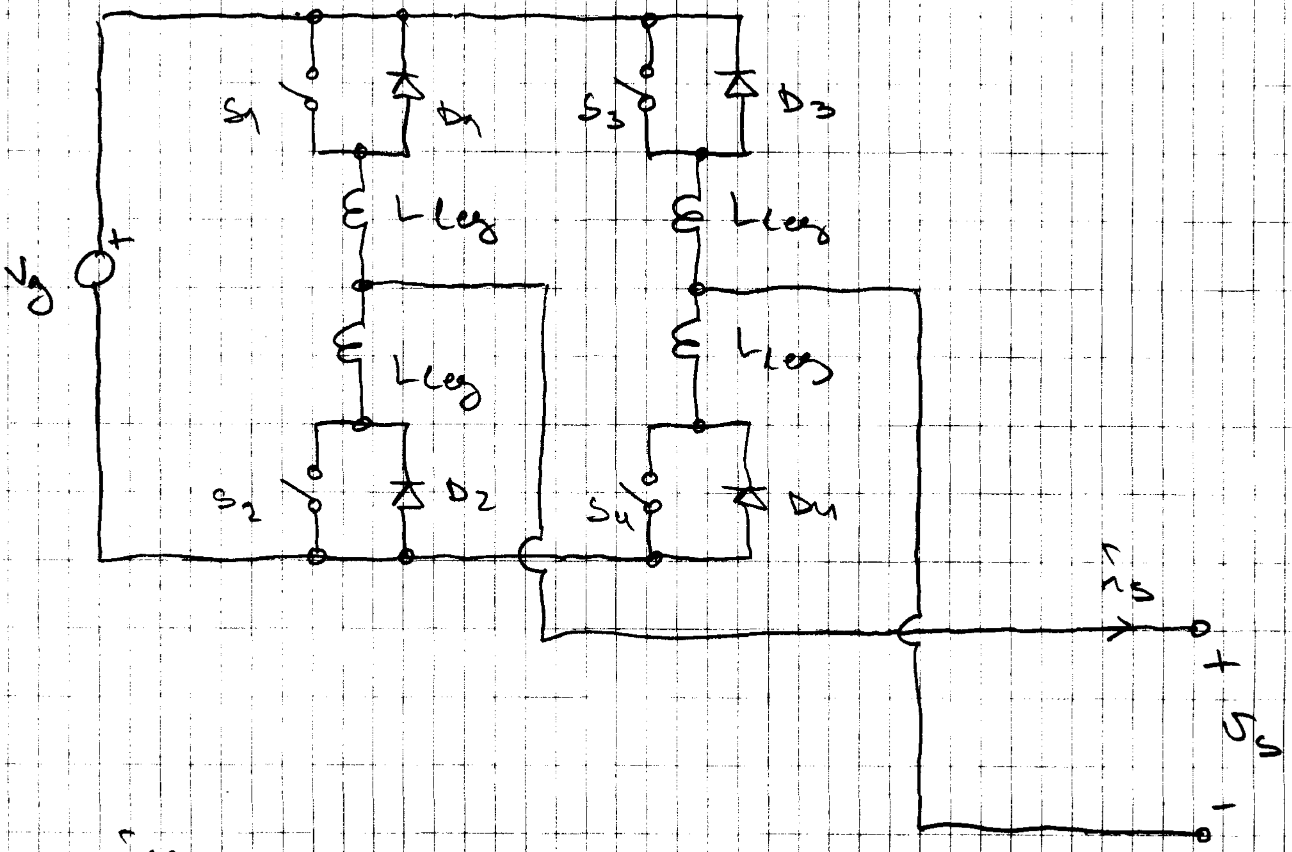


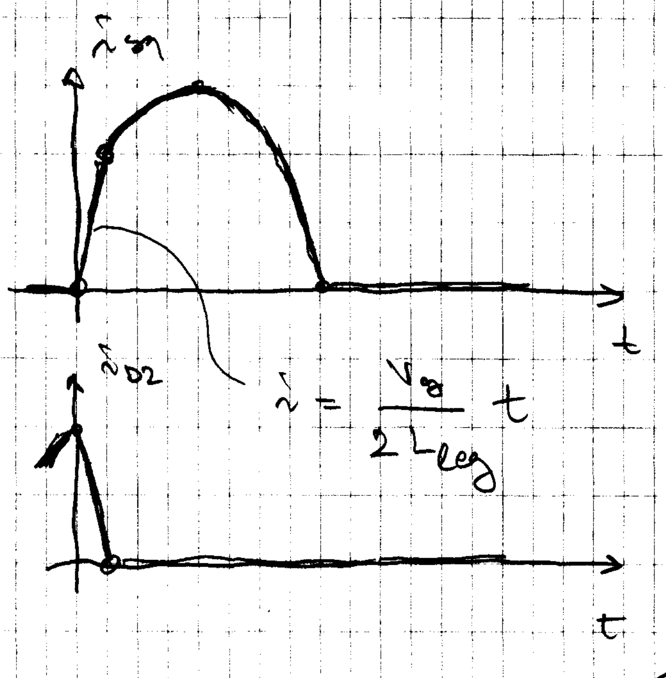




→ шиљак због нагомиланог наелектрисања у D2

– исе, мала модификација





$$i = \frac{V_g}{2L_{leg}} t$$

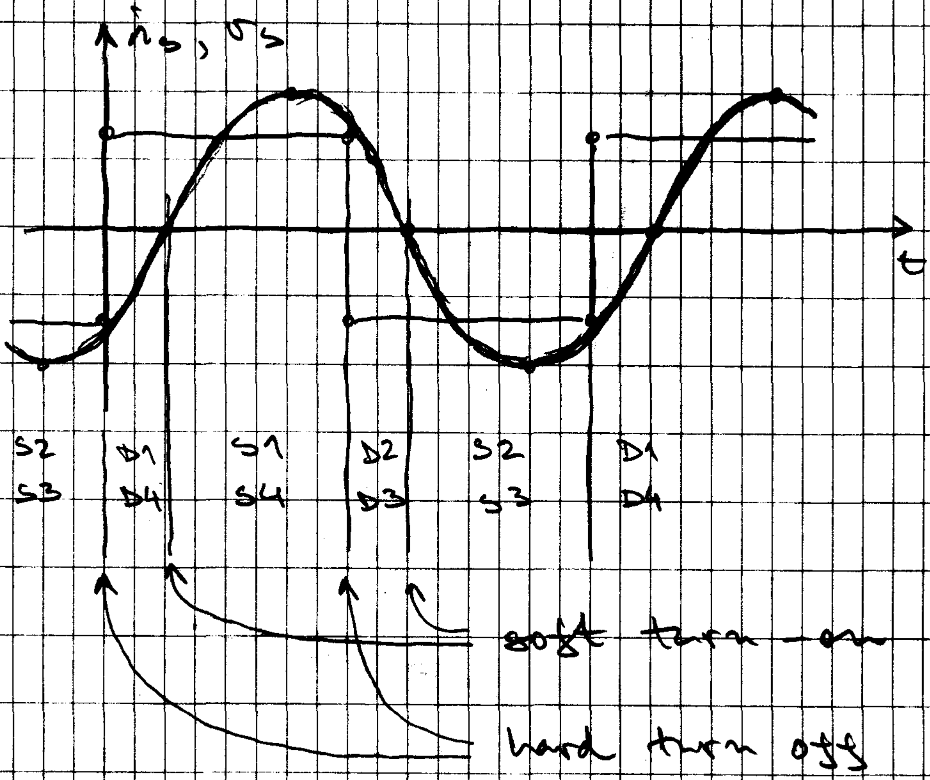
$$L_{eff} = L + 2L_{leg}$$

трајање комутационог интервала

$$\boxed{t_0 = \frac{2L_{leg}}{V_g} \cdot i_s(0)}$$

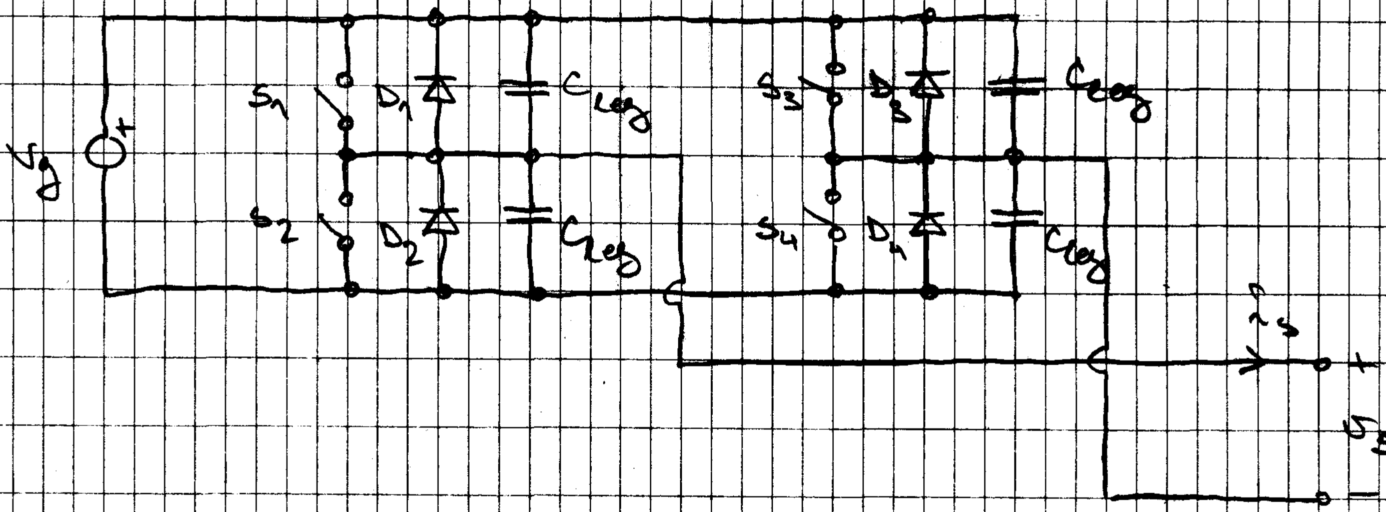
на основу овог се бира L_{leg}

2). $f_s > f_0$, [illegible] [illegible]



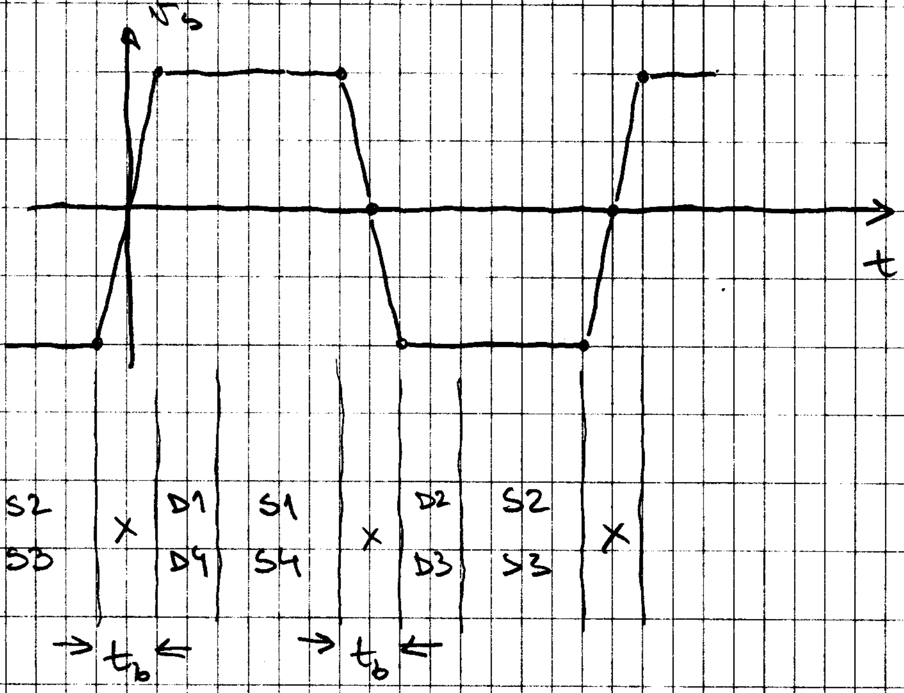
i_s, v_s
t
S2
S3
D1
D4
S1
S4
D2
D3
S2
S3
D1
D4
soft turn-on
hard turn off

i_{S1}
t

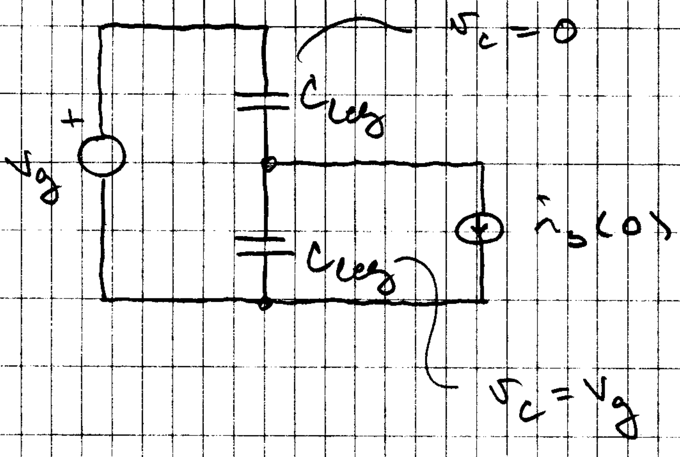


v_g
+
S_1
D_1
C_{leg}
S_2
D_2
C_{leg}
S_3
D_3
C_{leg}
S_4
D_4
C_{leg}
i_s
+
v_s
−

- када се гаси S1 S2 ~~не~~ сме одмах да се пали



трајање прелазног периода



$$\frac{dv_c}{dt} = \frac{1}{C_{leg}} \frac{1}{2} |i_s(0)|$$

$$\frac{\Delta v_c}{\Delta t} = \frac{V_g}{t_b} = \frac{1}{2C_{leg}} |i_s(0)$$

$$\boxed{t_b = \frac{2C_{leg}V_g}{|i_s(0)|}}$$

Егзактна анализа резонантних конвертора

- базирана на phase-plane - у
- једноставна, али компликоване једначине

Одзив LC кола у фазној равни

1. Нормализација и нотација

- суштина: поједноставити геометрију

V_{base} - бира се произвољно, што је најзгодније у нашем случају

$Z_{base} = Z_0 = \sqrt{\frac{L}{C}}$ - карактеристична импеданса LC кола

$V_{base} = V_g$ - усвајамо

$I_{base} = V_{base} / Z_{base} = V_g / Z_0$

$P_{base} = V_{base} I_{base} = \frac{V_g^2}{Z_0}$

$M = \frac{V}{V_g}$ - нормализовани излазни напон, dc conversion ratio

$m_C(t) = \frac{v_C(t)}{V_g}$ – нормализован напон резонантног кондензатора

$J = \frac{I}{I_{base}} = \frac{R_0 I}{V_g}$ – нормализована струја потрошача

$j_L(t) = \frac{i_L(t)}{I_{base}} = \frac{R_0 i_L(t)}{V_g}$ – нормализована струја резонантног калема

$f_{base} = f_0 = \frac{1}{2\pi\sqrt{LC}}$ – резонантна учестаност

$\omega_0 = \frac{1}{\sqrt{LC}}$

$F = f_s / f_0$ – нормализована учестаност прекидања

$\alpha_x = \omega_0 t_x$ – нормализовано трајање интервала t_x

$\gamma = \frac{\omega_0 T_s}{2} = \frac{\pi}{F}$ – нормализовано трајање полупериоде

$\alpha = \omega_0 t_\alpha$ – угао провођења диоде

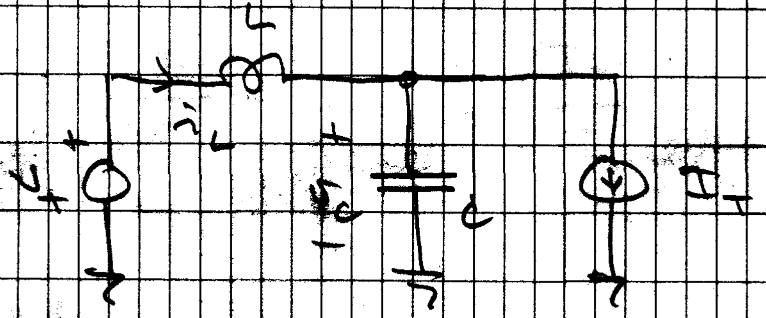
$\beta = \omega_0 t_\beta$ – угао провођења транзистора

$Q = R_0 / R$ – за серијски резонантни конвертор

$Q = R / R_0$ – за паралелни резонантни конвертор

R – actual load resistance

2. Интерпретација решења LC кола у фазној равни



$$L \frac{di_L}{dt} = V_T - v_C$$

$$C \frac{dv_C}{dt} = i_L - I_T$$

код нормализације

V_g – "неки" напон

$$\frac{L}{V_g} \frac{di_L}{dt} = \underbrace{\frac{V_T}{V_g}}_{M_T} - \underbrace{\frac{v_C}{V_g}}_{m_C}$$

$$\frac{L}{V_g} \frac{di_L}{dt} = \frac{L}{V_g} \frac{d\left(\frac{i_L}{I_{base}}\right)}{dt} \cdot I_{base} =$$

$$= \frac{L}{\cancel{V_g}} \frac{\cancel{V_g}}{r_0} \frac{dj_L}{dt} = \frac{L}{\sqrt{\frac{L}{C}}} \frac{dj_L}{dt} = \sqrt{LC} \frac{dj_L}{dt} =$$

$$= \frac{1}{\omega_0} \frac{dj_L}{dt}$$

$$\boxed{\frac{1}{\omega_0} \frac{dj_L}{dt} = M_T - m_c}$$

$$\underbrace{\frac{r_0}{V_g}}_{\frac{1}{I_{base}}} C \frac{dv_c}{dt} = \underbrace{\frac{i_L}{I_{base}}}_{j_L} - \underbrace{\frac{I_T}{I_{base}}}_{J_T}$$

$$\frac{r_0}{V_g} C \frac{dv_c}{dt} = \sqrt{\frac{L}{C}} \cdot C \frac{dm_c}{dt} = \sqrt{LC} \frac{dm_c}{dt} = \frac{1}{\omega_0} \frac{dm_c}{dt}$$

$$\boxed{\frac{1}{\omega_0} \frac{dm_c}{dt} = j_L - J_T}$$

Равнотежно решење

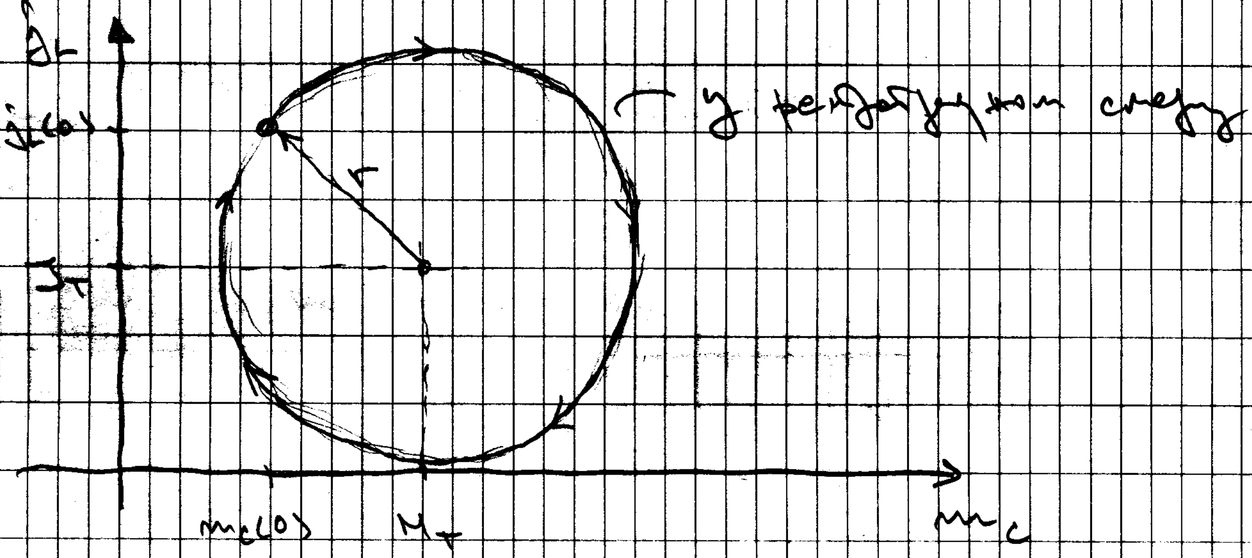
$$\boxed{j_{LP} = J_T}$$

$$\boxed{m_{cP} = M_T}$$

— тачка у phase-plane-у око које се све врти решење:

$$\boxed{\begin{aligned} m_c(t) = M_T + (m_c(0) - M_T)\cos(\omega_0 t \cancel{-\varphi}) + \\ + (\hat{j}_L(0) - J_T)\sin(\omega_0 t \cancel{-\varphi}) \end{aligned}}$$

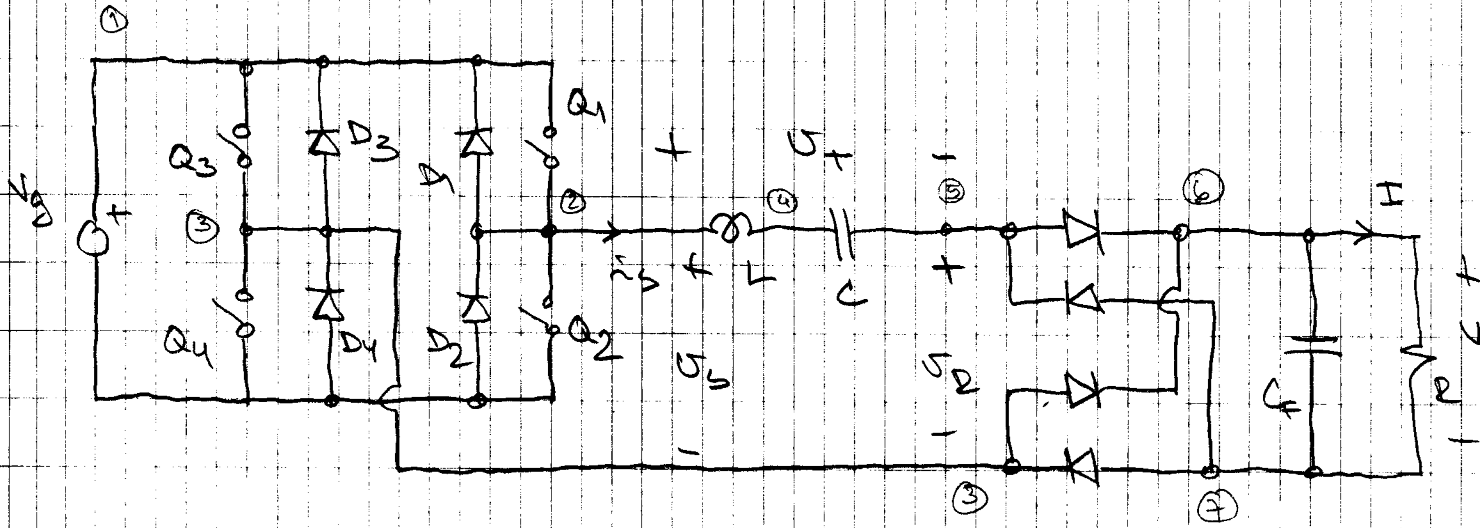
$$\boxed{\begin{aligned} \hat{j}_L(t) = J_T + (\hat{j}_L(0) - J_T)\cos(\omega_0 t \cancel{-\varphi}) - \\ - (m_c(0) - M_T)\sin(\omega_0 t \cancel{-\varphi}) \end{aligned}}$$



Γ у негативном смеру

$$\boxed{r = \sqrt{(m_c(0) - M_T)^2 + (\hat{j}_L(0) - J_T)^2}}$$

Анализа серијског резонантног конвертора

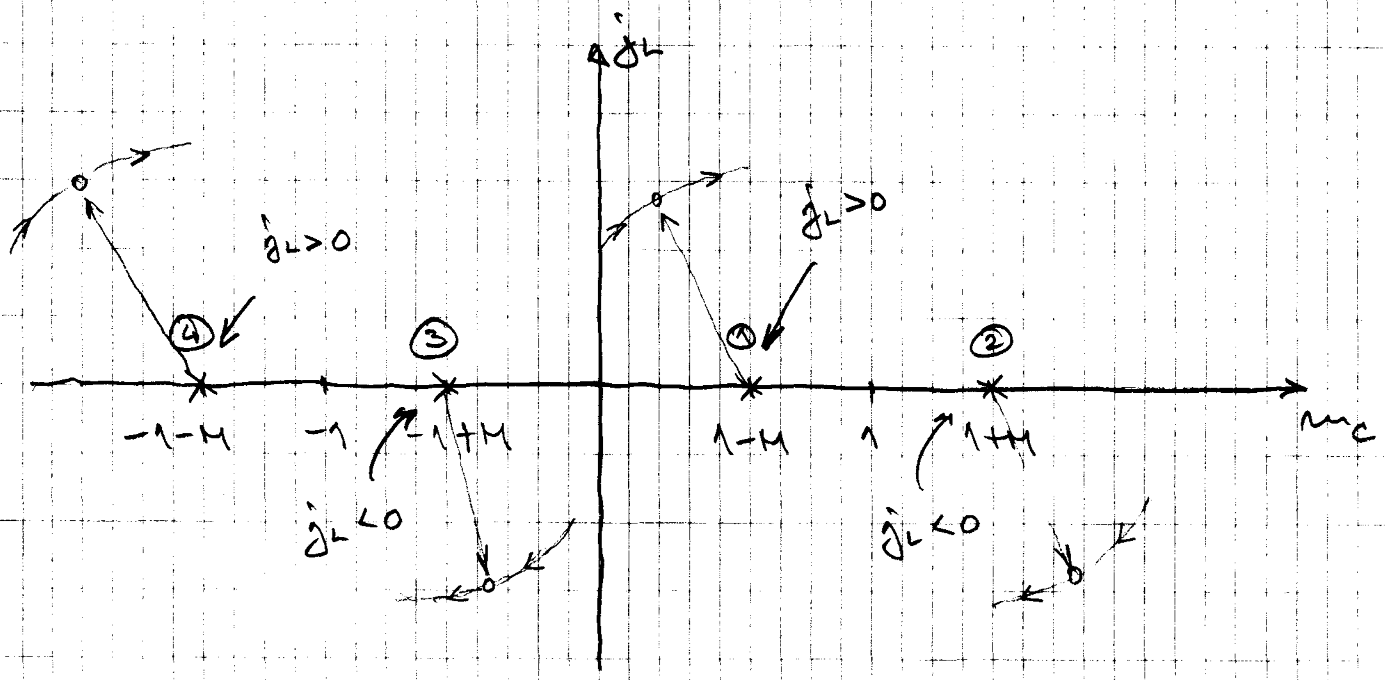


Због редног кондензатора $V_T = 0$ за било који мод рада

$\boxed{g_L}$

mode	on	D_1, D_4	D_2, D_3	i_s	v_s	v_R	M_T
1	Q_1, Q_4	off	off	+	V_g	V	$1 - M$
2	Q_1, Q_4	on	off	−	V_g	$-V$	$1 + M$
3	Q_2, Q_3	off	off	−	$-V_g$	$-V$	$-1 + M$
4	Q_2, Q_3	off	on	+	$-V_g$	V	$-1 - M$

нацртати таласне облике и скице



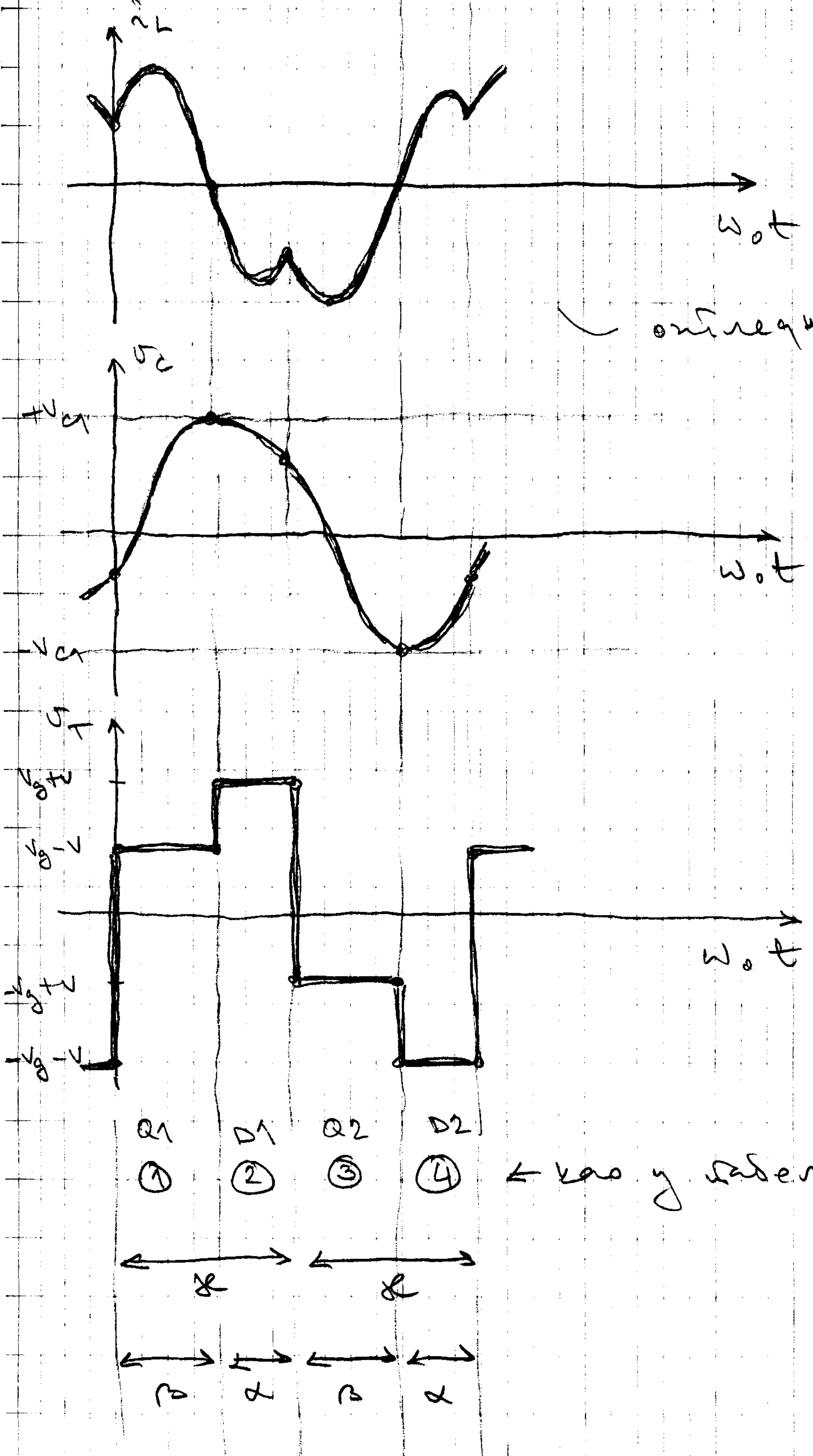
Под одређеним условима може да се деси ДЦМ (discontinuous conduction mode), х интервал, када не води. Карактеристика је

$$\boxed{\dot{j}_L = 0, \quad m_c = const}$$

Ако се користи и непотпуно компензована стална инвертора, могуће је и $v_s = 0$, па се као центри могу јави и

$\dot{j}_L > 0$	$-M$
$\dot{j}_L < 0$	M

Типични таласни облици



$i_L > 0 \quad v_C \uparrow$

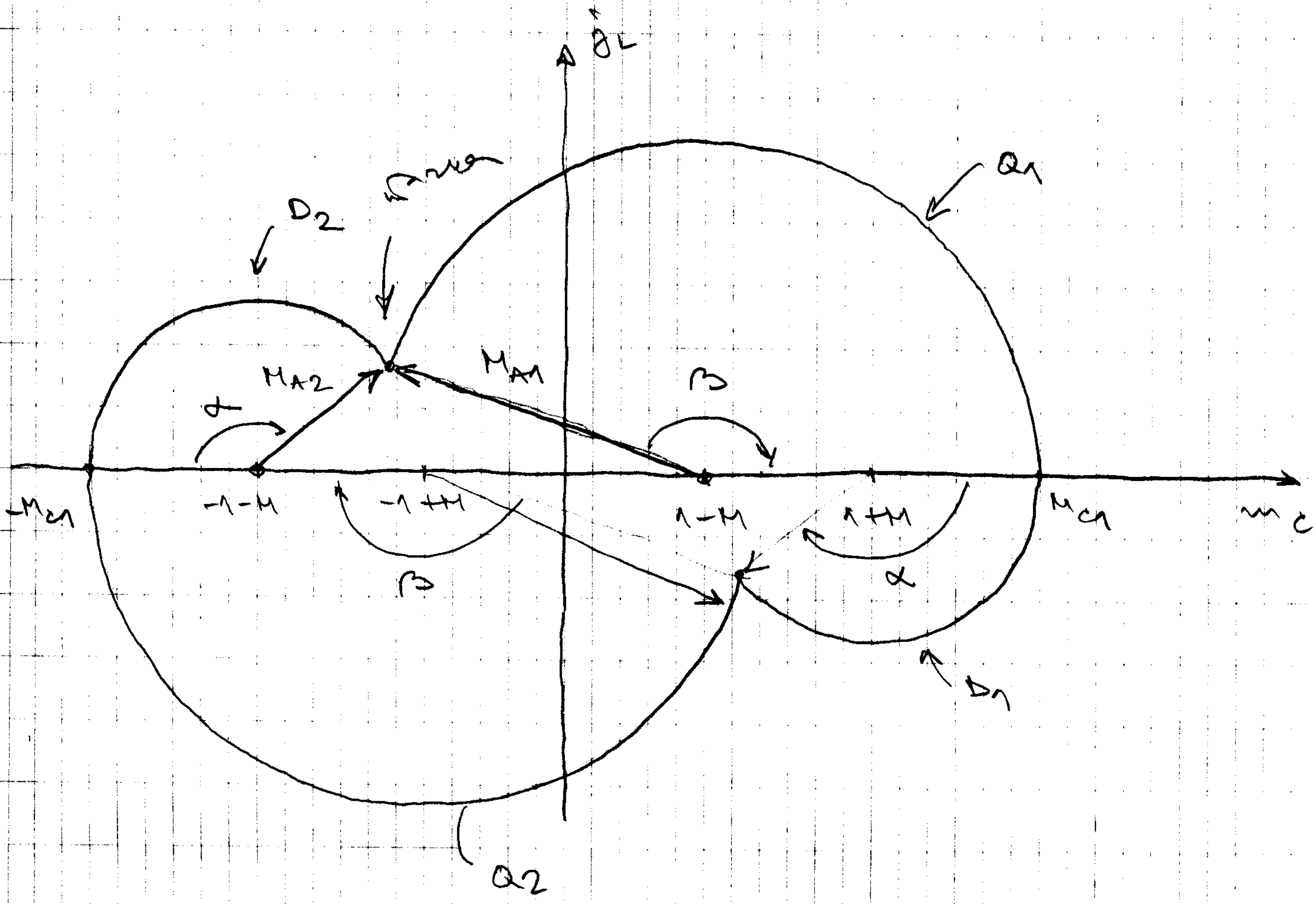
$i_L = 0 \quad v_C$ - extrem

$i_L < 0 \quad v_C \downarrow$

очигледно $f_0 > f_s$

← као у табели

дијаграм у фазној равни



- количина електрицитета која протекне у једној полупериоди

$$q = C \cdot 2 V_{C1}$$

- струја потрошача

$$I = \overline{|i_L|} = \frac{1}{\frac{T_s}{2}} \int_0^{T_s/2} |i_L(\tau)| \, d\tau = \frac{2}{T_s} \cdot q$$

$$I = \frac{2}{T_s} \cdot C \cdot 2 V_{сл} = \frac{4 \cdot C V_{сл}}{T_s}$$

$$V_{сл} = \frac{I T_s}{4C}$$

$$\frac{V_{сл}}{V_g} = M_{сл} = \underbrace{\frac{Z_0 I}{V_g}}_{J} \cdot \overset{\frac{1}{Z_0}}{\sqrt{\frac{C}{L}}} \cdot T_s \cdot \frac{1}{4C} =$$

$$= J \frac{T_s}{\underbrace{\sqrt{LC}}_{\omega_0}} \cdot \frac{1}{4} = \frac{1}{4} J \omega_0 T_s$$

$$\boxed{\frac{1}{2} T_s \omega_0 = \varkappa}$$

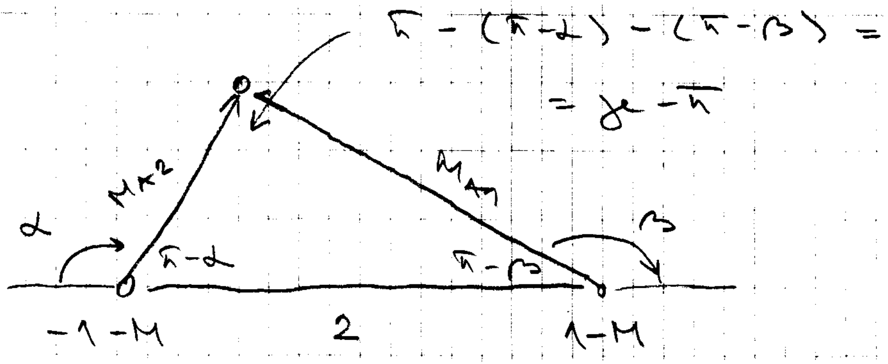
— дефиниција $\varkappa$, фазна дужина померача

$$\boxed{M_{сл} = \frac{1}{2} \varkappa J}$$

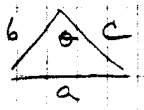
Циљ: ~~на~~ наћи $M(J, F)$

$$M_{A1} = M_{сл} - (1 - M) = \frac{\varkappa J}{2} - 1 + M$$

$$M_{A2} = M_{сл} - (1 + M) = \frac{\varkappa J}{2} - 1 - M$$



косинусов теорема



$$a^2 = b^2 + c^2 - 2bc\cos\theta$$

$$2^2 = M_{A2}^2 + M_{A1}^2 - 2M_{A1}M_{A2}\cos(\gamma - \pi)$$

$$4 = \left(\frac{J\gamma}{2} - 1 - M\right)^2 + \left(\frac{J\gamma}{2} - 1 + M\right)^2 +$$

$$+ 2\left(\frac{J\gamma}{2} - 1 - M\right)\left(\frac{J\gamma}{2} - 1 + M\right)\cos\gamma$$

подставляем

$$4 = 2\left(\frac{J\gamma}{2} - 1\right)^2 + 2M^2 + 2\left(\left(\frac{J\gamma}{2} - 1\right)^2 - M^2\right)\cos\gamma$$

$$1 = \left(\frac{J\gamma}{2} - 1\right)^2\left(\frac{1+\cos\gamma}{2}\right) + M^2\left(\frac{1-\cos\gamma}{2}\right)$$

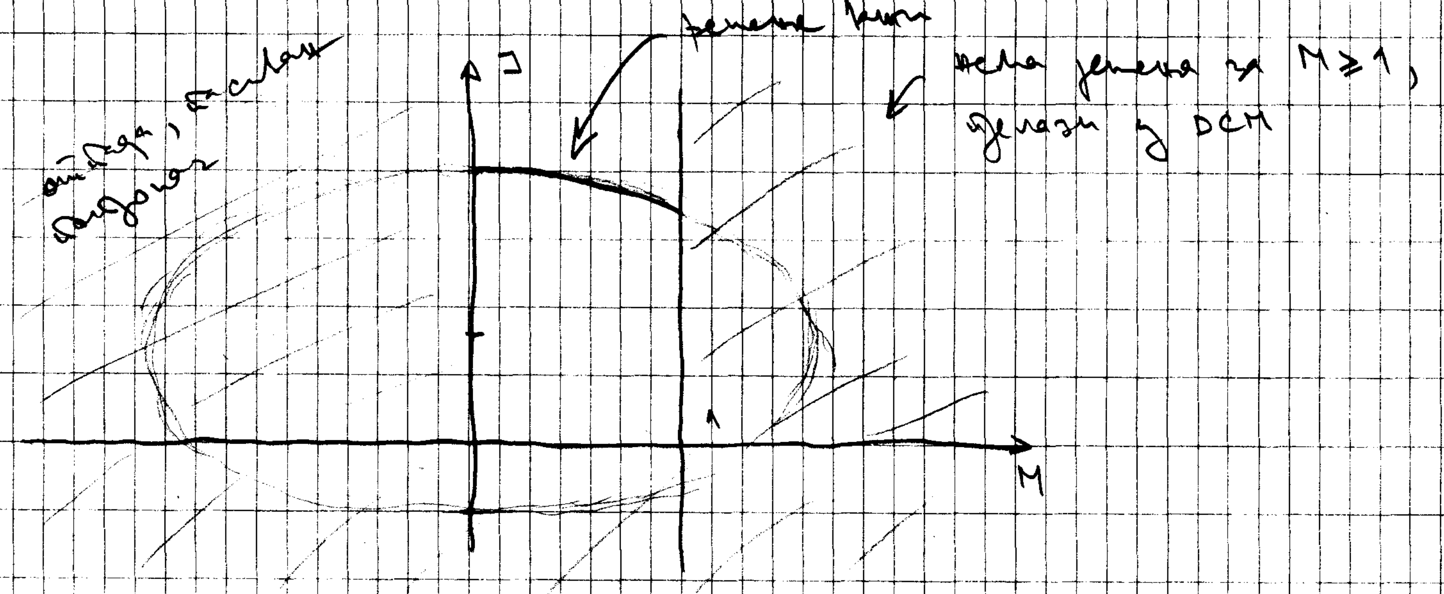
$$\boxed{M^2 \sin^2\frac{\gamma}{2} + \left(\frac{J\gamma}{2} - 1\right)^2 \cos^2\frac{\gamma}{2} = 1}$$

ово је згодно оно што ми треба

$$\gamma = \omega_0 \frac{T_s}{2} = 2\pi f_0 \frac{1}{2 f_s} = \frac{\pi}{\frac{f_s}{f_0}} = \frac{\pi}{F}$$

$$\boxed{\gamma = \frac{\pi}{F}}$$ – често се користи

Излазне карактеристике (output plane characteristics)



отпада, јер не излазимо за $J < 0$

посебни случајеви:

$F = 0.5$ (half resonance)

$\gamma = \frac{\pi}{F} = 2\pi$

$M^2 \cdot 0 + (\pi J - 1)^2 \cdot 1 = 1$

$\boxed{J = \frac{2}{\pi}}$ – није функција M, права се [illegible] [illegible] [illegible]

$F = 1.0$ (resonance)

$\gamma = \frac{\pi}{F} = \pi$

$M^2 \cdot 1 + (\frac{J\pi}{2} - 1)^2 \cdot 0 = 1$

$\boxed{M = 1}$ – није функција J, права се [illegible] [illegible] [illegible]

Вредност J за M = 1

$$\left(\frac{J\gamma}{2} - 1\right)^2 = \frac{1 - \sin^2\frac{\gamma}{2}}{\cos^2\frac{\gamma}{2}} = 1$$

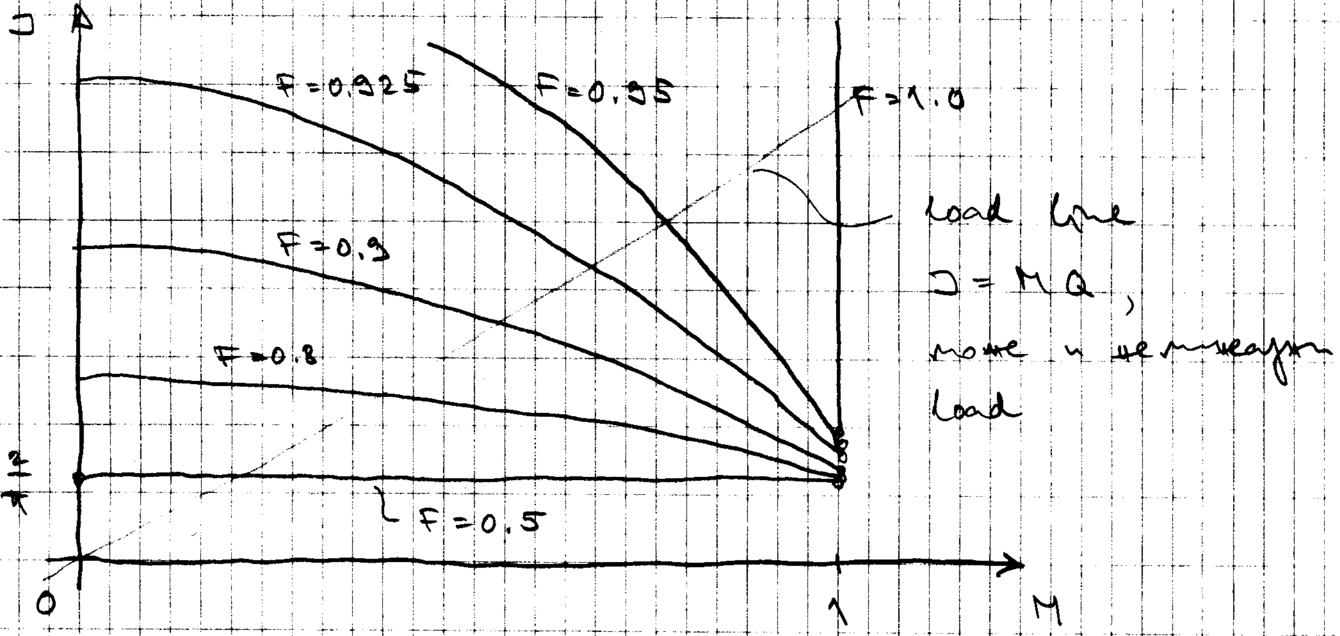
$$\frac{J\gamma}{2} = 2 \qquad \boxed{J = \frac{4}{\gamma}}$$

Струја кратког споја на излазу

$$\left(\frac{J_{sc}\,\gamma}{2}-1\right)^2\cos^2\frac{\gamma}{2}=1$$

$$\frac{J_{sc}\,\gamma}{2}-1=\left|\sec\frac{\gamma}{2}\right|$$

$$\boxed{J_{sc}=\frac{2}{\gamma}\left(1+\left|\sec\frac{\gamma}{2}\right|\right)=\frac{2F}{\pi}\left(1+\left|\sec\frac{\pi}{2F}\right|\right)}$$



load line

$J = MQ$,

може и нелинеаран load

карактеристика потрошача

$$R=\frac{V}{I}=\frac{\frac{V}{V_g}}{\frac{1}{R_0}\frac{R_0 I}{V_g}}=R_0\frac{M}{J}\qquad\frac{M}{J}=\frac{R}{R_0}$$

$$Q=\frac{R_0}{R}\qquad\qquad\boxed{J=MQ}$$

Карактеристике у контролној равни
(control plane characteristics)

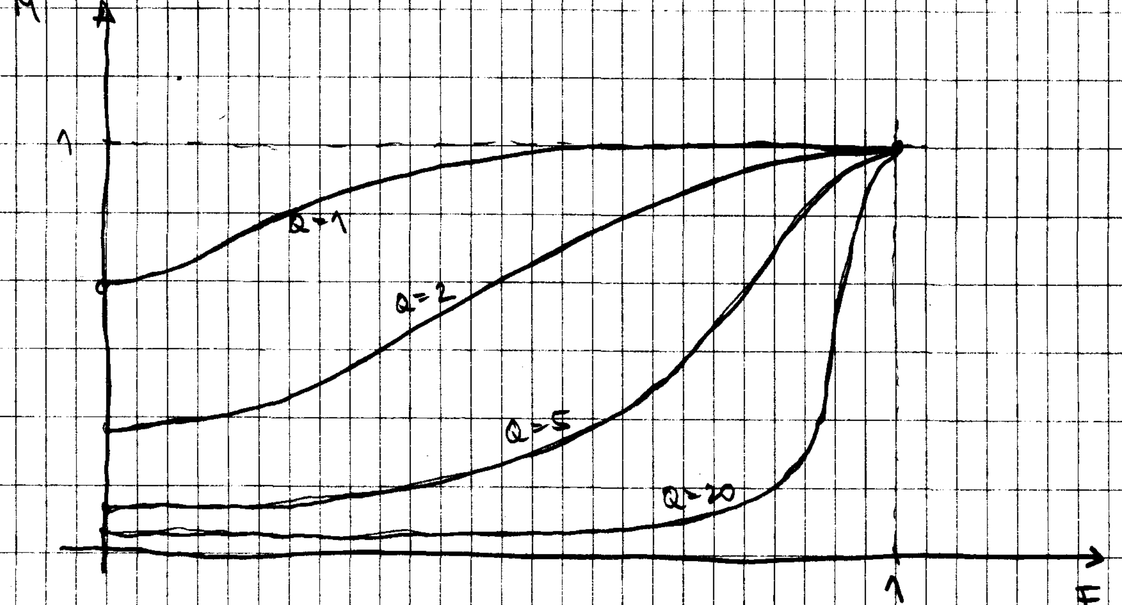
$$M^2 \sin^2\frac{\gamma}{2} + \left(\frac{MQ\gamma}{2} - 1\right)^2 \cos^2\frac{\gamma}{2} = 1$$

$(J = MQ$ - упрошћено$)$

$$M^2\left(\sin^2\frac{\gamma}{2} + \left(\frac{Q\gamma}{2}\right)^2\cos^2\frac{\gamma}{2}\right) -$$

$$- MQ\gamma\cos^2\frac{\gamma}{2} + \left(\cos^2\frac{\gamma}{2} - 1\right) = 0$$

$$\boxed{M = \frac{\frac{Q\gamma}{2}}{tg^2\frac{\gamma}{2} + \left(\frac{Q\gamma}{2}\right)^2}\left(1 \pm \sqrt{1 + \left(\frac{2}{Q\gamma}\right)^2 tg^2\frac{\gamma}{2}\left(tg^2\frac{\gamma}{2} + \left(\frac{Q\gamma}{2}\right)^2\right)}\right)}$$



ово је below resonance case, $f_s < f_0$, CCM 1

Модови рада

- до сада анализирали $k=1$ CCM
- мод рада одређује секвенца вођења, за $k=1$ CCM то је $Q_1 - D_1 - Q_2 - D_2$, за $k=0$ CCM то је $D_1 - Q_1 - D_2 - Q_2$
- ДИСКОНТИНУАЛНИ МОДОВИ
- за light loads, $J < \frac{2}{\pi}$, јавља се стање кад не воде диоде у тренутку, означава се са X.
- $k=1$ DCM $\quad Q_1 - X - Q_2 - X$

$$I = \overline{|i_L|} = \frac{2q}{T_s}$$

q - наелектрисање које прође кроз tank у једној полупериоди

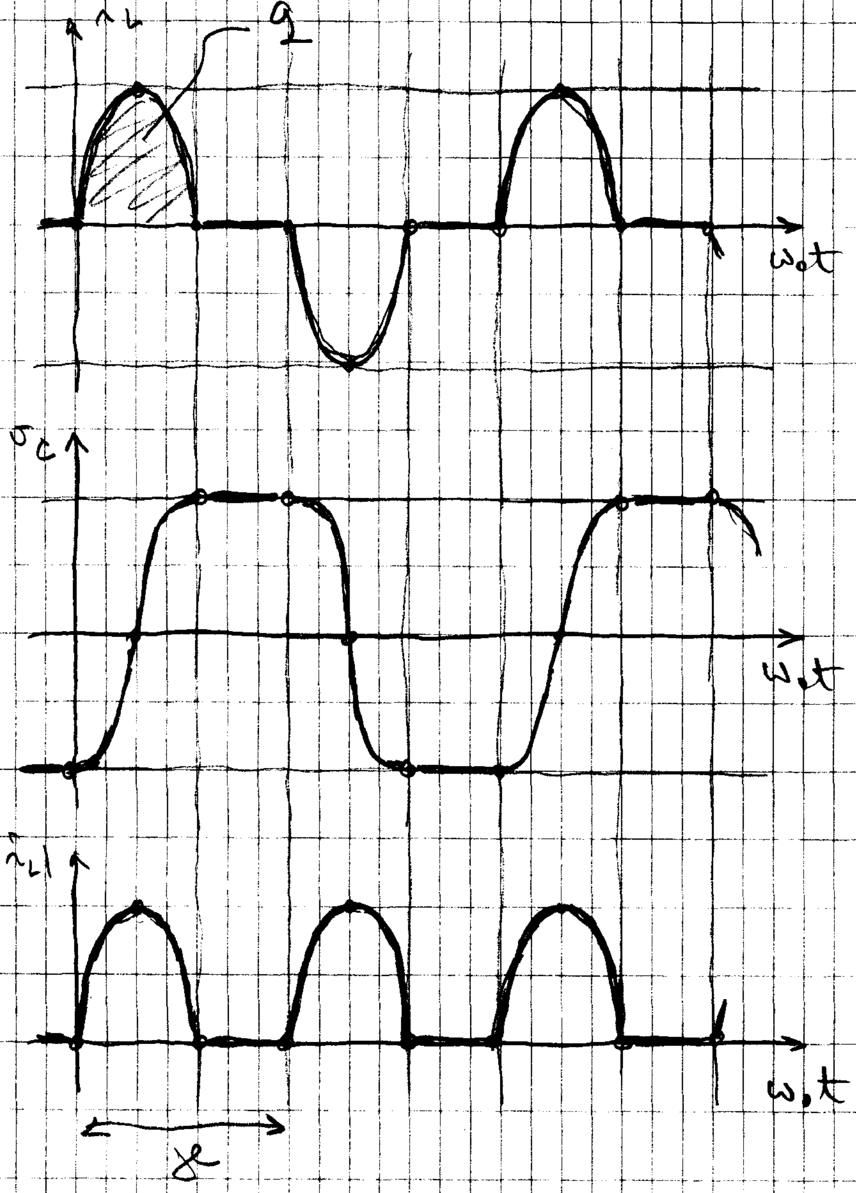
$$q = \int_0^{\frac{T_s}{2}} i_L(t)\,dt$$

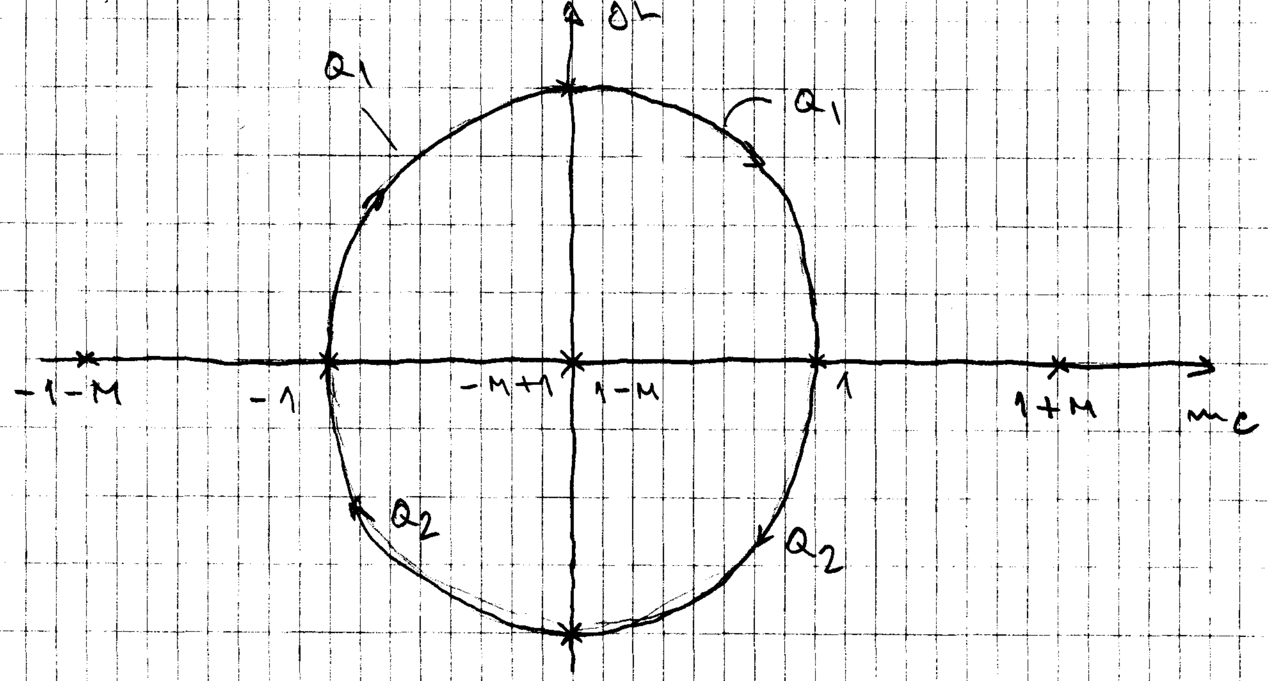
$$i_g = \frac{2}{T_s}\int_0^{\frac{T_s}{2}} i_L(t)\,dt = \frac{2q}{T_s} = I$$

$$P_{in} = P_{out} \qquad V_g I = V I \qquad \boxed{V = V_g}$$

$$\boxed{M = 1}$$

$$q = C \cdot 2V_{ca} \qquad V_{ca} = \frac{I T_s}{4C} \rightarrow \boxed{M_{ca} = \frac{J\gamma}{2}}$$


$\hat{i}_L$
Q_1
$\omega_0 t$
v_C
$\omega_0 t$
$|\hat{i}_L|$
$\omega_0 t$
γ


$\hat{i}_L$
Q_1
Q_1
$-1-M$
-1
$-M+1$
$1-M$
1
$1+M$
m_C
Q_2
Q_2

Ограничења за DCM k=1 mode

k - број half-cycles in one half period

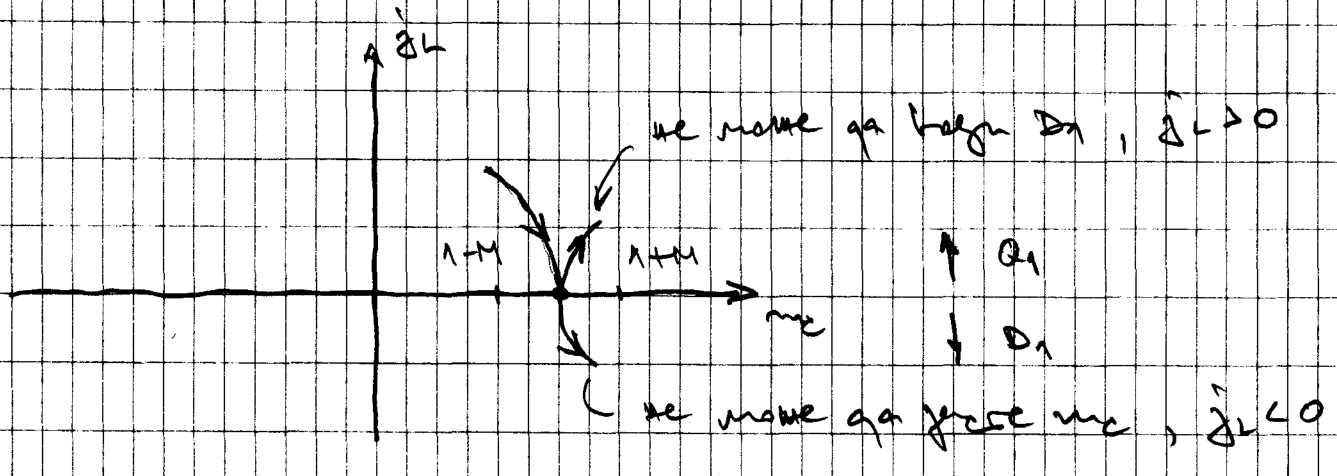
овај мод не може изнад резонансе, за $f_0 < f_s$

$\boxed{F < 1}$ $\boxed{\varkappa > \pi}$

$1 + M > M_{C1}$

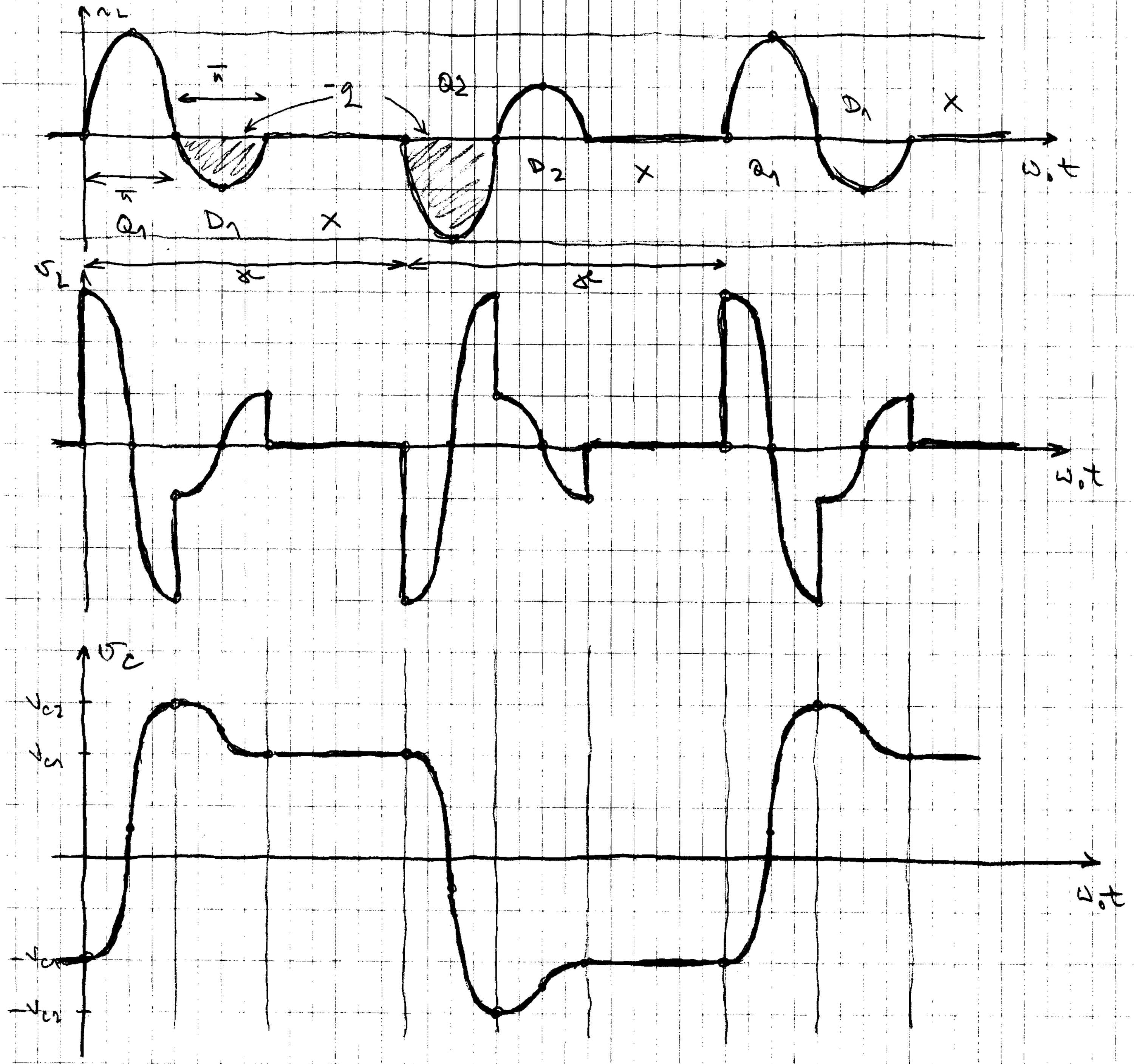
$1 + 1 > \frac{J\varkappa}{2}$ $\boxed{J < \frac{4}{\varkappa}}$

ограничење у фазној равни



$k=2$ Дисконтинуални режим

$(k=2$ DCM$)$



$$2V_{c2} = \frac{q}{C} \rightarrow q = 2CV_{c2}$$

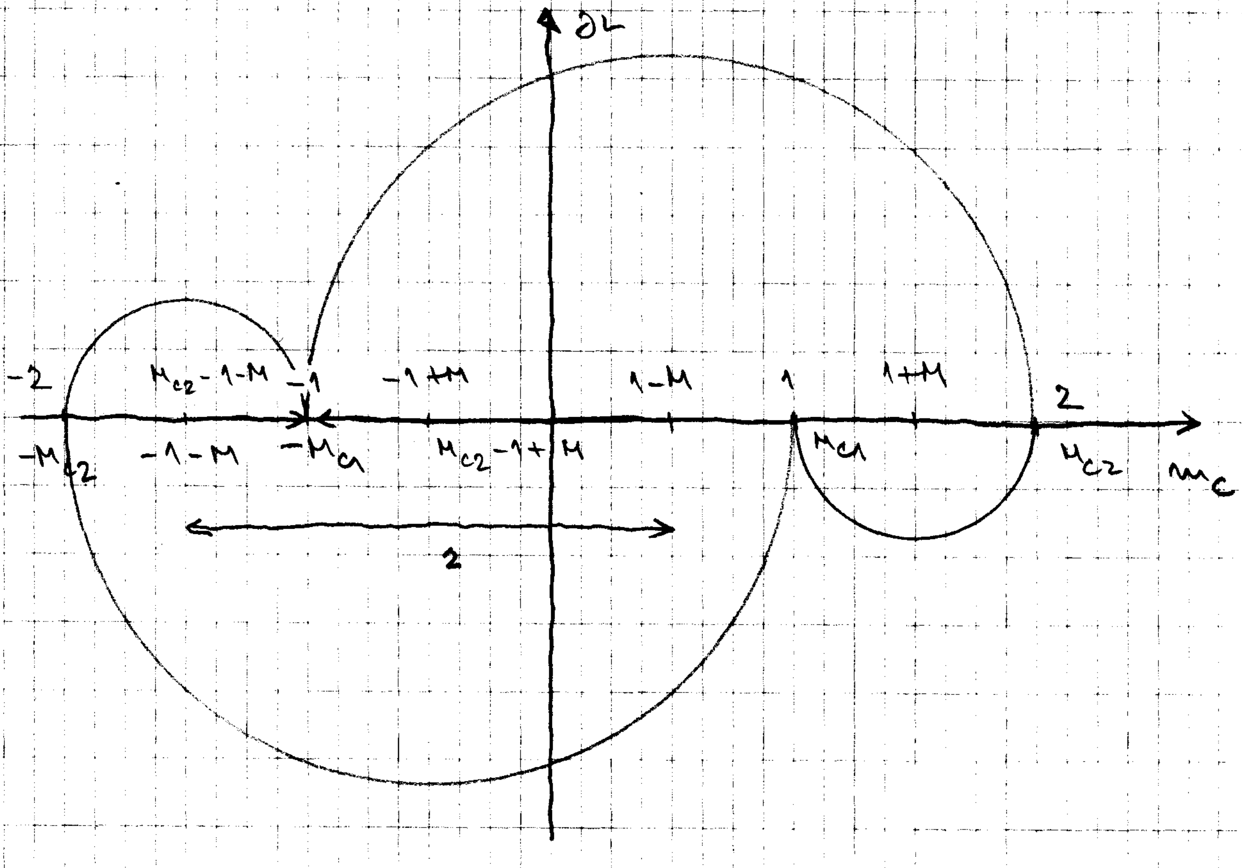
$$I = \overline{|\hat{i}_L|} = \frac{2q}{T_s} = \frac{4CV_{c2}}{T_s}$$

$$J = \frac{R_0 I}{V_g} = \sqrt{\frac{L}{C}} \cdot C \cdot \frac{4V_{c2}}{V_g} \cdot \frac{1}{T_s} =$$

$$= \frac{1}{\omega_0} 4M_{c2} \cdot \frac{1}{T_s} = \frac{4M_{c2}}{2\gamma} = \frac{2M_{c2}}{\gamma}$$

$$\boxed{M_{c2} = \frac{J\gamma}{2}}$$

- ово је увек лако одавде, да се q поклопе са геометријом



$$2 = (M_{c2} - (1+M)) + (M_{c2} - (1-M)).$$

$$2 = 2M_{c2} - 2 \rightarrow M_{c2} = 2 = \frac{J\gamma}{2}$$

$$\boxed{J = \frac{4}{\gamma}} \qquad \boxed{J = \frac{4}{\pi} F}$$

за резистиван потрошач:

$$J = MQ = \frac{4}{\pi} F$$

$$\boxed{M = \frac{4}{\pi} \frac{F}{Q}}$$ - output plane characteristics

битно погледно: колико је M_{c1}?

$$M_{c1} = M_{c2} - 2(M_{c2} - (1+M)) =$$

$$= M_{c2} - 2M_{c2} + 2 + 2M = 2M \qquad (M_{c2} = 2)$$

$$\boxed{M_{c1} = 2M}$$

Тражимо за $k = 2$ DCM

да би одредио два полуимпулса у полупериоди

$$f_s < \frac{f_0}{2}$$

$$\boxed{F \leq \frac{1}{2}} \qquad \boxed{\varkappa \geq 2\pi}$$

Ово одаје и горњу границу за J

$$J = \frac{4}{\pi} F \leq \frac{4}{\pi} \cdot \frac{1}{2} = \frac{2}{\pi}$$

$$\boxed{J \leq \frac{2}{\pi}}$$

Да не би дошао трећи полуимпулс

$$(1+M) - (1-M) > M_{c2} - (1+M)$$

$$2M > 2 - 1 - M$$

$$3M > 1$$

$$\boxed{M > \frac{1}{3}}$$

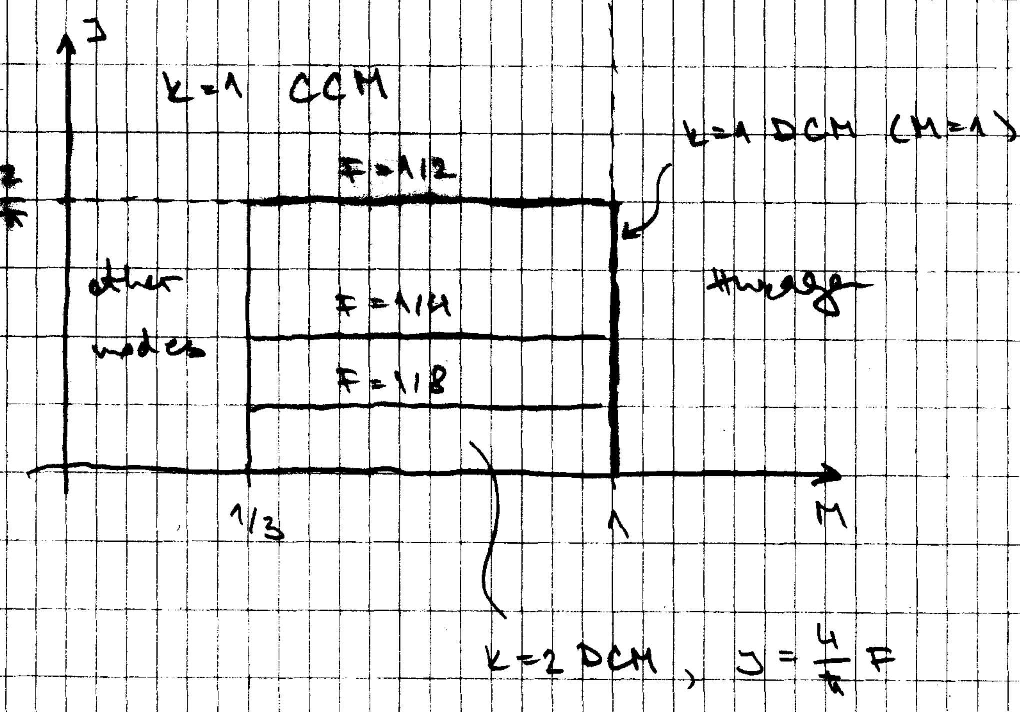
- Имамо 3 ограничења, за F, M и J

$0 < F < \frac{1}{2}$

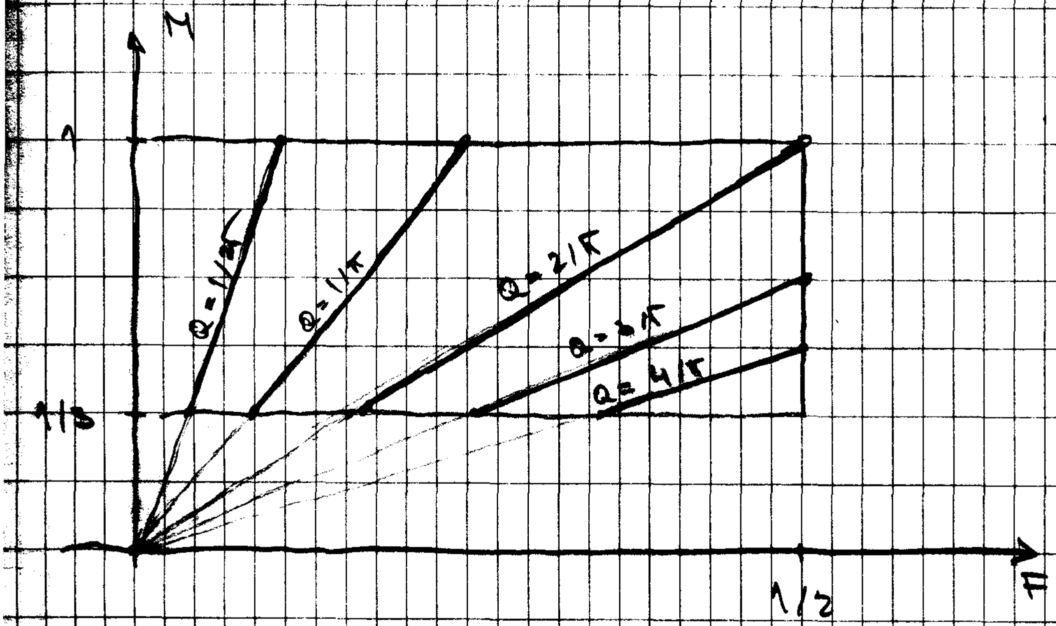
$0 < J < \frac{2}{\pi}$

$\frac{1}{3} < M < 1$ - ово се може проширити на $0 < M < 1$ додавањем пригушења

Карактеристике DCM 2 у M - J домену



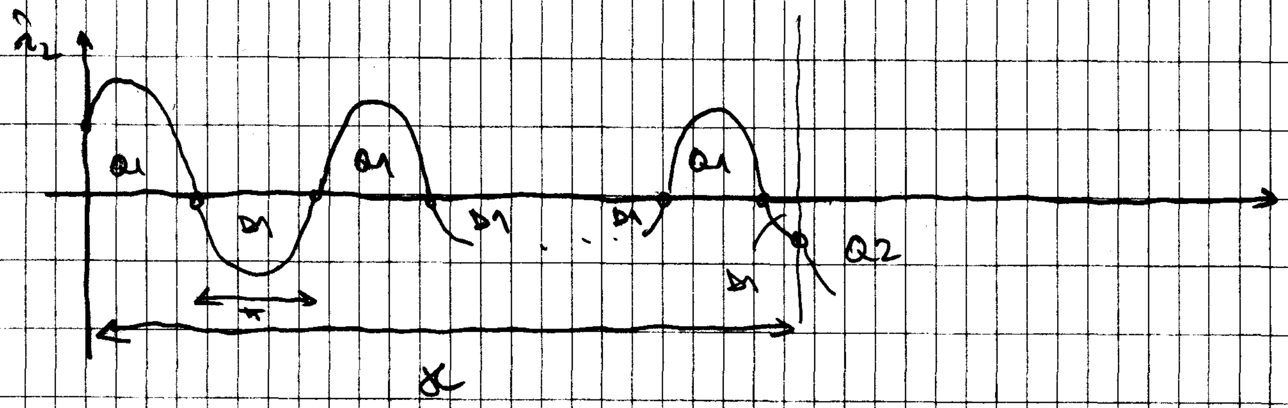
Карактеристике у контролној (M(F)) равни



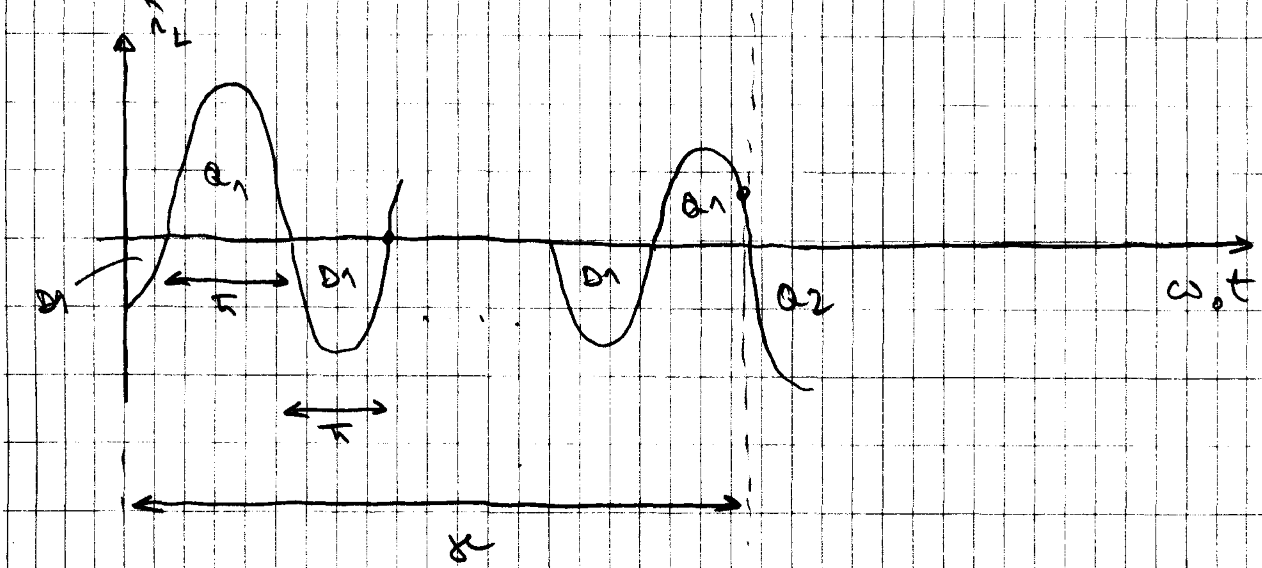
Опште решење

- k CCM

k непарно



k - парно



k CCM се може остварити за

$$\boxed{\frac{f_0}{k+1} < f_s < \frac{f_0}{k}}$$

subharmonic index

$$\boxed{\xi = k + \frac{1+(-1)^k}{2}}$$

k	ξ
0	1
1	1
2	3
3	3
4	5
5	5
6	7
7	7
⋮	⋮

M је ограничен са:

$$\boxed{0 \le M \le \frac{1}{\xi}}$$

Излазне карактеристике су елиптичне

$$\boxed{M^2 \xi^2 \sin^2 \frac{\gamma}{2} + \frac{1}{\xi^2}\left(\frac{J\gamma}{2} + (-1)^k\right)^2 \cos^2 \frac{\gamma}{2} = 1}$$

Steady-State Control Plane Characteristics

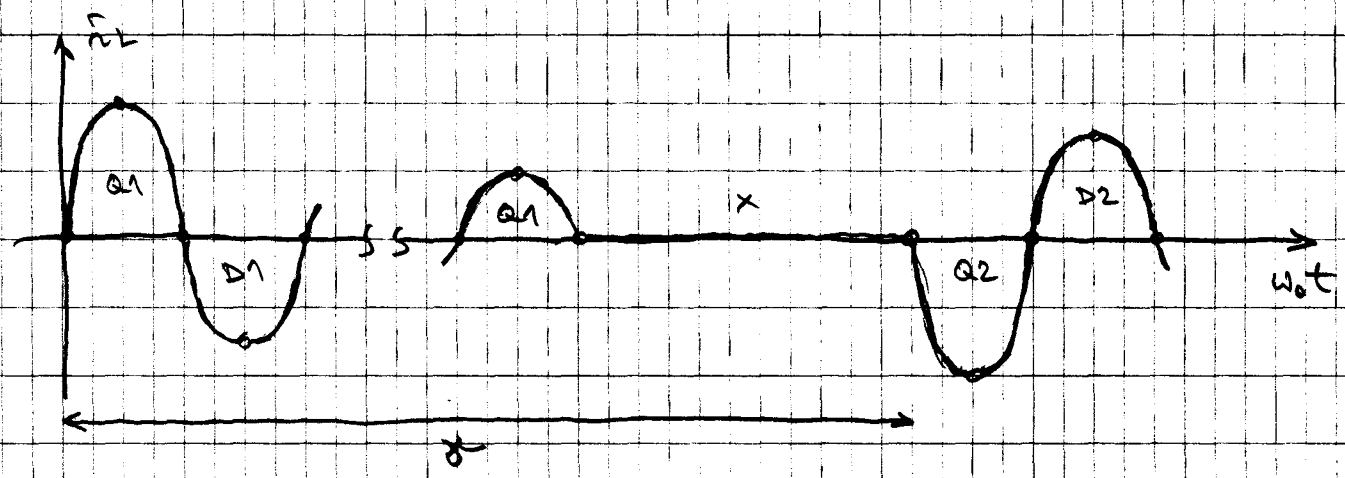
$$\boxed{M = \frac{\frac{Q\gamma}{2}}{\xi^4 \operatorname{tg}^2 \frac{\gamma}{2} + \left(\frac{Q\gamma}{2}\right)^2}\left((-1)^{k+1} + \sqrt{1 + \frac{\left(\xi^2 - \cos^2 \frac{\gamma}{2}\right)\left(\xi^4 \operatorname{tg}^2 \frac{\gamma}{2} + \left(\frac{Q\gamma}{2}\right)^2\right)}{\left(\frac{Q\gamma}{2}\right)^2 \cos^2 \frac{\gamma}{2}}}\right)}$$

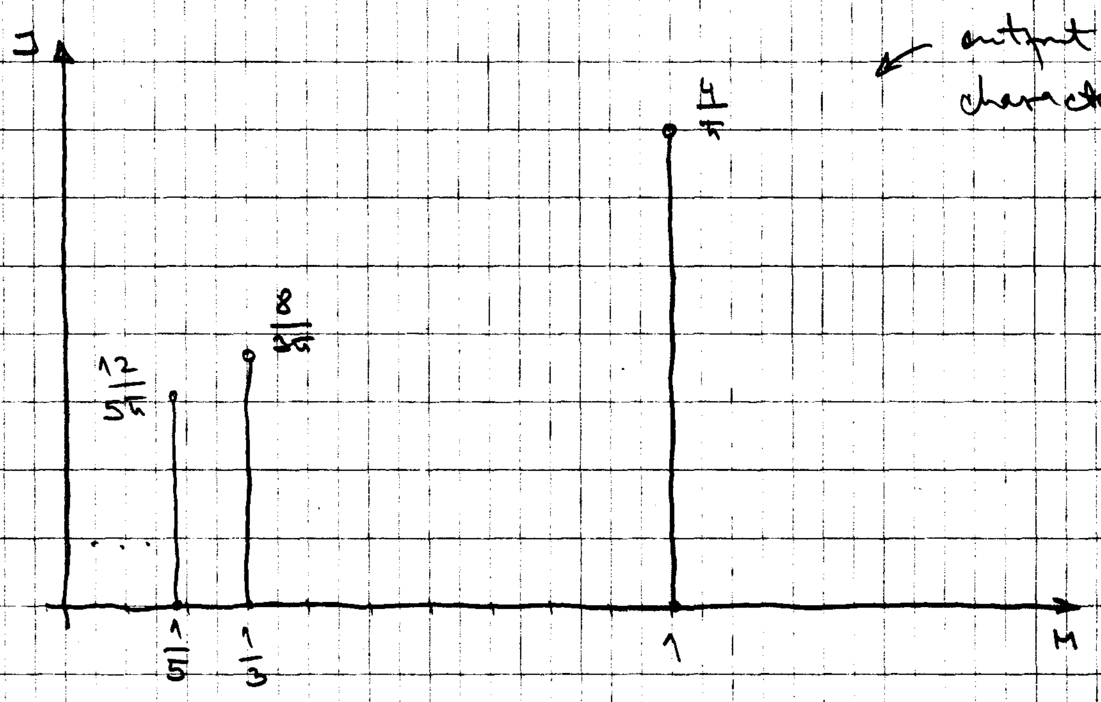
k DCM , k непарно

$$\boxed{f_s < \frac{f_0}{k}} \qquad \boxed{M = \frac{1}{k}}$$

$$\boxed{\frac{2(k+1)}{\gamma} > J > \frac{2(k-1)}{\gamma}}$$

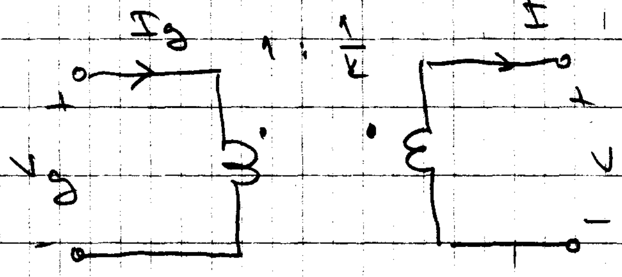
Типични таласни облици





output plane characteristics

- модел конвертора у устаљеном стању, за средње вредности струја и напона

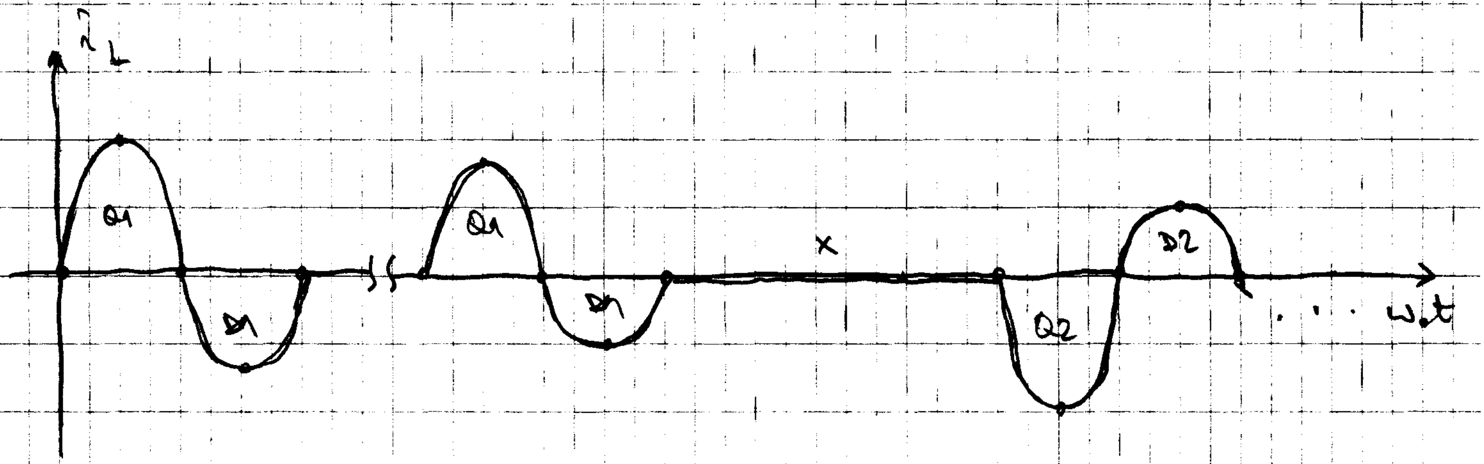


k DCM, k neparno

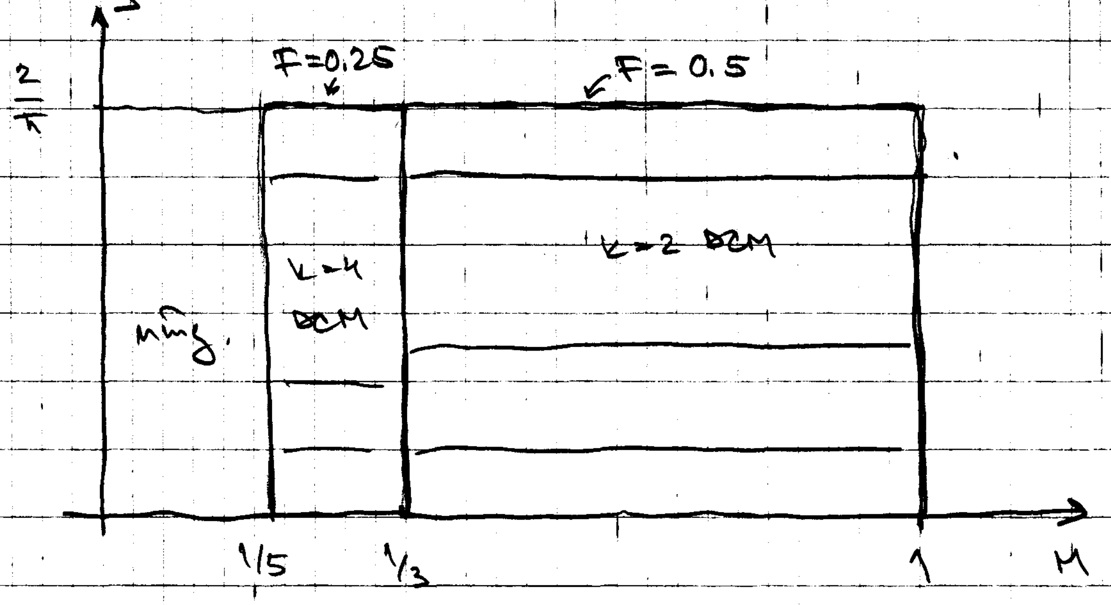
$$\boxed{f_s < \frac{f_0}{k}}$$

$$\boxed{J = \frac{2k}{\gamma}}$$

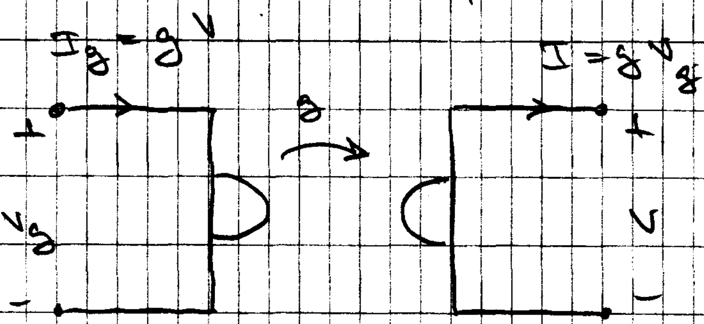
$$\boxed{\frac{1}{k+1} < M < \frac{1}{k-1}}$$



output plane characteristics



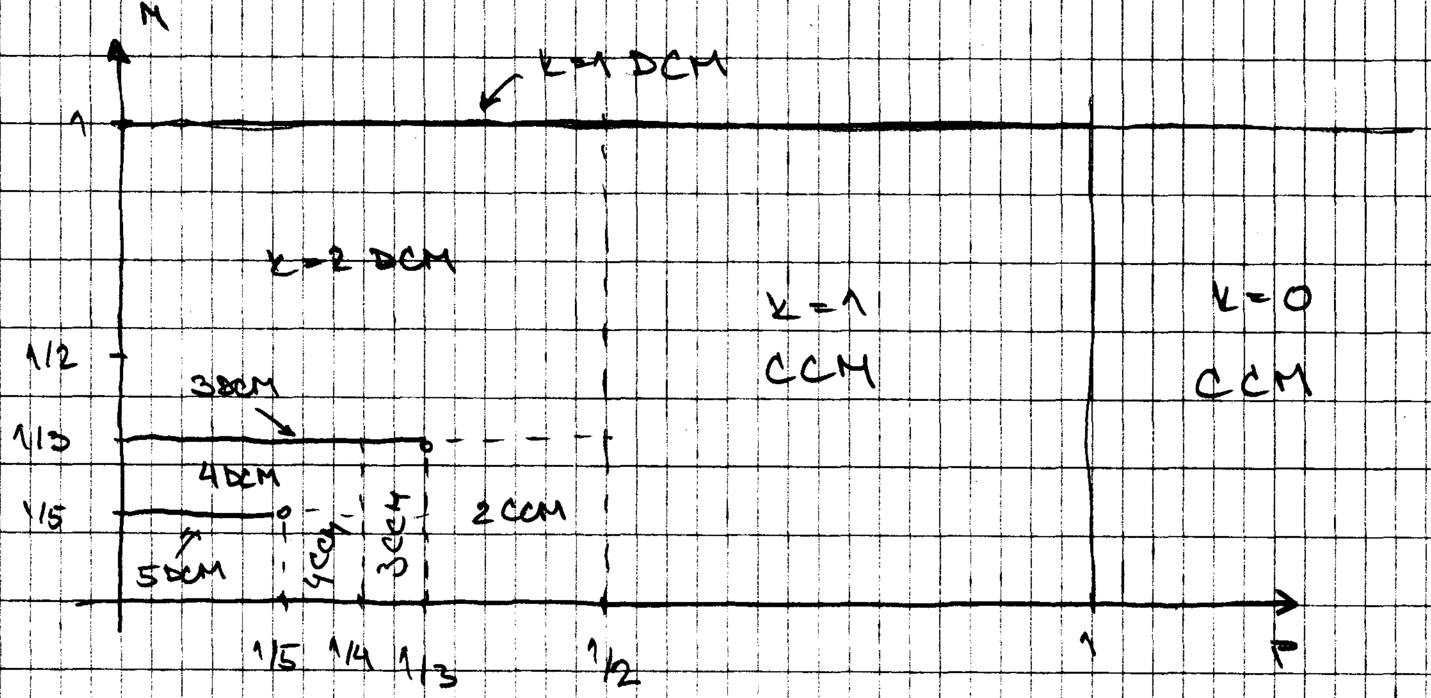
- dc model, averaged voltages and currents



$$\boxed{g = \frac{2k}{\gamma R_0}}$$

Комплетне карактеристике

- у компактној (M(F)) облику, а функцији



[illegible] , [illegible] , [illegible] 76

[illegible]

F, Q → [illegible]

$k = \text{int}\left(\frac{1}{F}\right)$

$k_1 = \text{int}\left(\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{Q\pi}{2F}}\right)$

$k_1 > k \longrightarrow k$ CCM

$k_1 < k \longrightarrow k_1$ DCM

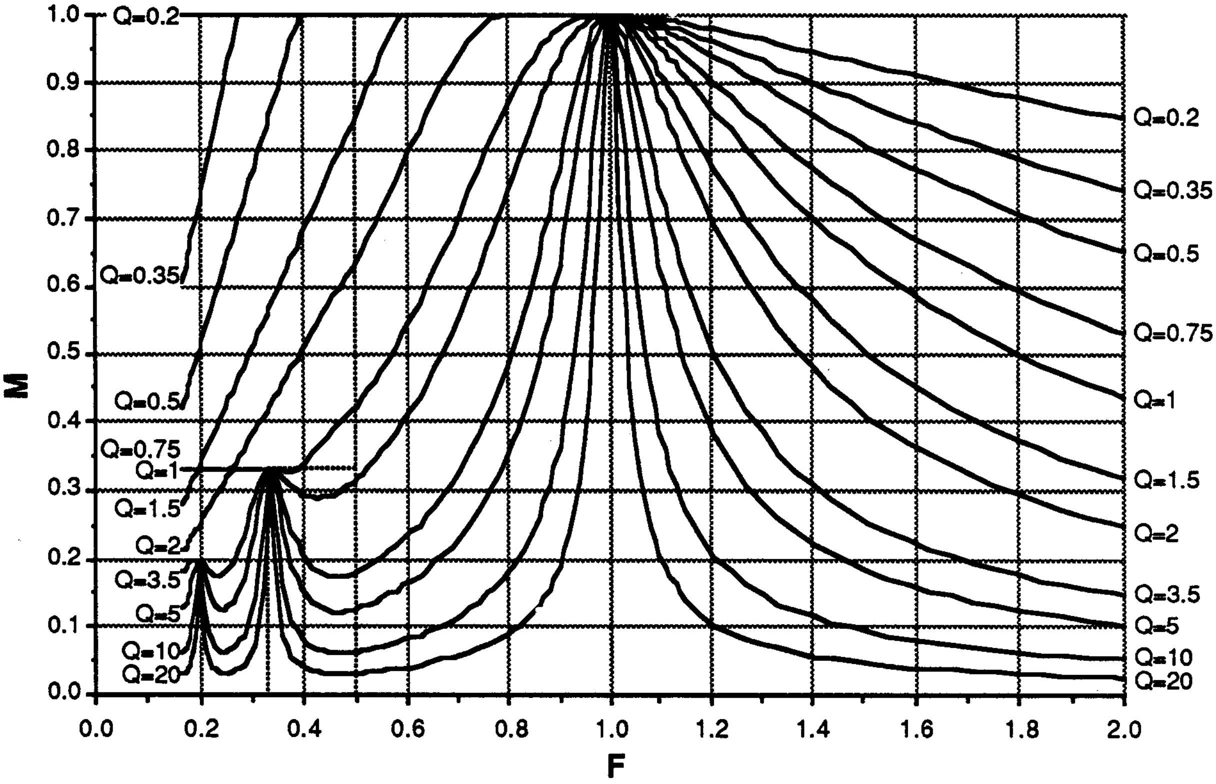


Fig. 4.36. Complete control plane characteristics of the series resonant converter, for the range $0.2 \leq F \leq 2$.

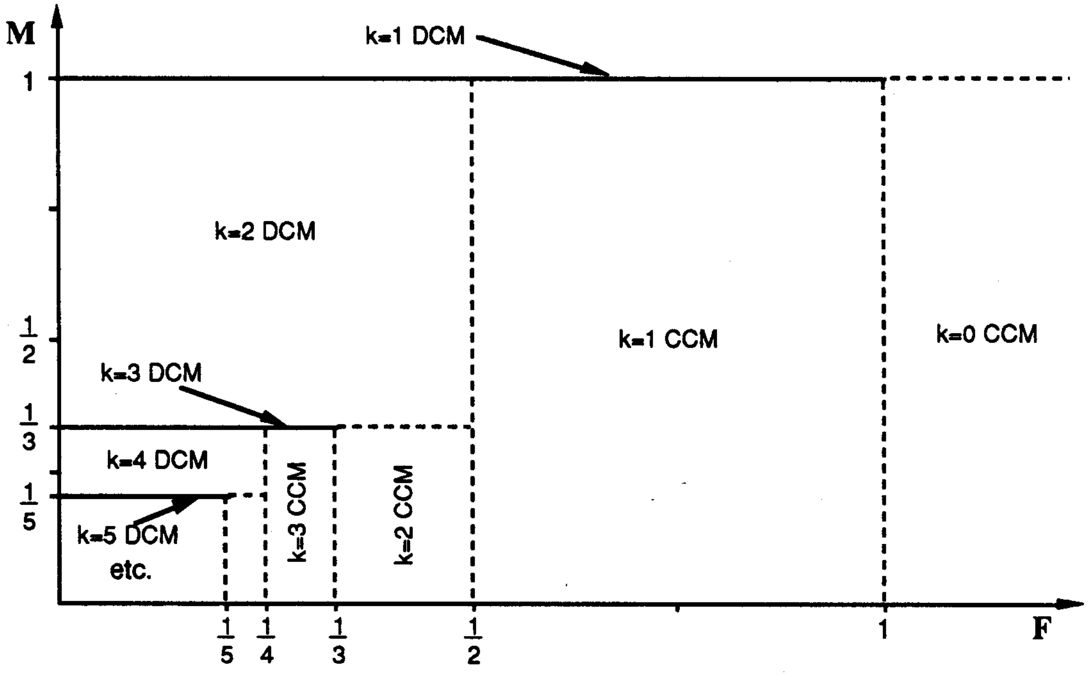


Fig. 4.37. Complete control plane characteristics for continuous and discontinuous conduction mode boundaries.

Dynumesser

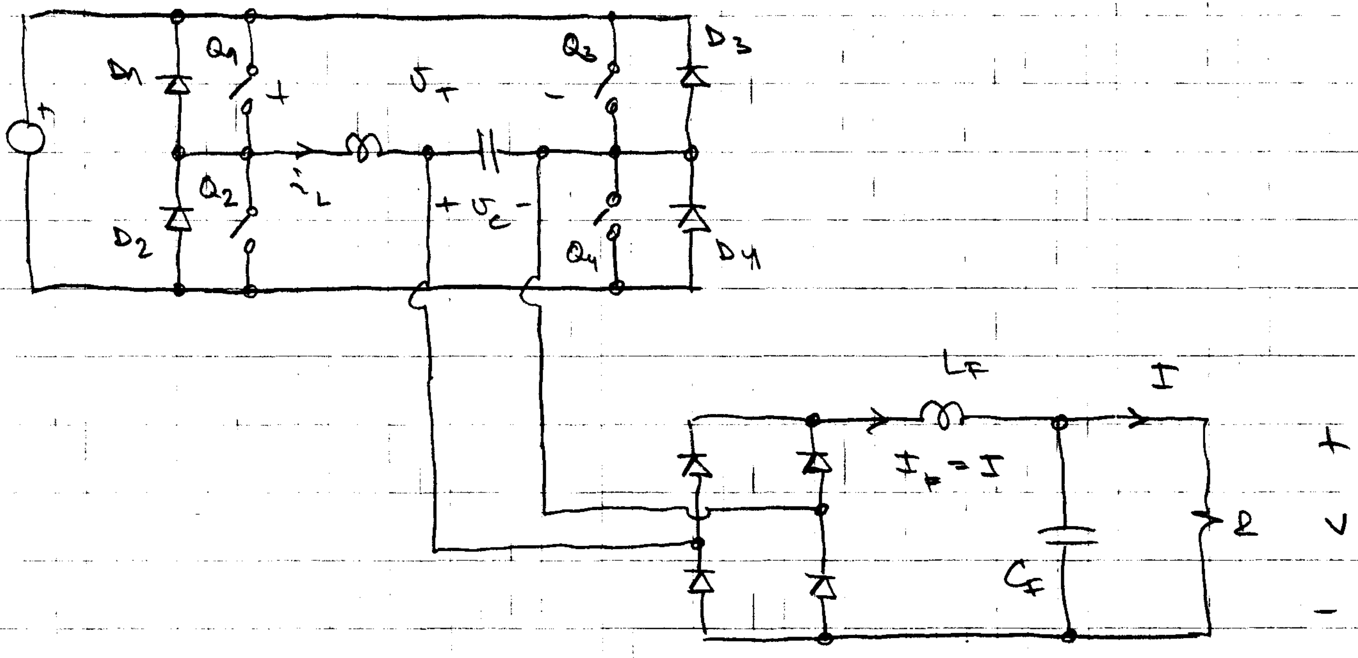
k = 0 CCM ZV turn on

k = 1 CCM ZC turn off

k = 2 DCM ZC turn on, ZC turn off

output plane characteristics, Bolt, eq. 78.

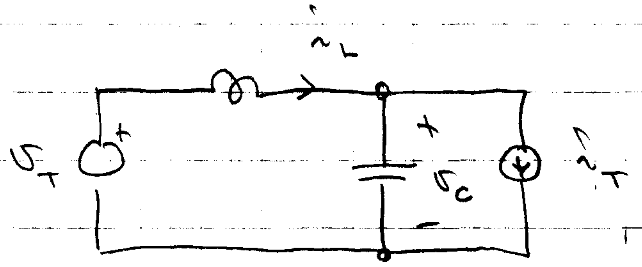
Parallel Resonant Converter



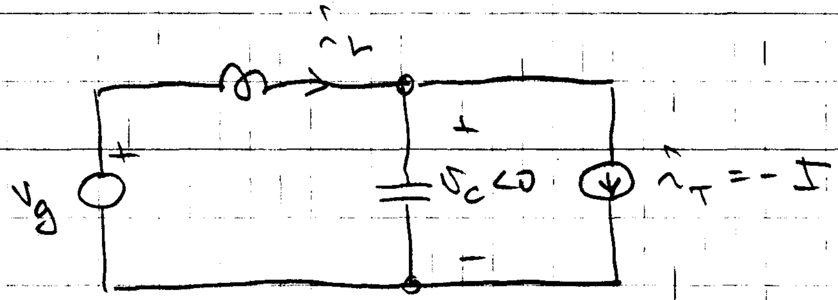
$$I_F = I$$

$$V = \overline{|v_c|}$$

interval	V_T	M_T	I_T	J_T	v_c
1	V_g	$+1$	$-I$	$-J$	$-$
2	V_g	$+1$	$+I$	$+J$	$+$
3	$-V_g$	-1	$+I$	$+J$	$+$
4	$-V_g$	-1	$-I$	$-J$	$-$

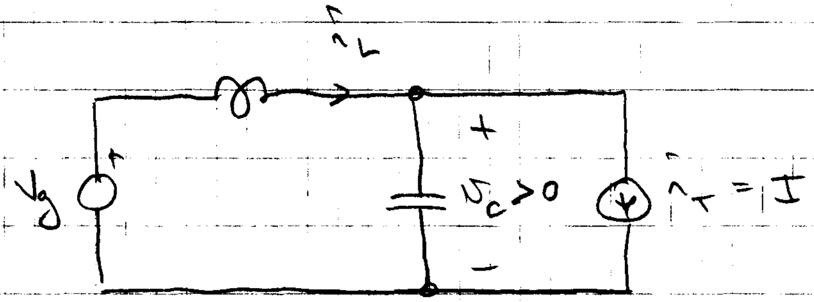


①



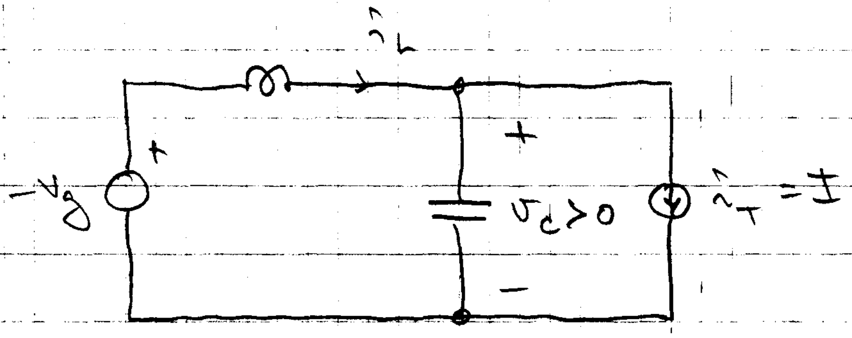
Q1, Q4 / D1, D4 $0 < \omega_0 t < \alpha$

②



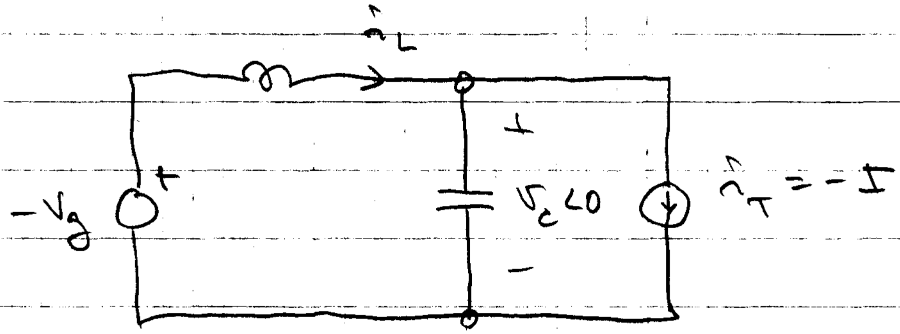
Q1, Q4 / D1, D4 $\alpha < \omega_0 t < \alpha + \beta = \gamma$

③

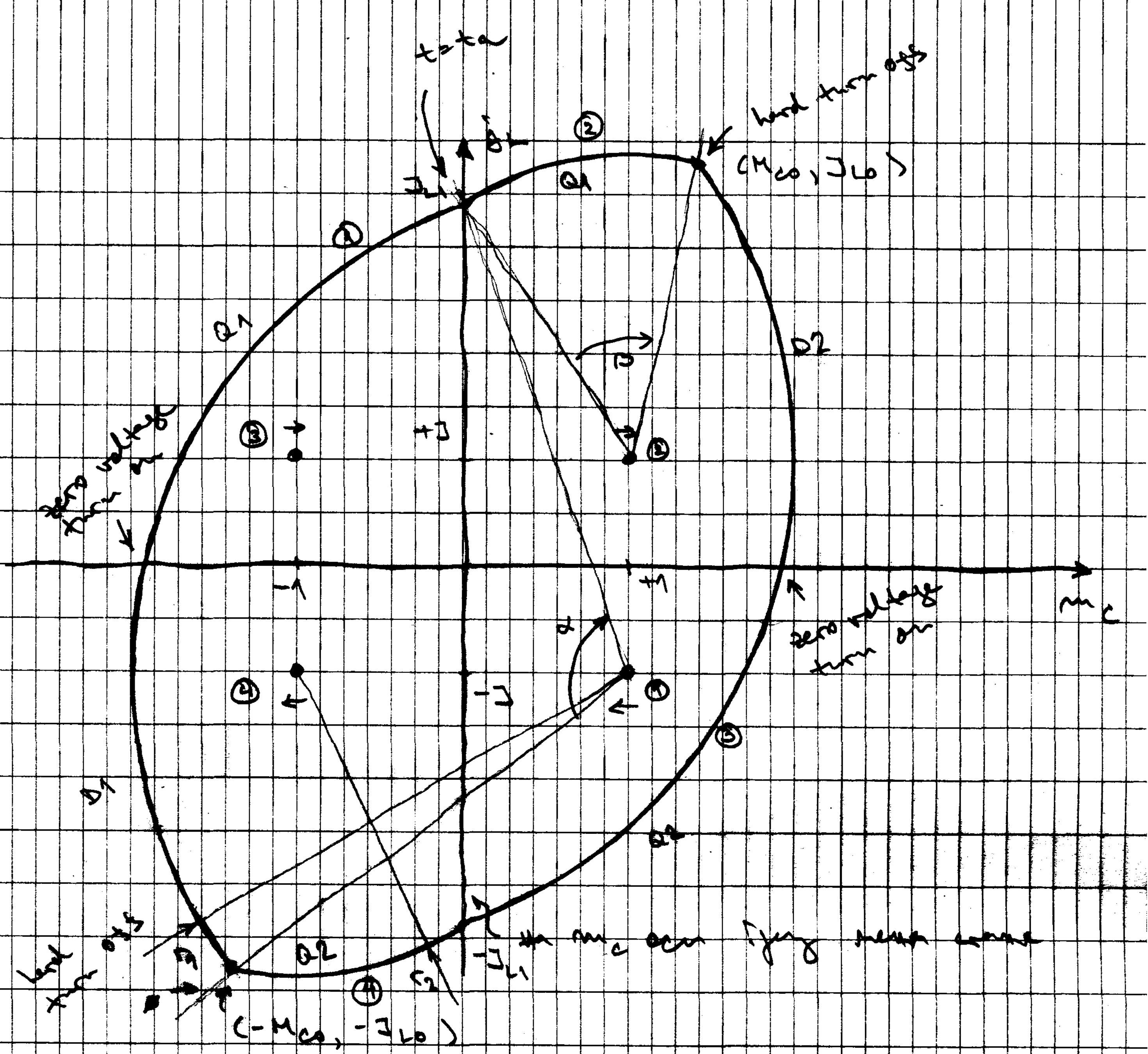


Q2, Q3 / D2, D3 $\gamma < \omega_0 t < \gamma + \alpha$

④



Q2, Q3 / D2, D3 $\gamma + \alpha < \omega_0 t < 2\gamma$

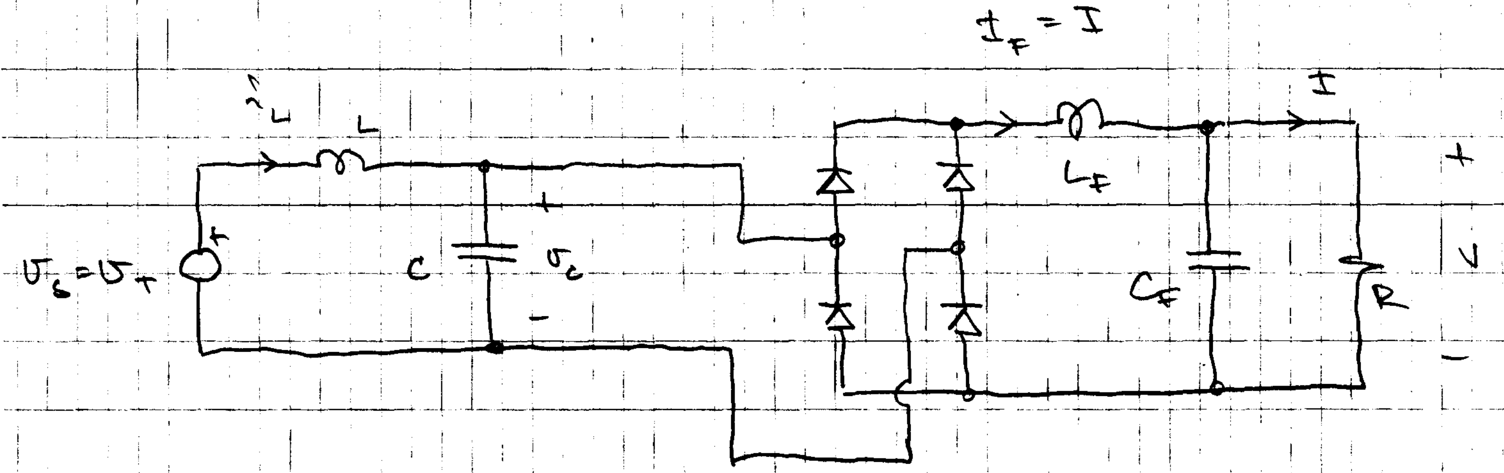


① - центар за леву полуталас

④ - центар за леву полуталас

② - центар за десну полуталас

③ - центар за десну полуталас



$$V = \overline{|v_C|}$$

$$V = \frac{2}{T_S}\int_{t_a}^{t_a+\frac{1}{2}T_S} v_C(t)\,dt$$

$t = t_a$ – почетак интервала полупериода v_C

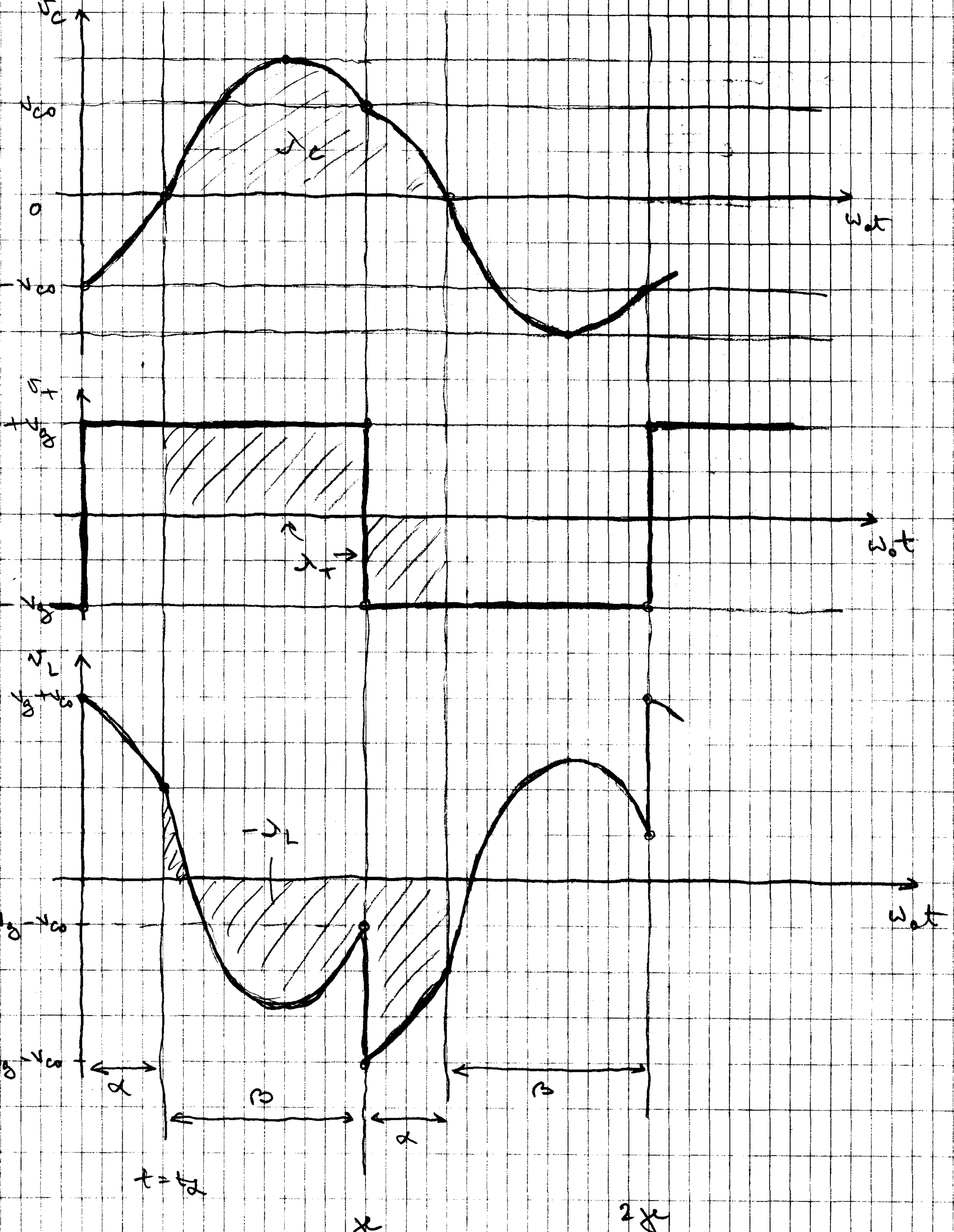
$$V = \frac{2}{T_S}\lambda_C$$

$$\lambda_C = \int_{t_a}^{t_a+\frac{1}{2}T_S} v_C(t)\,dt$$

$$\lambda_C = \lambda_T - \lambda_L$$

$$\lambda_T = \int_{t_a}^{t_a+\frac{1}{2}T_S} v_T(t)\,dt$$

$$\lambda_L = \int_{t_a}^{t_a+\frac{1}{2}T_S} v_L(t)\,dt$$


v_C
v_C0
0
-v_C0
ω0t
v_C
v_t
+V_g
-V_g
i_t
ω0t
v_L
V_g+v_C0
-i_L
ω0t
V_g-v_C0
-V_g-v_C0
α
β
β
α
t = t_2
π
2π

$$V = \frac{2}{T_s}(\lambda_T - \lambda_L)$$

v_T - конечния ~~[illegible]~~

$$\lambda_T = V_g\left(\frac{1}{2}T_s - 2t_a\right)$$

$$\lambda_T = V_g\left(\frac{\beta}{\omega_0} - \frac{\alpha}{\omega_0}\right)$$ - са същите !

$$\lambda_T = V_g\,\frac{\beta-\alpha}{\omega_0}$$

$$\lambda_L = L(-2I_{L1})$$

$$V = \frac{2}{T_s}\left(V_g\frac{\beta-\alpha}{\omega_0} + 2LI_{L1}\right)$$

$$\frac{V}{V_g} = M = \frac{2}{\boxed{T_s}}\left(\frac{\beta-\alpha}{\boxed{\omega_0}} + \frac{1}{\boxed{\omega_0}}\cdot 2\overset{J_{L1}}{\overbrace{\overset{R_0}{\boxed{\omega_0 L}}\cdot\frac{I_{L1}}{V_g}}}\right)$$

2γ

$$M = \frac{1}{\gamma}(\beta - \alpha + 2J_{L1})$$

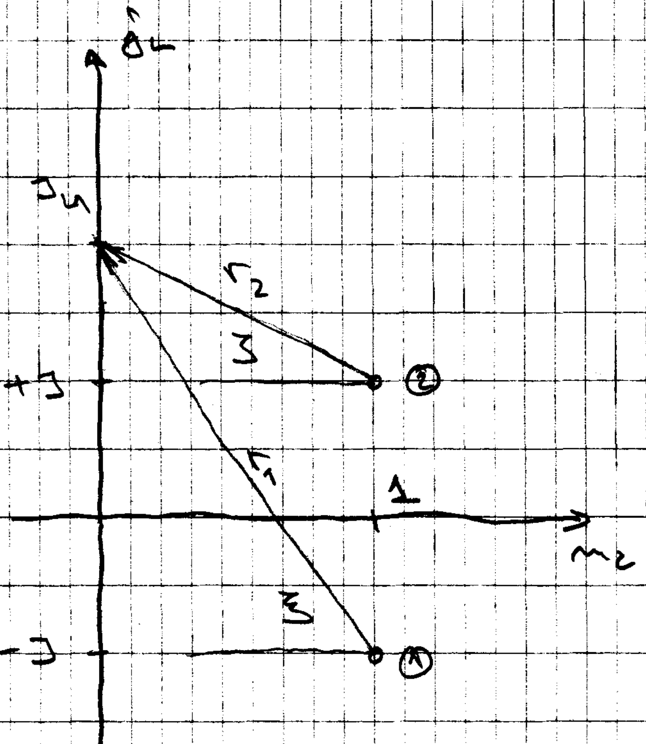
$$M = \frac{2}{\gamma}\left(\frac{\beta-\alpha}{2} + J_{L1}\right)$$

$$\boxed{\frac{M\gamma}{2} = (\varphi + J_ц)} \qquad (0)$$

$$\boxed{\varphi \overset{деф}{=} \frac{\beta - \alpha}{2}}$$

– Геометрия: заменяме $J_ц$, β и α с J и γ

① определяме r_1 и r_2



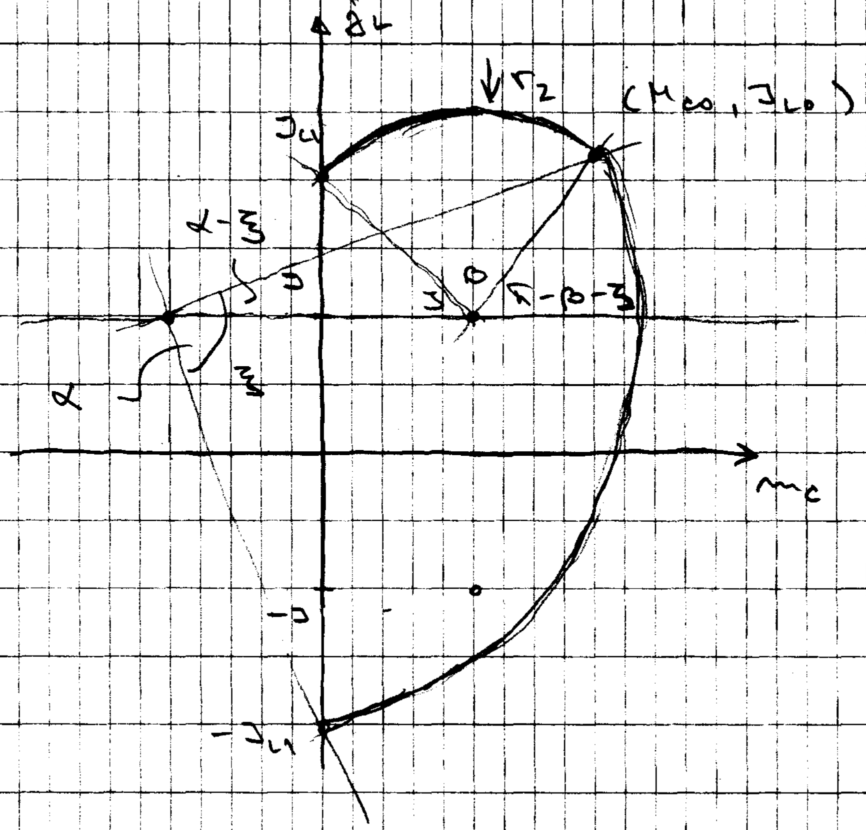
$$r_2 \sin\zeta = J_ц - J \qquad (1)$$

$$r_2 \cos\zeta = 1 \qquad (2)$$

$$r_1 \sin\xi = J_ц + J \qquad (3)$$

$$r_1 \cos\xi = 1 \qquad (4)$$

② одредимо граничне услове за J_{L0}



$$J_{L0} = J + r_2 \sin(\pi - \beta - \xi)$$

$$\underline{J_{L0} = J + r_2 \sin(\beta + \xi)} \qquad (5)$$

$$\underline{J_{L0} = J + r_1 \sin(\alpha - \xi)} \qquad (6)$$

$$r_2 \sin(\beta + \xi) = r_1 \sin(\alpha - \xi)$$

$$r_2 (\sin\beta \cos\xi + \cos\beta \sin\xi) =$$

$$= r_1 (\sin\alpha \cos\beta - \cos\alpha \sin\beta) \qquad (7)$$

елиминишемо $r_2\cos\zeta$, $r_2\sin\zeta$, $r_1\cos\xi$, $r_1\sin\xi$ користећи (1) − (4)

$$\sin\alpha - (J_{L1}+J)\cos\alpha = \sin\beta + (J_{L1}-J)\cos\beta \qquad (8)$$

имамо, уз $\frac{\alpha+\beta}{2} = \frac{\gamma}{2}$, $\frac{\beta-\alpha}{2} = \varphi$

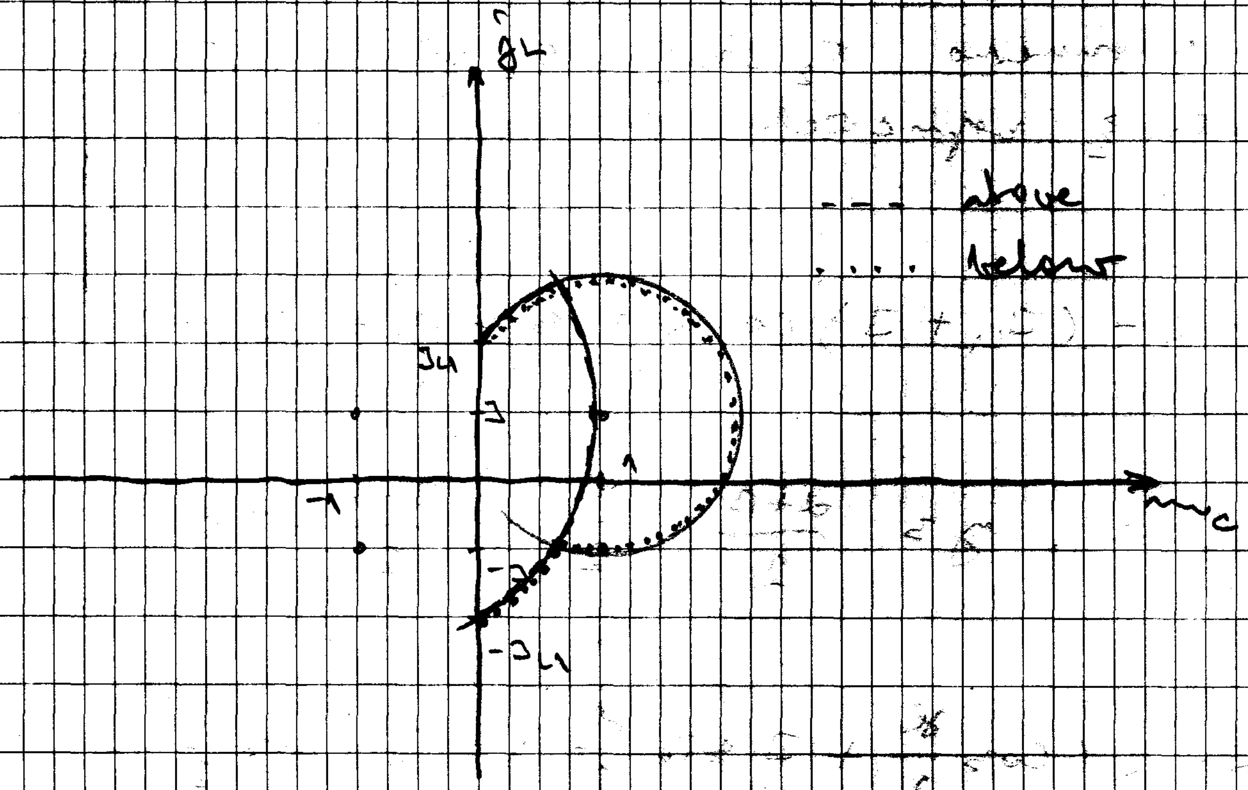
$$-\sin\varphi\left(\cos\frac{\gamma}{2} + J\sin\frac{\gamma}{2}\right) = J_{L1}\cos\frac{\gamma}{2}\cos\varphi \qquad (9)$$

③ одредимо другачије услове за M_{co}

$$M_{co} = 1 + r_2\cos(\pi-\beta-\zeta)$$

$$M_{co} = 1 - r_2\cos(\beta+\zeta) \qquad (10)$$

$$M_{co} = r_1\cos(\alpha-\xi) - 1 \qquad (11)$$

Im
--- above
... below
2i
i
-1
1
Re
-i
-2i

и з приравняваме

$$1 - r_2 \cos(\beta + \xi) = r_1 \cos(\alpha - \xi) - 1$$

$$\cos\varphi \left(\cos\frac{\gamma}{2} + J \sin\frac{\gamma}{2} \right) = J_{L1} \cos\frac{\gamma}{2} \sin\varphi + 1$$

(12)

из (12) и (9) елиминираме $J_{L1} \cos\frac{\gamma}{2}$

$$\cos\varphi = \cos\frac{\gamma}{2} + J \sin\frac{\gamma}{2} \qquad (13)$$

$$\boxed{\varphi = \pm \arccos\left(\cos\frac{\gamma}{2} + J \sin\frac{\gamma}{2} \right)} \qquad (14)$$

above resonance $\varphi = \frac{\beta - \alpha}{2} < 0$

below resonance $\varphi = \frac{\beta - \alpha}{2} > 0$

защо?. виж стр

$$\varphi = \begin{cases} -\arccos\left(\cos\frac{\chi}{2} + J\sin\frac{\chi}{2}\right) & 0 < \chi < \pi \quad \text{(above)} \\ \arccos\left(\cos\frac{\chi}{2} + J\sin\frac{\chi}{2}\right) & \pi < \chi < 2\pi \quad \text{(below)} \end{cases} \qquad (15)$$

$$J_u = -\frac{\sin\varphi}{\cos\frac{\chi}{2}} \qquad (16)$$

$$J_{\omega} = -(J^2 - 1)\, tg\, \frac{\chi}{2} \qquad (17)$$

$$M_{\omega} = -\frac{J\sin\varphi}{\cos\frac{\chi}{2}} \qquad (18)$$

заменом (16) у (0)

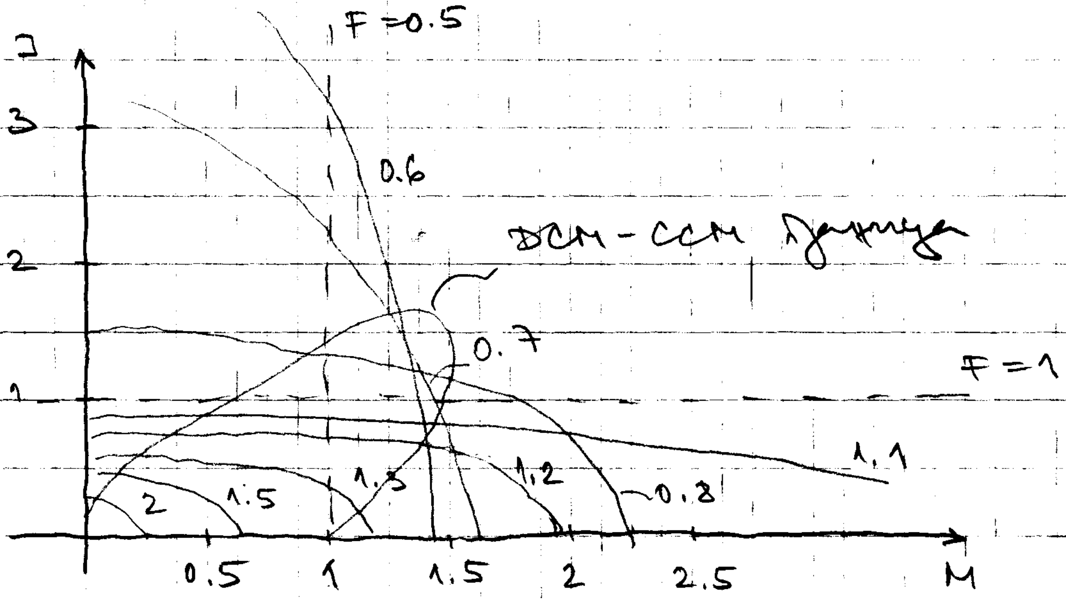
$$\boxed{M = \frac{2}{\chi}\left(\varphi - \frac{\sin\varphi}{\cos\frac{\chi}{2}}\right)} \quad - \quad (17)$$

(ово је $M(J,\chi)$, т.ј. $M(J,F)$,

експлицитно, без интеграла.

Иначе - highly transcedental

– излазне карактеристике J(M) F – задат,



апроксимација елипсама

$$\frac{M^2}{a^2} + \frac{J^2}{b^2} = 1$$

$$a^2 = M^2\Big|_{J=0} = \left(1 - \left(\frac{2}{\gamma}\right) \operatorname{tg}\frac{\gamma}{2}\right)^2$$

↳ open-circuit voltage

$$b^2 = J^2\Big|_{M=0} = \left(\frac{1 - \cos\frac{\gamma}{2}}{\sin\frac{\gamma}{2}}\right)^2$$

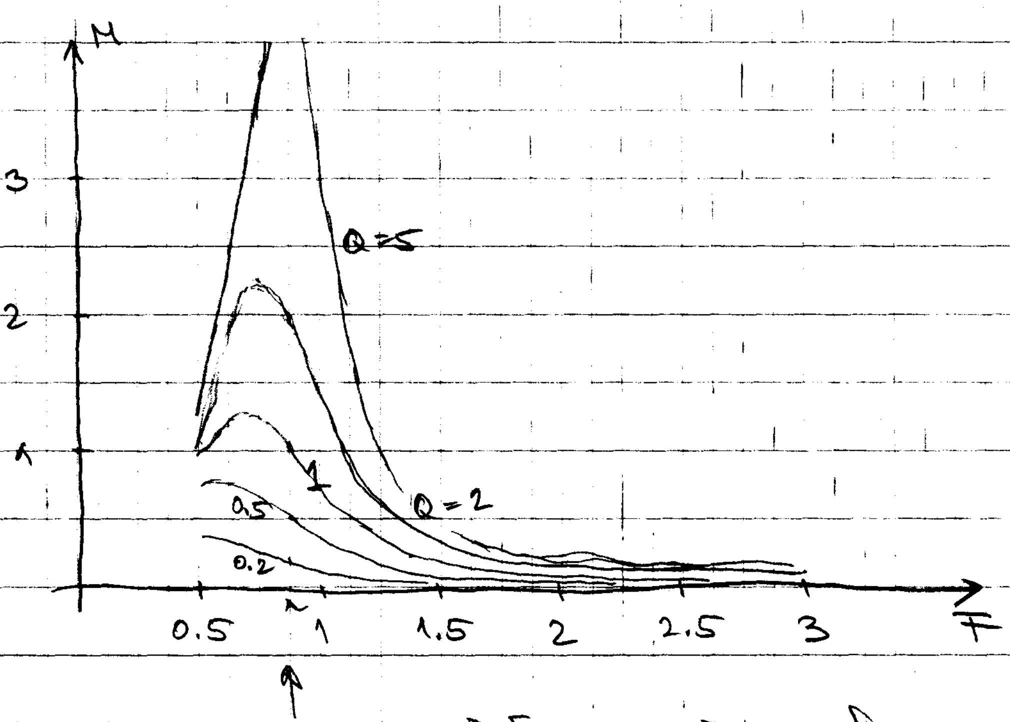
↳ "short"-circuit current

Control Plane Characteristics $M(F)$,
Q - param

$Q = \frac{R}{R_0}$ (parallel!)

$J = \frac{M}{Q}$

$$M = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{Q^2 b^2}}}$$



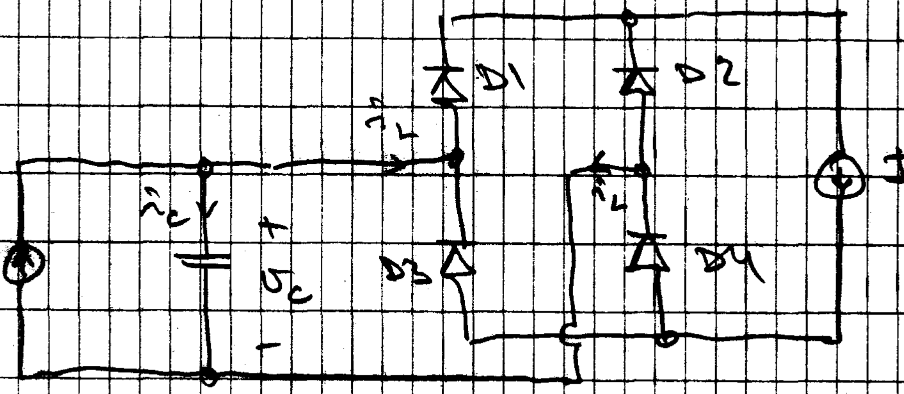
Fm, gyártójuk [illegible] adja

Discontinuous Conduction Mode

- слично SRC, кад је напон кондензатора нула - све 4 диоде у једну воде

- добија се и излаз и ниже резонансе ако је инвертор деблокиран

- разматрање:



$v_C(0) < 0$ - увек

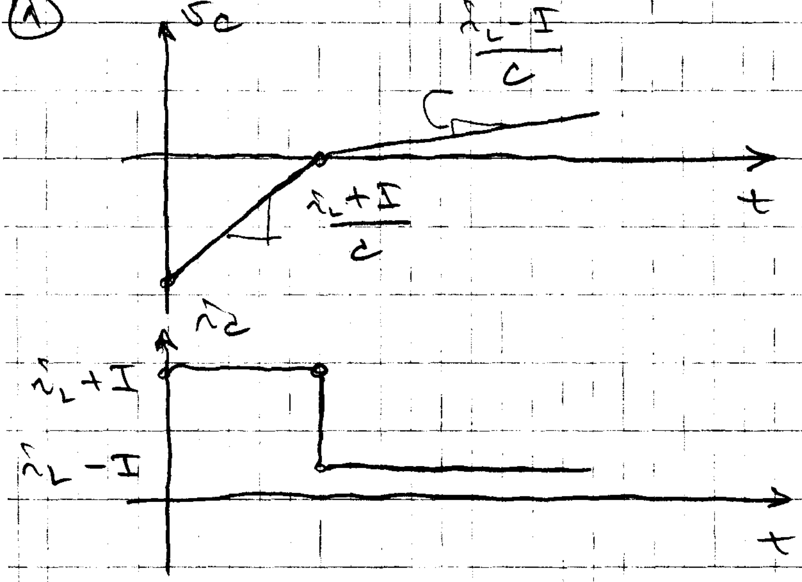
i_L = const током комутације

случајеви

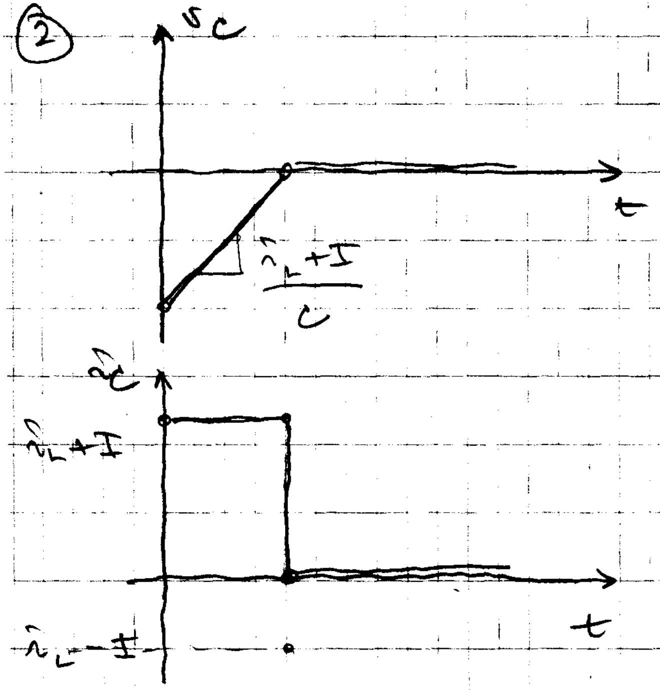
① $i_L > I$

② $i_L < I$

①



②



негативно i_C, $v_C < 0$, требало би да проводно сада све диоде воде!

$i_{D1} + i_{D2} = I$ $\qquad$ $i_{D1} = i_{D3} + i_L$

$i_{D3} + i_{D4} = I$ $\qquad$ $i_{D4} = i_{D2} + i_L$

сингуларни систем

додатни симетрију

$$\left.\begin{aligned}\hat{i}_{D1} &= \hat{i}_{D4}\\ \hat{i}_{D3} &= \hat{i}_{D2}\end{aligned}\right\}$$ због симетричних једнаких R_{on}

$$\hat{i}_{D1} + \hat{i}_{D2} = I$$

$$\hat{i}_{D1} - \hat{i}_{D3} = \hat{i}_L$$

$$\hat{i}_{D1} - \hat{i}_{D2} = \hat{i}_L$$

$$2\,\hat{i}_{D1} = I + \hat{i}_L$$

$$\hat{i}_{D1} = \hat{i}_{D4} = \frac{1}{2}\left(I + \hat{i}_L\right)$$

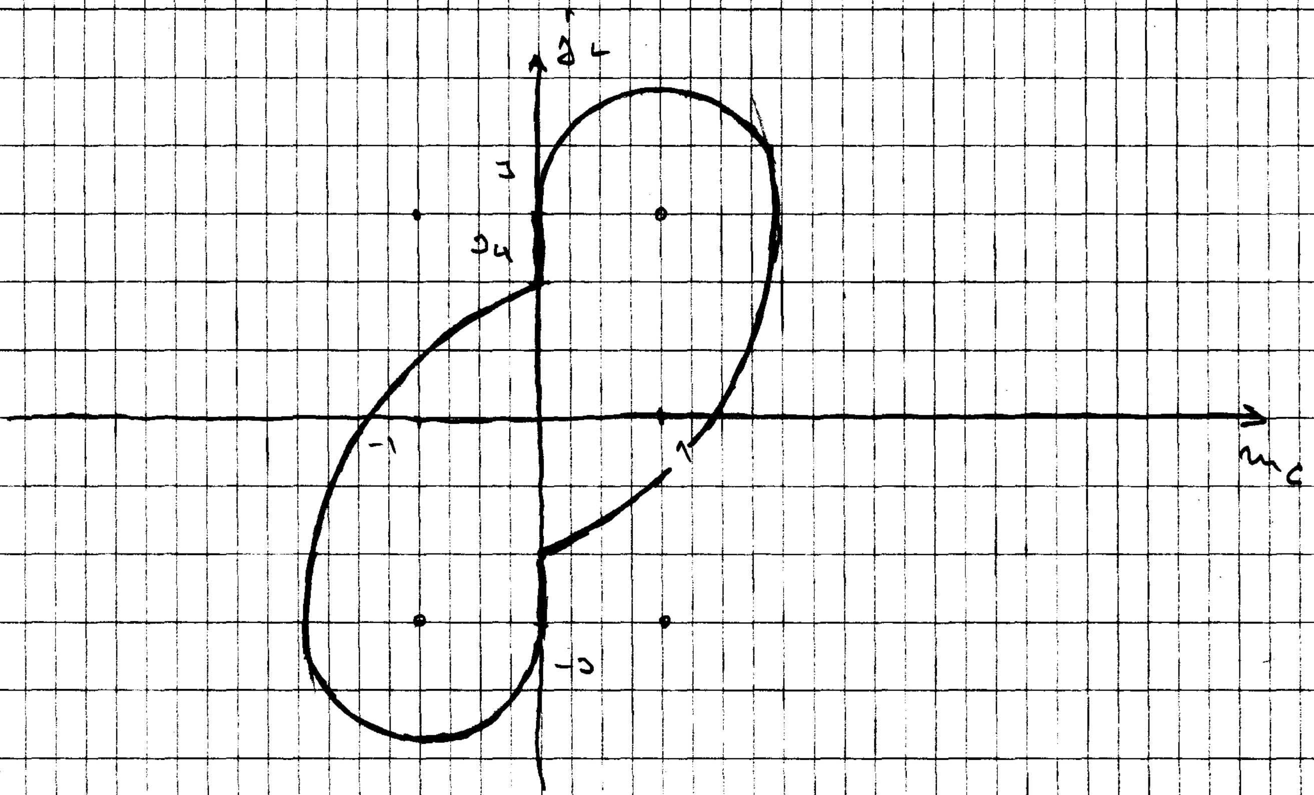
$$2\,\hat{i}_{D2} = I - \hat{i}_L$$

$$\hat{i}_{D2} = \hat{i}_{D3} = \frac{1}{2}\left(I - \hat{i}_L\right)$$

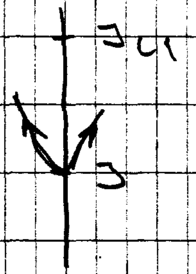
$\hat{i}_L < I$ - да $\hat{i}_D > 0$!

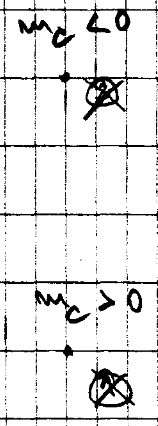
Значи да док $\hat{i}_L$ не порасте преко I !

— фазна [illegible]



не може





⇐

Q2 Q3 off,
не може [illegible]
[illegible] − 1

[illegible] [illegible] $m_c = 0$ $J_{L1} < |\hat{j}_L| < J$

CCM - DCM boundary

$J_{L1} < J$

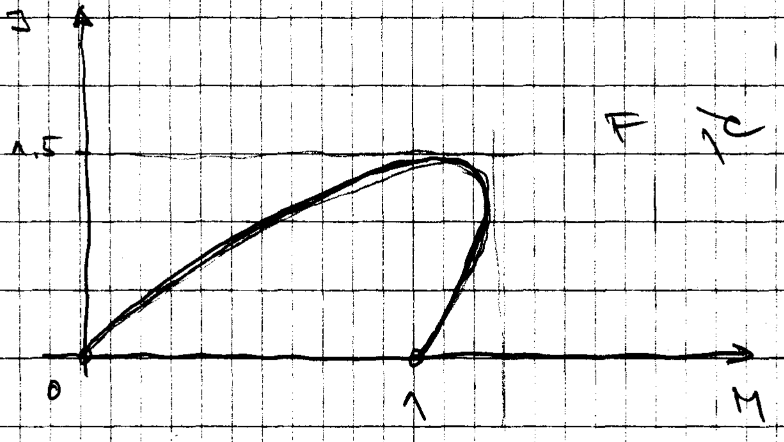
$J > J_{crit}$ за DCM

$J < J_{crit}$ за CCM

$$J_{crit} = -\frac{1}{2}\sin\gamma + \sqrt{\sin^2\frac{\gamma}{2} + \frac{1}{4}\sin^2\gamma}$$

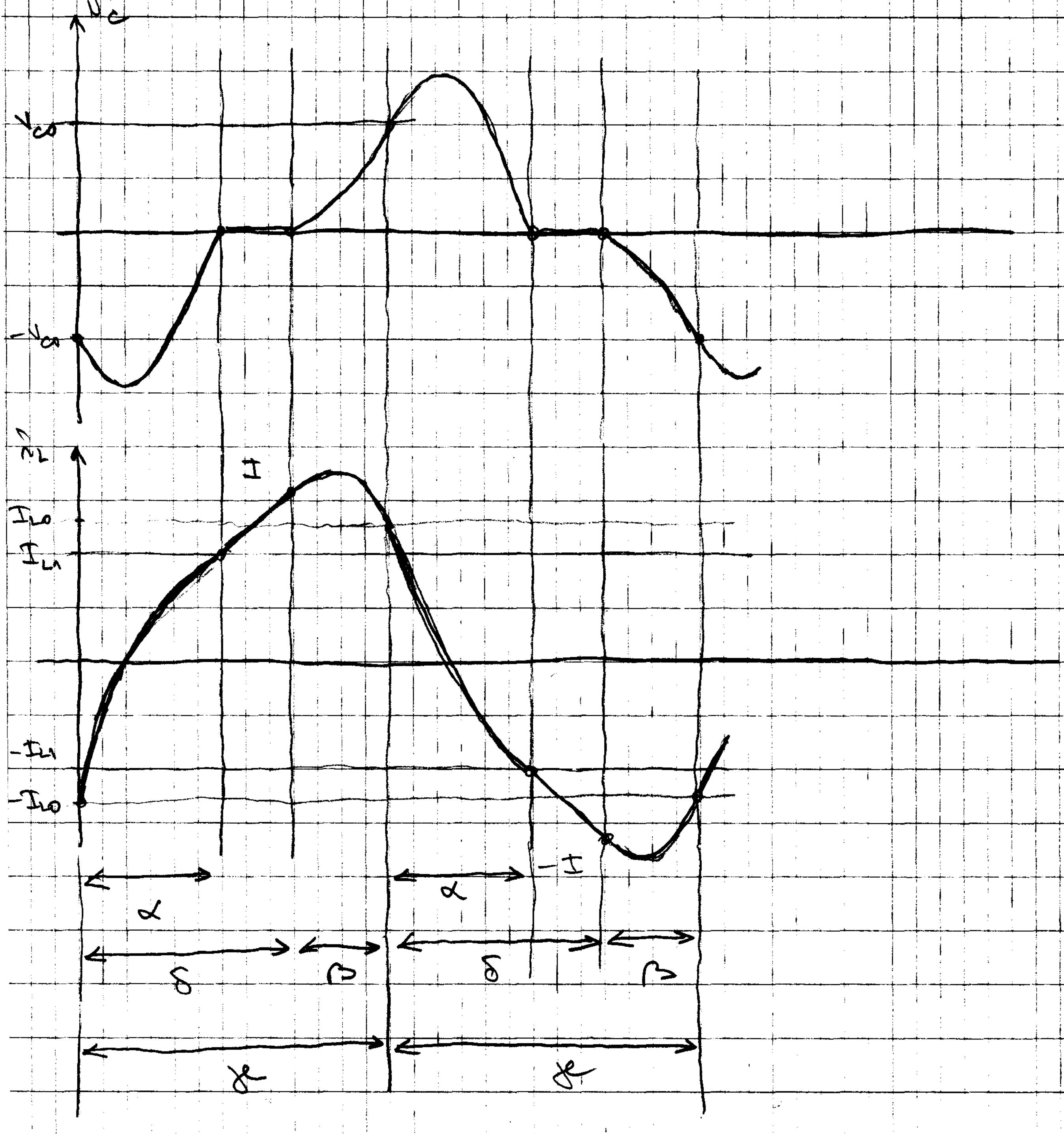
$$J_{L1} = -\frac{\sin\varphi}{\cos\frac{\gamma}{2}} \qquad \varphi = - - - .$$

+ нулеване



F /c dependency

Решение за DCM



једначине:

$$M_{co} = 1 - \cos\beta$$

$$J_{co} = J + \sin\beta$$

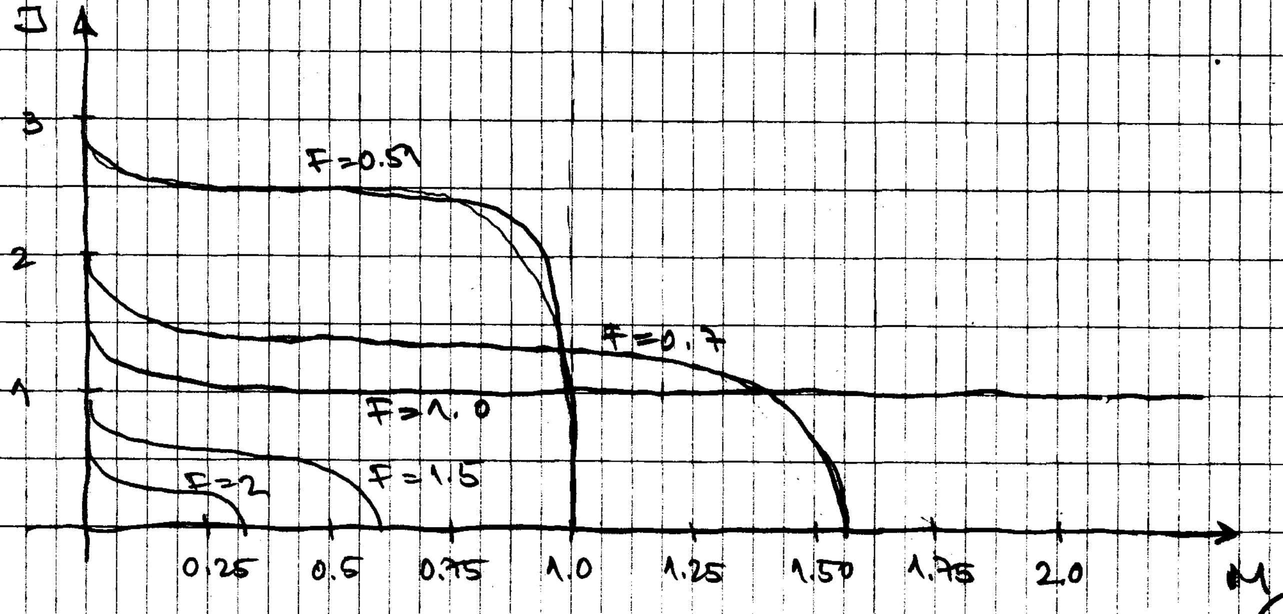
$$\cos(\alpha+\beta) - 2\cos\alpha = -1$$

$$-\sin(\alpha+\beta) + 2\sin\alpha + (\delta-\alpha) = 2J$$

$$\beta + \delta = \gamma$$

$$M = 1 + \frac{2}{\gamma}(J - \delta)$$

- не може се решити аналитички, мора нумерички



Квазирезонантни конвертори

- модификација PWL конвертора са циљем да се смањи switching loss.
- смањен (значајно) switching, повећан conduction

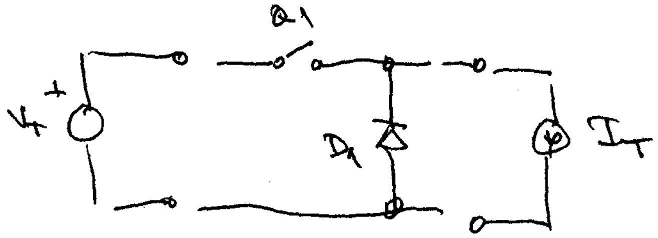
(1) мањи трансформатори и/или филтери са повећаним f_s

(2) мањи sw loss од неидеалних компоненти L_σ, C_{stray} капацитивности и сл., као и semiconductor device capacitances

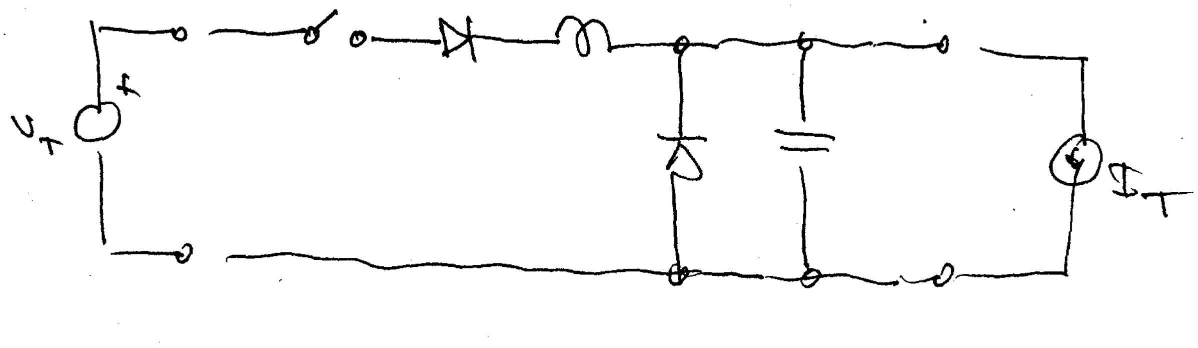
- веома пожељно, може у многу апликација

Type a Zero Current Switch

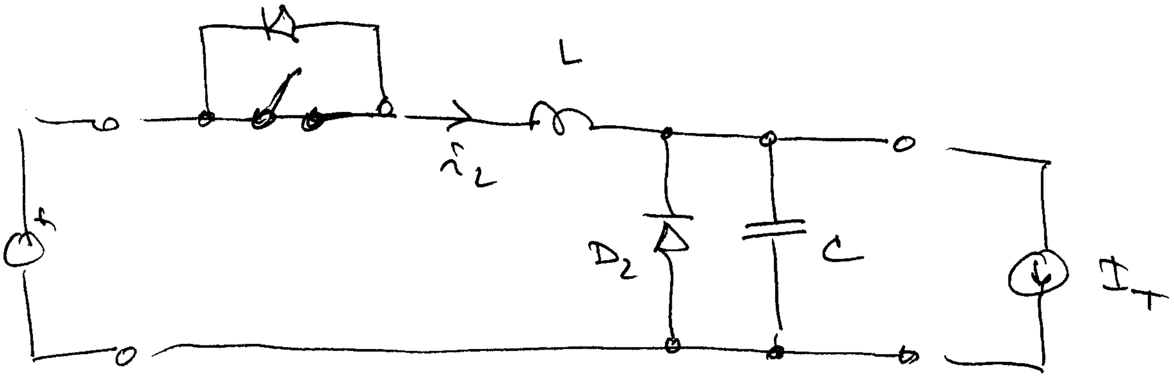
Standard PWM



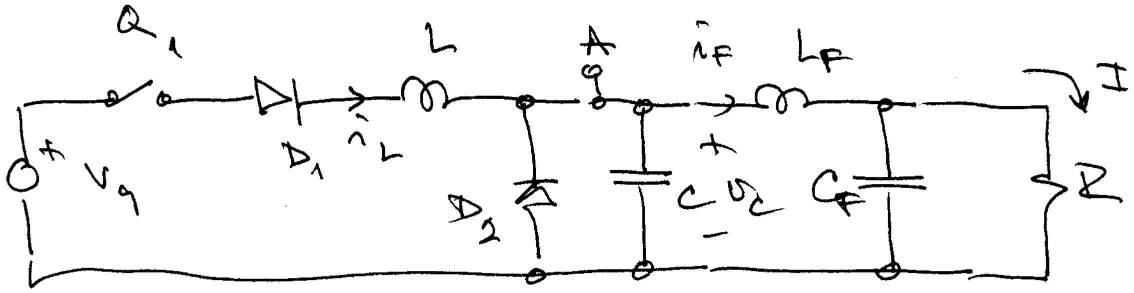
Half-wave ZCS



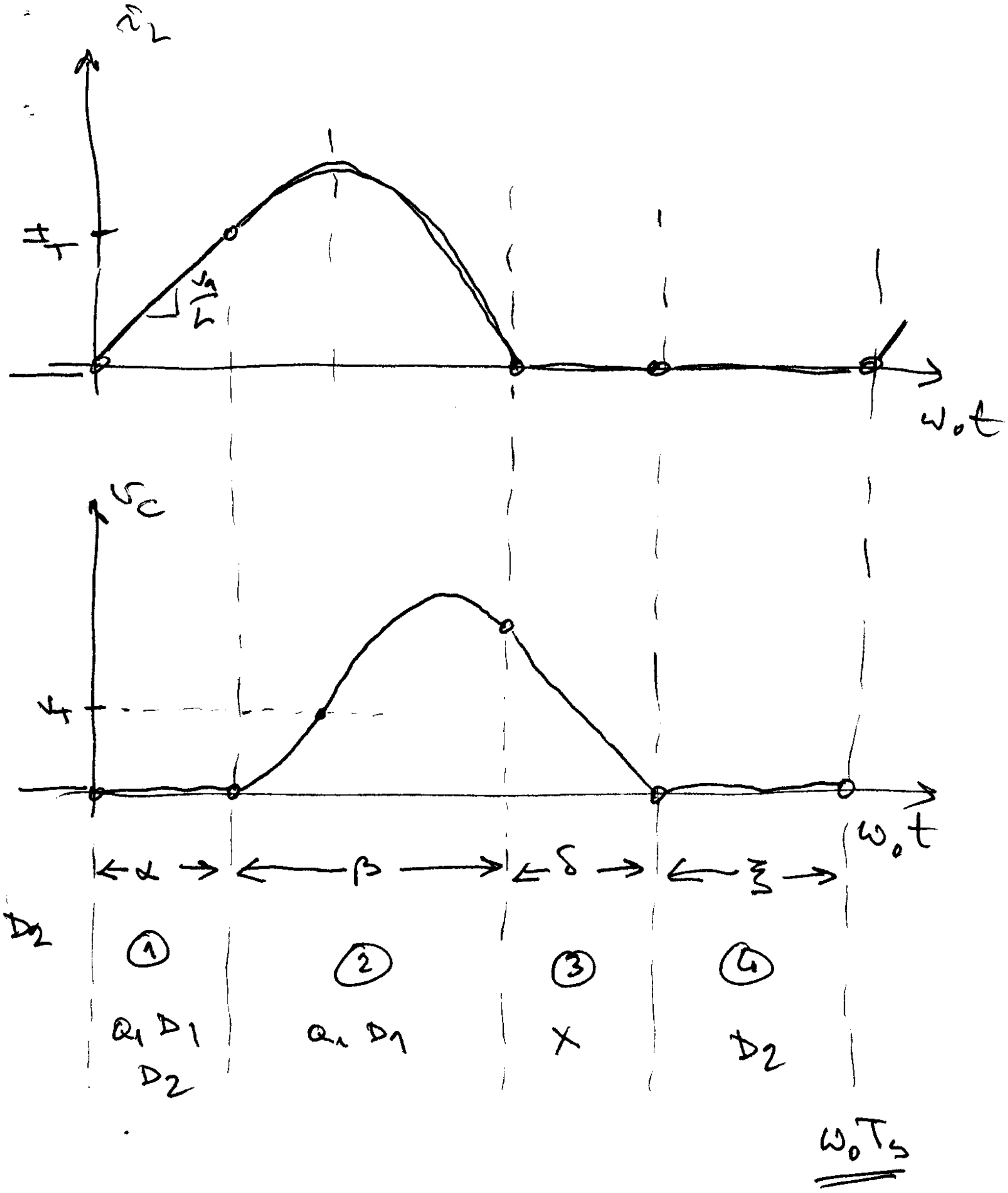
Full wave ZCS



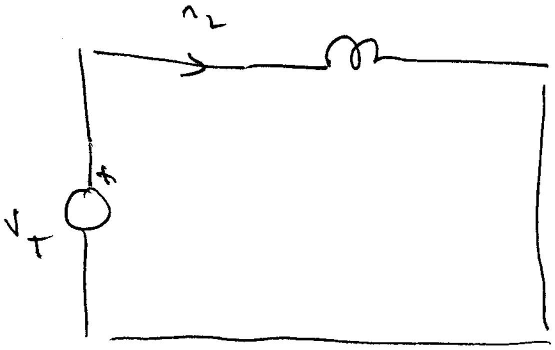
Buck converter Example



$i_F = I$



①

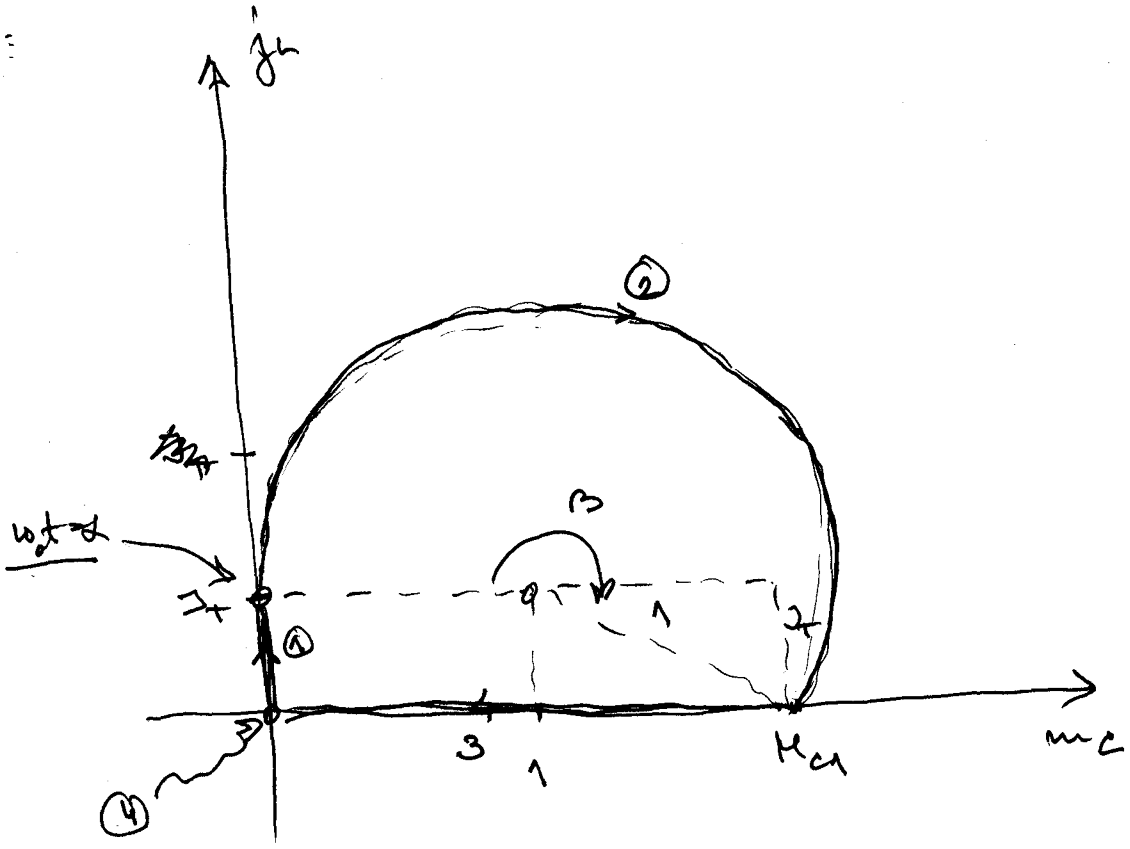


$$V_{base} = V_T$$

$$R_{base} = R_0 = \sqrt{\frac{L}{C}}$$

$$I_{base} = V_T / R_0$$

$$f_{base} = f_0 = \frac{1}{2\pi\sqrt{LC}}$$


jh
②
ωt=α
Jr
①
B
1
2r
3
1
Hc1
mc
④

$$\frac{1}{\omega_0}\frac{d\hat{j}_L}{dt} = 1 \qquad \hat{j}_L(0) = 0$$

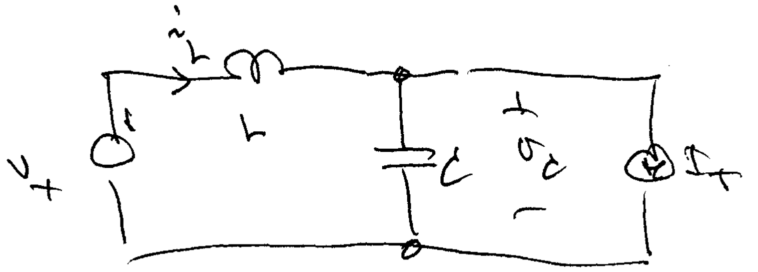
$$\Rightarrow \underline{\hat{j}_L(\omega_0 t) = \omega_0 t} \qquad \underline{m_C(\omega_0 t) = 0} \quad \text{II ј-на}$$

D_2 се гаси кад $\hat{i}_L$ достигне I_T

$\hat{i}_L = I_T \quad \omega_0 t = \alpha$ – interval ends

$$\boxed{\hat{j}_L(\alpha) = J_T = \alpha}$$ (solution for α)

Interval ②



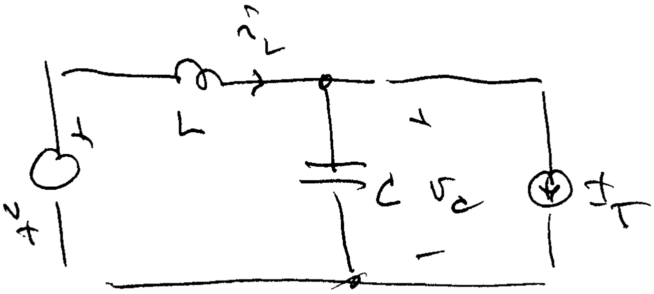
$$L\frac{di_L}{dt} = V_T - v_C \qquad \frac{1}{\omega_0}\frac{d\hat{j}_L}{dt} = 1 - m_C$$

$$C\frac{dv_C}{dt} = \hat{i}_L - I_T \qquad \frac{1}{\omega_0}\frac{dm_C}{dt} = \hat{j}_L - J_T$$

Трајекторија је кружни лук са центром на

$(m_C, \hat{j}_L) = (1, J_T)$

$$: q_2 = \int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha+\beta}{\omega_0}} i_L \, dt$$



$$q_2 = \underbrace{\int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha+\beta}{\omega_0}} i_C \, dt}_{= q_C = C\Delta v_C} + \underbrace{\int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha+\beta}{\omega_0}} I_T \, dt}_{= I_T \frac{\beta}{\omega_0}} = C V_{ca} + I_T \frac{\beta}{\omega_0}$$

$$= C(V_{ca} - 0) =$$

$$= C V_{ca}$$

$$\bar{i}_L = \frac{1}{T_s}\left(\frac{1}{2}\frac{\alpha}{\omega_0} I_T + C V_{ca} + I_T \frac{\beta}{\omega_0}\right) =$$

$$= \frac{1}{\omega_0 T_s}\left(\frac{1}{2}\alpha I_T + \omega_0 C V_{ca} + \beta I_T\right)$$

$$\bar{J}_L = \frac{F}{2\pi} J_T \left(\frac{1}{2}\alpha + \beta + \frac{M_{ca}}{J_T}\right)$$

$$\bar{J}_L = J_T F \frac{1}{2\pi}\left(\frac{1}{2} J_T + \pi + \arcsin J_T + \frac{1}{J_T}\left(1 + \sqrt{1 - J_T^2}\right)\right)$$

$$\boxed{j_{L\,peak} = 1 + J_T}$$

$$\boxed{m_{c\,peak} = 2}$$

$$\boxed{\beta = \pi + \sin^{-1} J_T}$$

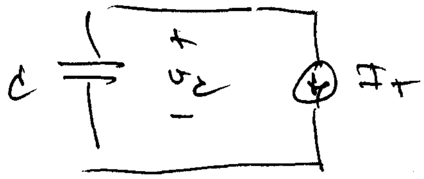
$j_L(\alpha+\beta) = 0$ - гаси се Q1 D1

$m_c(\alpha+\beta) = M_{c1} = 1 + \sqrt{1-J_T^2}$

$$\boxed{J_T \leq 1}$$

Interval ③

$C\dfrac{dv_c}{dt} = -I_T, \quad v_c(\alpha+\beta) = V_{c1}$



$\dfrac{1}{\omega_0}\dfrac{dm_c}{dt} = -J_T, \quad m_c(\alpha+\beta) = M_{c1}$

$m_c(\omega_0 t) = M_{c1} - J_T(\omega_0 t - \alpha - \beta)$

завршава се када $v_c = 0$, D_2 почне да проводи

$m_c(\alpha+\beta+\delta) = 0 = M_{c1} - J_T\delta$

$$\delta = \frac{M_{c1}}{J_T} = \frac{1}{J_T}\left(1 + \sqrt{1-J_T^2}\right)$$

Interval ④

исто као $D'T_s$ интервал код PWL

$i_L = 0$ $\qquad$ $j_L = 0$

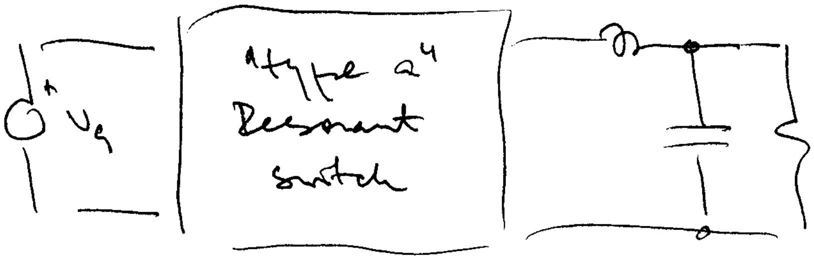
$v_c = 0$ $\qquad$ $m_c = 0$

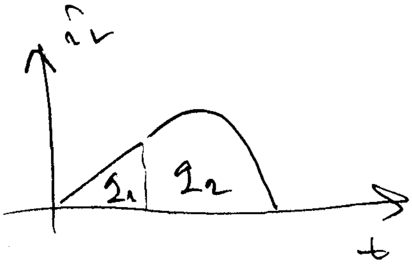
$$\omega_0 T_s = \alpha + \beta + \delta + \xi$$

minimum switching interval

$$\underline{\omega_0 T_s \geq \alpha + \beta + \delta} \qquad (\underline{\underline{\xi > 0}})$$

и средња вредност





$$\overline{i_L} = \frac{1}{T_s}\int_0^{T_s} i_L \, dt = \frac{q_1 + q_2}{T_s}$$

$$q_1 = \int_0^{\alpha/\omega_0} i_L \, dt = \frac{1}{2}\left(\frac{\alpha}{\omega_0}\right) I_T$$

$$P_{out} = P_{in}$$

$$\bar{I}_T \langle v_c \rangle = \bar{V}_T \langle \hat{i}_L \rangle$$

$$J_T \, \bar{m}_c = 1 \cdot \bar{\hat{j}}_L$$

$$\bar{m}_c = \frac{1}{J_T} \bar{\hat{j}}_L$$

$$\bar{m}_c = F \underbrace{\frac{1}{2\pi} \left(\frac{1}{2} J_T + \pi + \sin^{-1} J_T + \frac{1}{J_T} \left(1 + \sqrt{1 - J_T^2}\right)\right)}_{P(J_T)}$$

$$\boxed{\bar{m}_c = F \, P(J_T)}$$

Switch conversion ratio

$$\mu = \frac{\langle v_c \rangle}{V_T} = \frac{\langle \hat{j}_L \rangle}{J_T} = F P(J_T) \qquad \cancel{F = \frac{f_s}{f_0}}$$

controllable by $F = \frac{f_s}{f_0}$

depends on $J_T = \frac{I_T R_0}{V_T}$

Output Plane

Mode Boundaries:

1. $J_T \leq 1$ - otherwise no zero current switching

2. $\xi \geq 0 \Rightarrow \frac{2\pi}{F} \geqslant \alpha + \beta + \delta$

$$\frac{2\pi}{F} \geq J_T + \pi + \sin^{-1} J_T + \frac{1}{J_T}\left(1 + \sqrt{1 - J_T^2}\right)$$

$$\frac{2\pi}{F} \geqslant \frac{2\pi}{F}\langle m_c \rangle + \frac{1}{2} J_T$$

$$\langle M_c \rangle \leq 1 - \frac{J_T F}{4\pi} < 1$$

$$0 \leq \langle m_c \rangle \leq 1 - \frac{J_T F}{4\pi}$$

$$0 \leq J_T \leq 1$$

3a buck

$$M = \frac{V}{V_g} = \langle m_c \rangle$$

$$J = \frac{I R_0}{V_g} = J_T$$

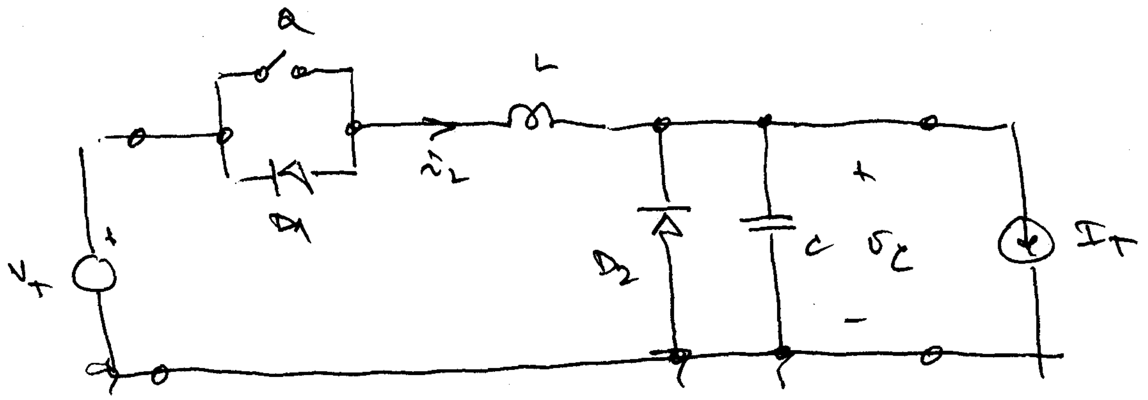
- Control Plane Characteristics – учење од рачуна.

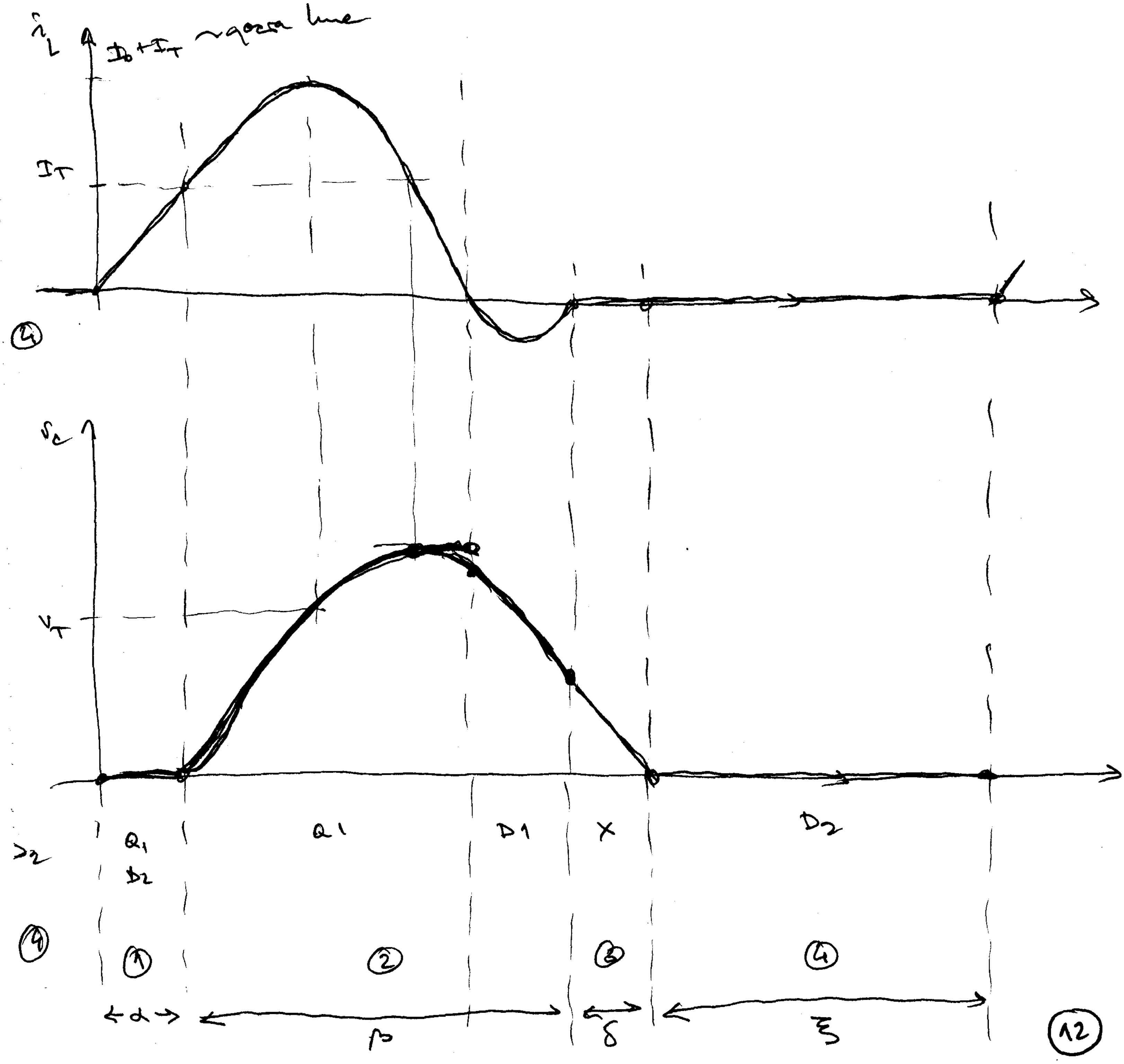
$M = JQ$ Q – сумарно R . . .

$M = FP\left(\frac{M}{Q}\right)$ – мора нумерички

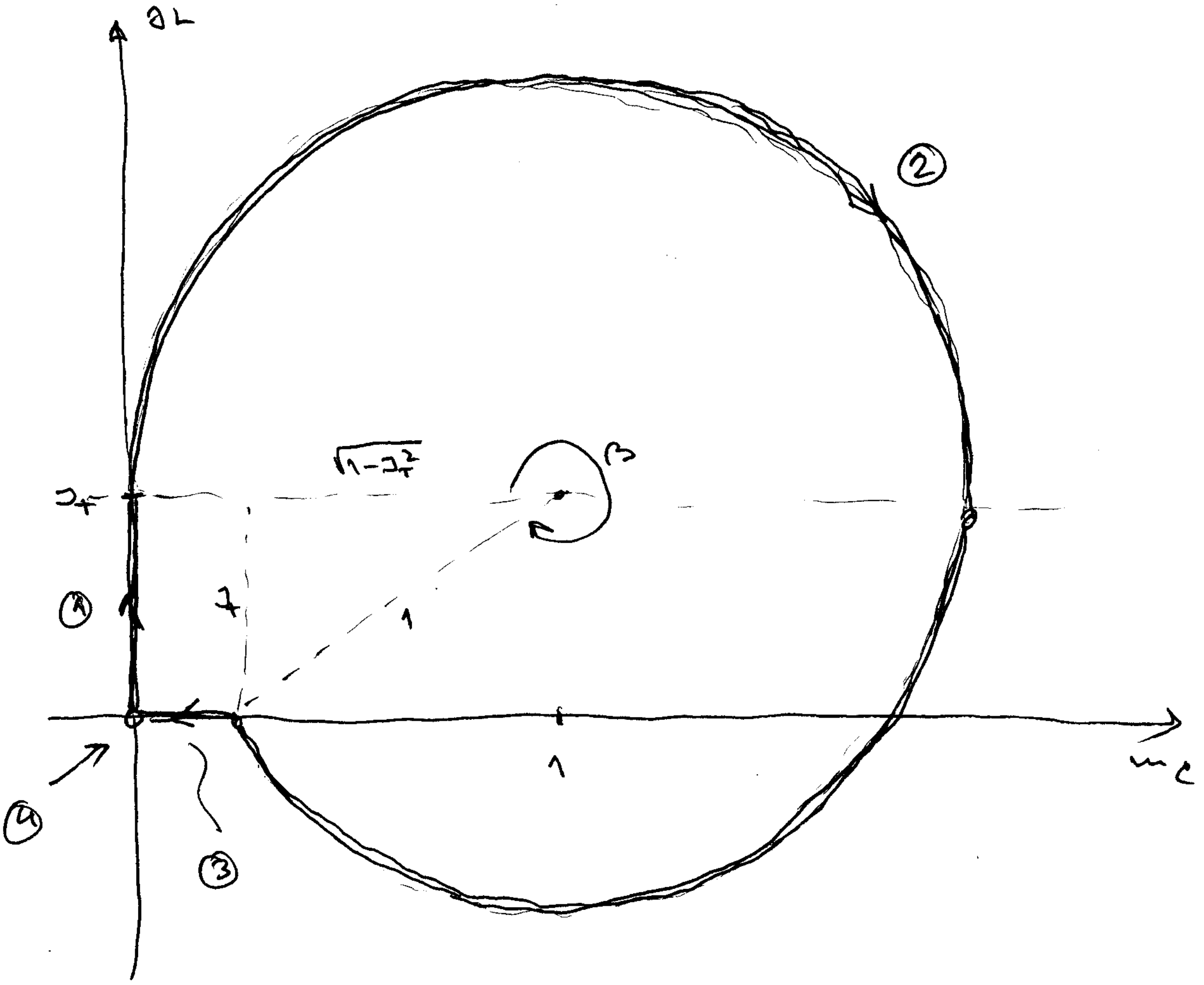
јако нелинеарно

Full wave : "type a" zero current switch





Normalized State Plane



Анализа је иста као и за half-wave case, осим за интеграл ②

$$\beta = \begin{cases} \pi + \arcsin J_T & \text{— half wave} \\ 2\pi - \arcsin J_T & \text{— full wave} \end{cases}$$

$$M_{cn} = \begin{cases} 1 + \sqrt{1 - J_T^2} & \text{— half wave} \\ 1 - \sqrt{1 - J_T^2} & \text{— full wave} \end{cases}$$

У оба случаја

$$\langle m_c \rangle = F \frac{1}{2\pi} \left(\frac{1}{2} \alpha + \beta + \frac{M_{cn}}{J_T} \right)$$

замена

$$\bar{m}_c = F \frac{1}{2\pi} \left(\frac{1}{2} J_T + 2\pi - \arcsin J_T + \frac{1}{J_T} (1 - \sqrt{1 - J_T^2}\right.$$

In general, switch conversion ratio can be written as

$P(J_T) \cong 1$ (унутар 4%)

$$\mu = \frac{\bar{\sigma}_c}{V_T} = \bar{m}_c = F\,P(J_T) \cong F \text{ (within 4\%)}$$

$P(J_T)$ се мења у ~~зависности~~ зависности од случаја (half / full wave).

Решение:

$$\mu = \frac{\hat{v}_c}{V_T} = \bar{m}_c = F\,P(J_T)$$

half-wave: $$P(J_T) = k_{1/2}(J_T) \triangleq \frac{1}{2\pi}\left(\frac{J_T}{2} + \pi + \arcsin J_T + \frac{1}{J_T}\left(1+\sqrt{1-J_T^2}\right)\right)$$

full-wave: $$P(J_T) = k_1(J_T) \triangleq \frac{1}{2\pi}\left(\frac{J_T}{2} + 2\pi - \arcsin J_T + \frac{1}{J_T}\left(1-\sqrt{1-J_T^2}\right)\right)$$

за full-wave case:

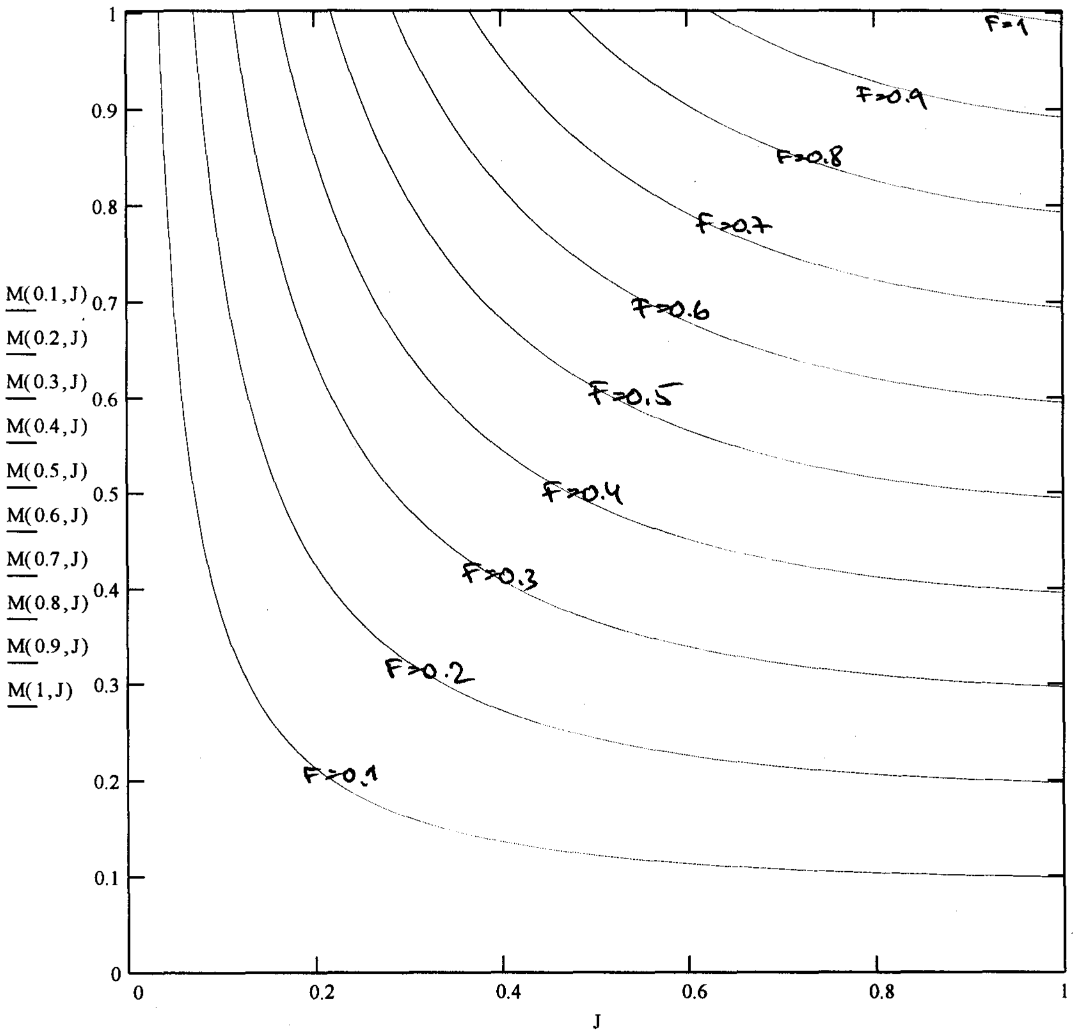
$\mu \cong F = \frac{f_s}{f_0}$ voltage source, controlable by F, исто како PWM са D → F

за switching regulator, full-wave треба многу помал frequency range него half-wave за да се регулира излез

$$P(J) := \frac{1}{2\cdot\pi}\cdot\left[\frac{1}{2}\cdot J + \pi + \operatorname{asin}(J) + \frac{1}{J}\cdot\left(1 + \sqrt{1 - J^2}\right)\right]$$

$$M(F,J) := F\cdot P(J)$$

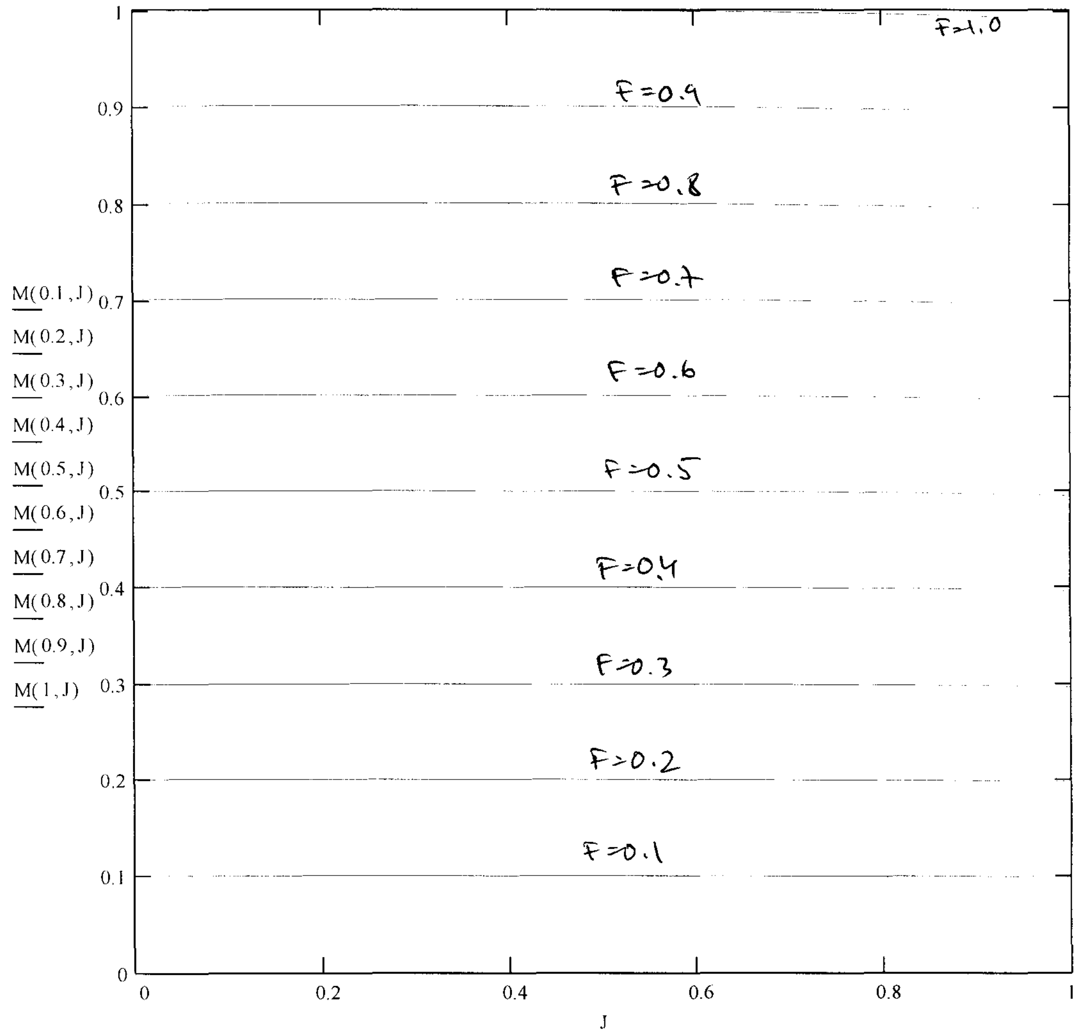
$$J := 0.01, 0.02 .. 1$$



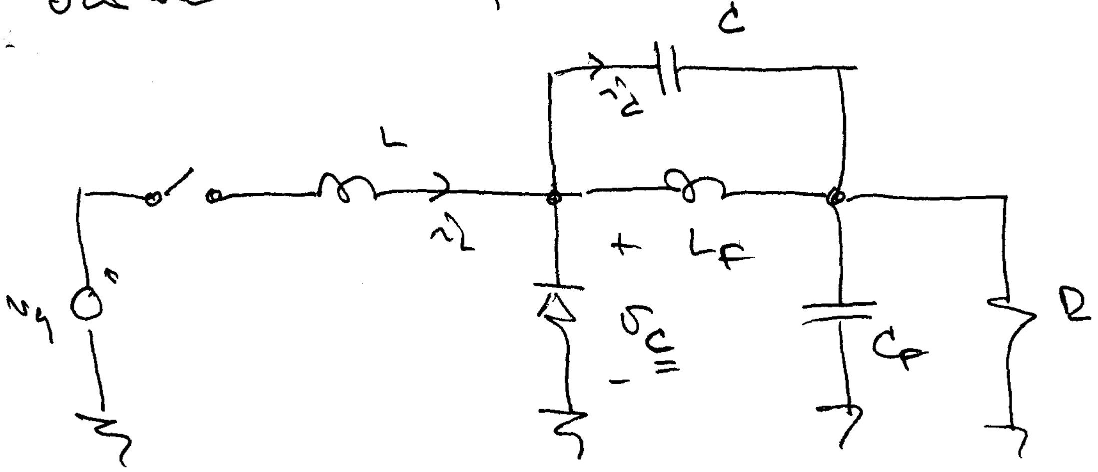
$$P(J)\quad \frac{1}{2\cdot\pi}\cdot\left[\frac{1}{2}\cdot J + 2\cdot\pi - \mathrm{asin}(J) + \frac{1}{J}\cdot\left(1 - \sqrt{1 - J^2}\right)\right]$$

$$M(F,J)\quad F\cdot P(J)$$

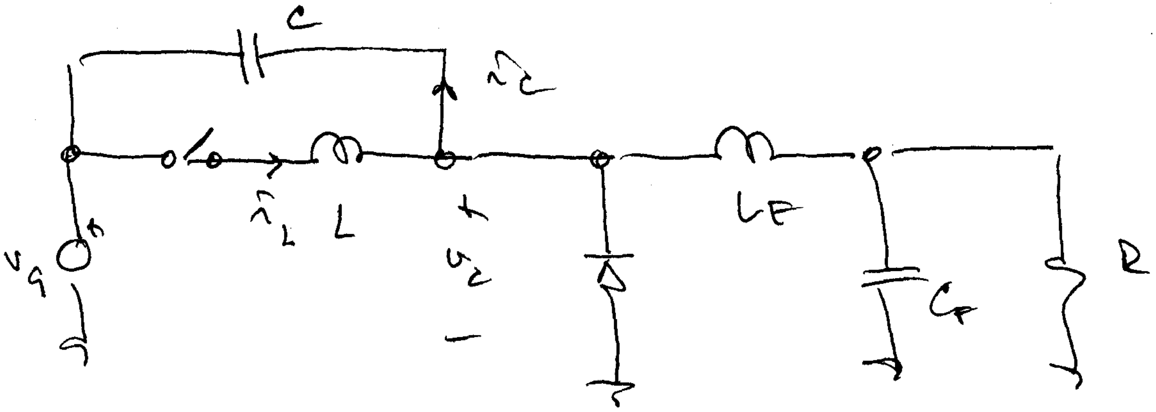
$$J : 0.01, 0.02 .. 1$$



Останалите топологии



исти и [illegible] обикн. нула и [illegible]

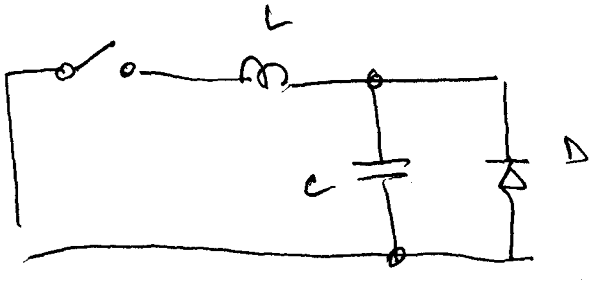


исто нула и [illegible], type a resonant switch

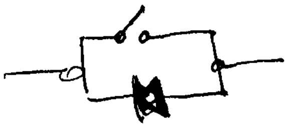
<u>Тест за тип резонантен прекидач</u>

V, C_R → кратко
I, L_F → отворено

type "a" се [illegible] [illegible] на



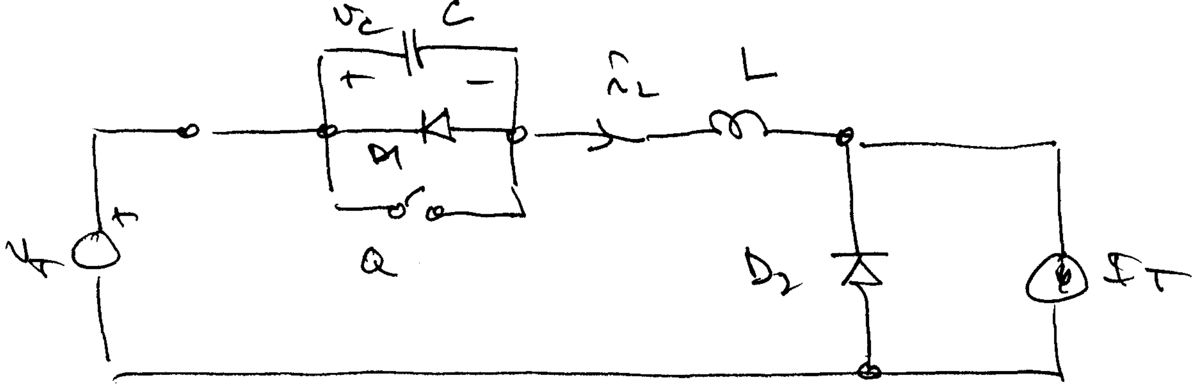

– half wave

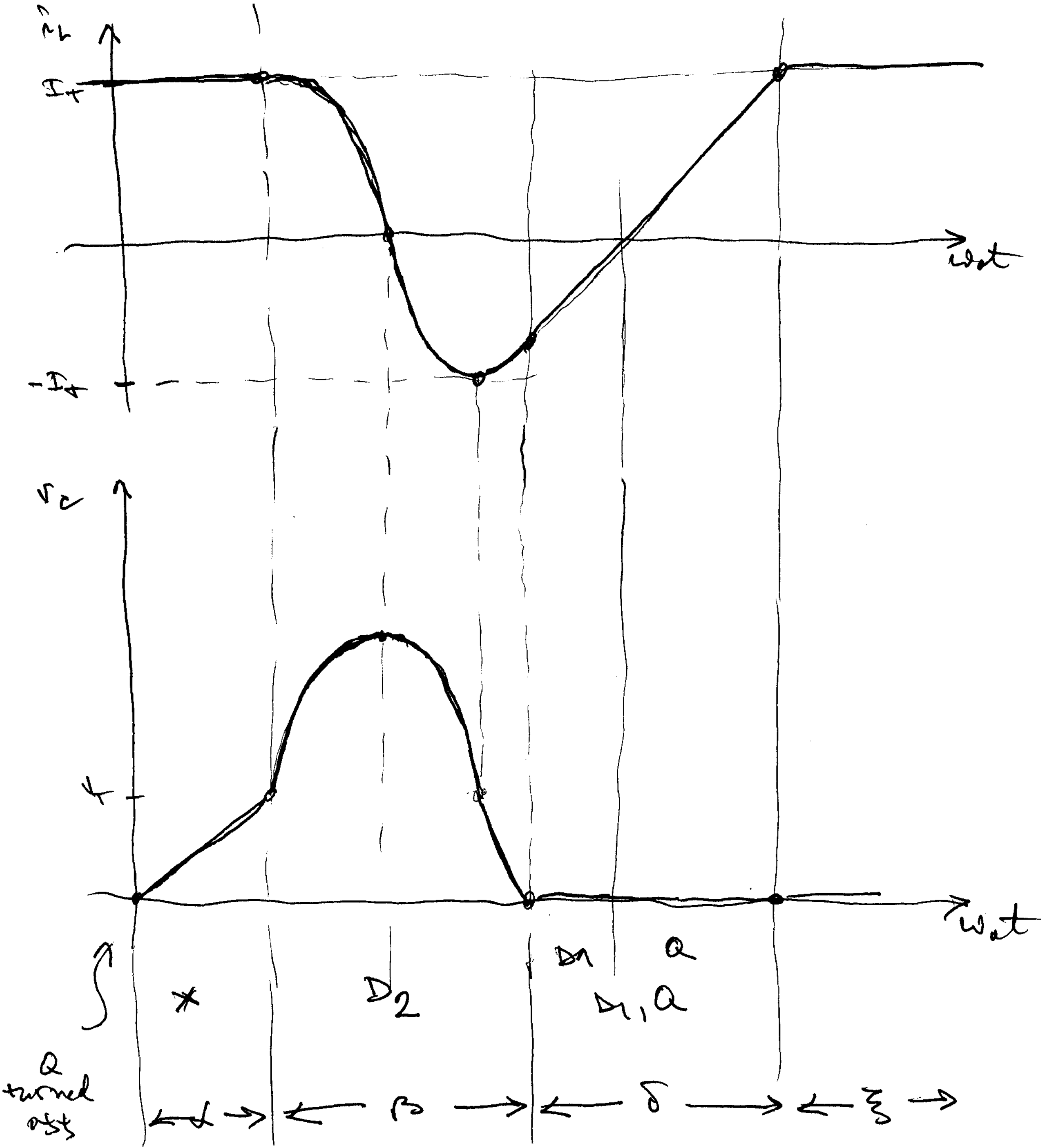

– full wave

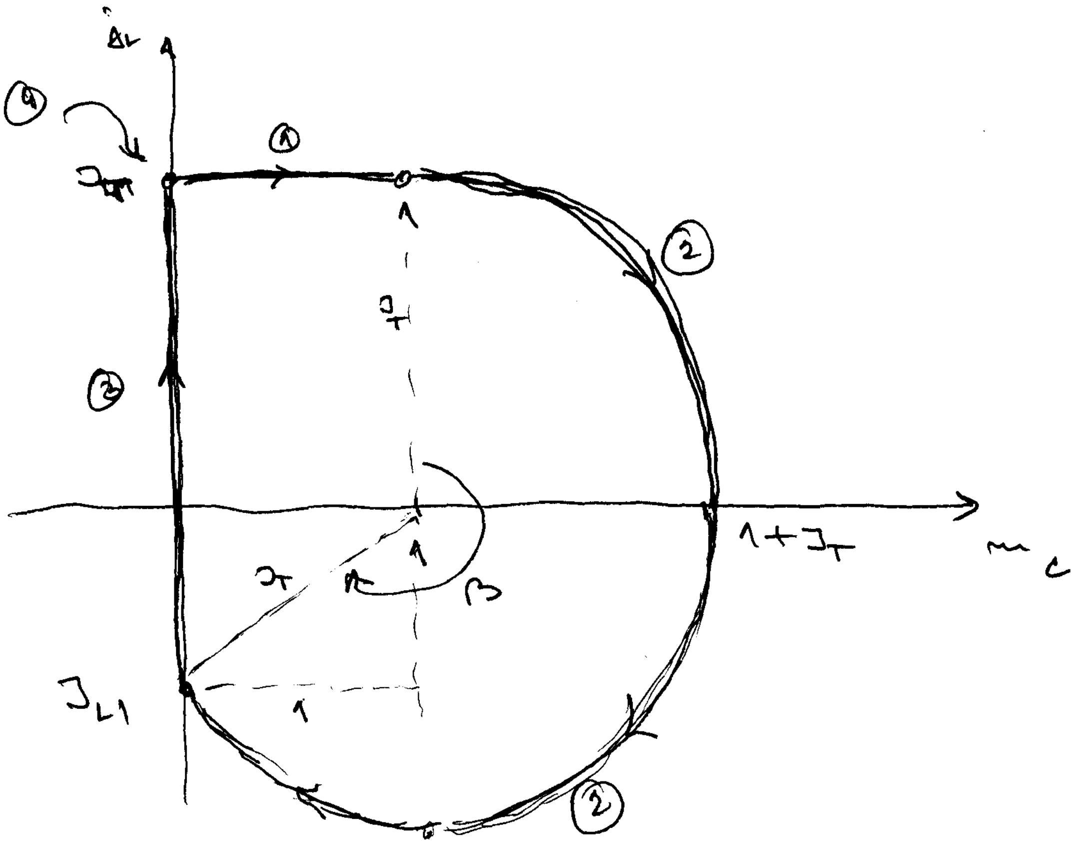
The Zero-Voltage resonant switch

"type 6" ZVS

Basic Network



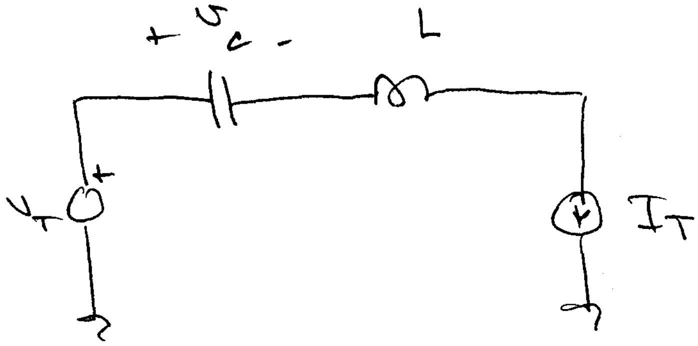




$$J_{L1} = \sqrt{J_T^2 - 1}$$

ⓐ – [illegible] када се Q искључи.

– у [illegible] De искључено



коло дегенерисано

$$v_c = 0 + \frac{I_T}{C} \cdot t = \frac{I_T}{C} \cdot t$$

$$\frac{1}{\omega_0} \dot{m}_c = I_T \qquad m_c(0) = 0$$

$$m_c = I_T \omega_0 t$$

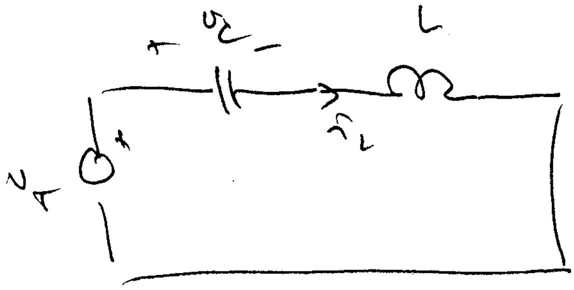
завршава се кад [illegible] D2

$$m_c(\alpha) = 1$$

$$1 = I_T \alpha$$

$$\boxed{\alpha = \frac{1}{I_T}}$$

② води D_2



улазак у $(m_C, j_L) = (1, 0)$

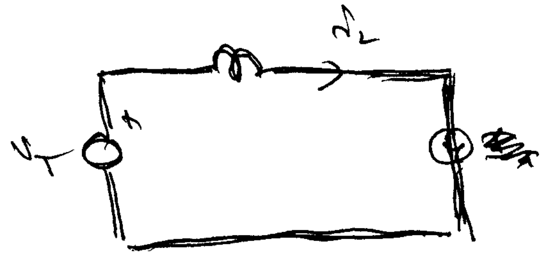
одмах даље не проводе диода D_1 $(m_C > 0)$

$$\beta = \pi + \arcsin \frac{1}{J_T}$$

$$J_{L1} = \sqrt{J_T^2 - 1}$$

$\underline{J_T \geq 1}$ – $\underline{\text{захтева се}}$

③ проводи D_1 (касније и Q_1), води D_2



$$\frac{1}{\omega_0} \frac{dj_L}{dt} = 1 \qquad j_L(\alpha+\beta) = -J_{L1}$$

$$j_L(\omega_0 t) = -J_{L1} + \omega_0 t - (\alpha+\beta)$$

задржава се док

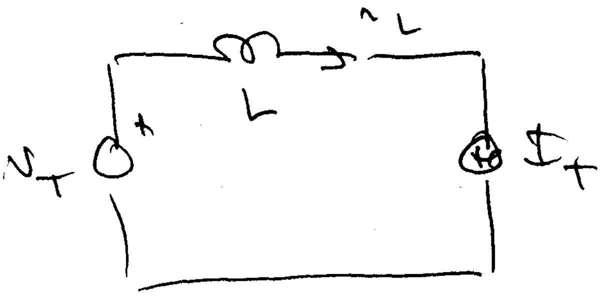
$$j_L(\alpha+\beta+\delta) = J_T$$

$J_T = - J_{L1} + \delta$

$$\boxed{\delta = J_T + J_{L1} - J_T + \sqrt{J_T^2 - 1}}$$

Interval ④ Q_2 води

$\alpha + \beta + \delta \le \omega_0 t \le \alpha + \beta + \delta + \zeta$)

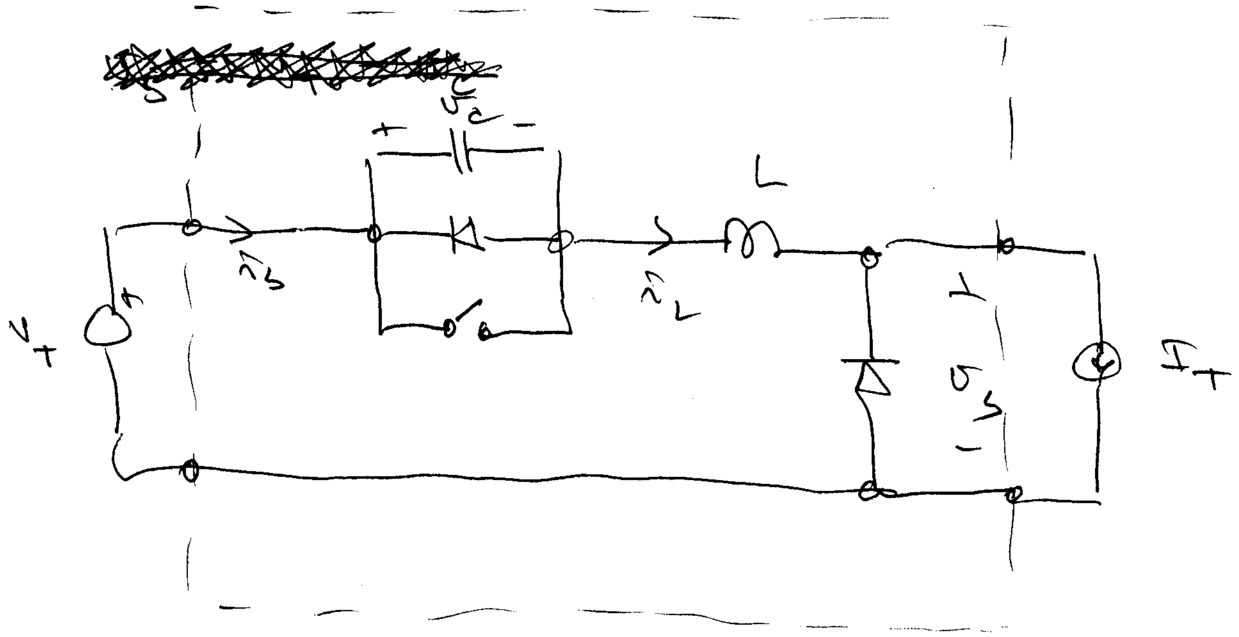


$i_L = I_T$

$v_C = 0$

а води, струја неодређено

Усредњавање



$$v_S = V_T - v_C - v_L$$

$$\overline{v_S} = \overset{V_T}{\cancel{\overline{V_T}}} - \overline{v_C} - \overset{0}{\cancel{\overline{v_L}}}$$

$$\overline{v_S} = V_T - \overline{v_C}$$

Нормализација:

$$\langle m_S \rangle = 1 - \langle m_C \rangle$$

циљ: наћи $\overline{m_C}$

$$\bar{v}_c = \frac{1}{T_s}\int_0^{T_s} v_c(t)\,dt = \frac{\lambda_1+\lambda_2}{T_s}$$

$$\lambda_1 = \frac{1}{2}\left(\frac{\alpha}{\omega_0}\right)(V_T)$$

λ_2 - flux-linkage arguments

$$v_c = V_T - v_L \quad \frown \text{ интеграл } ②$$

$$\lambda_2 = \int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha+\beta}{\omega_0}} v_c\,dt = \int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha+\beta}{\omega_0}} v_T\,dt + \int_{\frac{\alpha}{\omega_0}}^{\frac{\alpha+\beta}{\omega_0}} v_L\,dt = \lambda_T + \lambda_L$$

$$\boxed{\lambda_T = V_T\,\frac{\beta}{\omega_0}}$$

$\lambda_L = L\,\Delta\,\hat{i}_L$ - КЕ у интегралној форми

$$\boxed{\lambda_L = L\left(\hat{i}_L\left(\frac{\alpha+\beta}{\omega_0}\right) - \hat{i}_L\left(\frac{\alpha}{\omega_0}\right)\right) = L(-I_{L1} - I_T)}$$

$$\lambda_2 = V_T\,\frac{\beta}{\omega_0} - L(-I_{L1} - I_T)$$

$$\bar{\sigma}_2 = \frac{\lambda_1 + \lambda_2}{T_s} =$$

$$= \frac{1}{\omega_0 T_s}\left(\frac{1}{2}\alpha V_T + \beta V_T + \omega_0 L\,(I_{L1} + I_T)\right)$$

Нормализација:

$$\bar{m}_c = \frac{F}{2\pi}\left(\frac{1}{2}\alpha + \beta + J_{L1} + J_T\right)$$

замена за α, β и J_{L1}

$$\bar{m}_c = \frac{F}{2\pi}\left(\frac{1}{2}\,\frac{1}{J_T} + \pi + \arcsin\left(\frac{1}{J_T}\right) + \right.$$

$$\left. + J_T + \sqrt{J_T^2 - 1}\right)$$

$$= F\,P(J_T)$$

исто како и за half-wave zero current (type a) switch, само што на место J_T иде $\frac{1}{J_T}$

Средња вредност излазног напона

$$\overline{m_s} = 1 - \overline{m_c}$$

$$\overline{v_s} = \mu V_T \qquad \mu = 1 - FP(J_T)$$

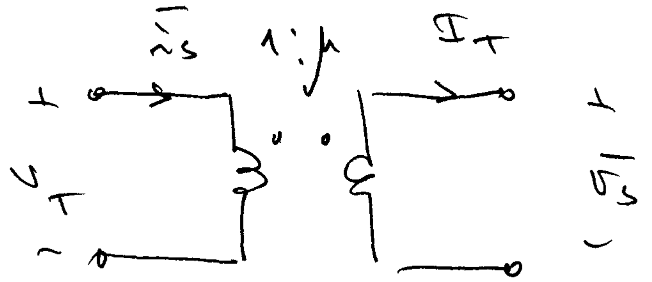
$$P = \frac{1}{2\pi}\left(\frac{1}{2}\,\frac{1}{J_T} + \pi + \arcsin\frac{1}{J_T} + J_T + \sqrt{J_T^2 - 1}\right)$$

Средња вредност улазне струје, ЗОЕ

$$V_T \langle i_s \rangle = \langle v_s \rangle I_T$$

$$\overline{v_s} = \mu V_T$$

$$\overline{i_s} = \mu I_T$$



Current формулите за full-wave case

$P \cong 1 \qquad \mu \cong 1 - F$

Резиме формулите за type a и type b

Switch	μ	$P(J_T)$	load range
PWM	D	—	∞
type a $\frac{1}{2}$ wave	$F P(J_T)$	$k_{1/2}(J_T)$	$0 \le J_T \le 1$
type a 1 wave	$FP(J_T) \cong F$	$k_1(J_T) \cong 1$	$0 \le J_T \le 1$
type b $\frac{1}{2}$ wave	$1 - FP(J_T)$	$k_{1/2}\left(\frac{1}{J_T}\right)$	$1 \le J_T \le \infty$
type b 1 wave	$1 - FP(J_T) \cong 1 - F$	$k_1\left(\frac{1}{J_T}\right) \cong 1$	$1 \le J_T \le \infty$

за dc је $0 \le \mu \le 1$

$$k_{1/2}(x) = \frac{1}{2\pi}\left(\frac{1}{2}x + \pi + \arcsin x + \frac{1}{x}\left(1 + \sqrt{1 - x^2}\right)\right)$$

$$k_1(x) = \frac{1}{2\pi}\left(\frac{1}{2}x + 2\pi - \arcsin x + \frac{1}{x}\left(1 - \sqrt{1 - x^2}\right)\right)$$

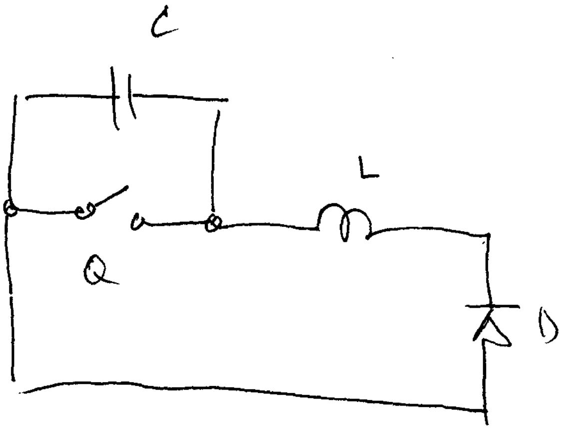
карактеристике овог типа [illegible] "type a" резонантног прекидача

1) zero-current switching
2) peak switch currents знатно веће од I_F
3) peak switch voltages — V_q, исто као у PWM

Остале топологије:

Type b resonant switch

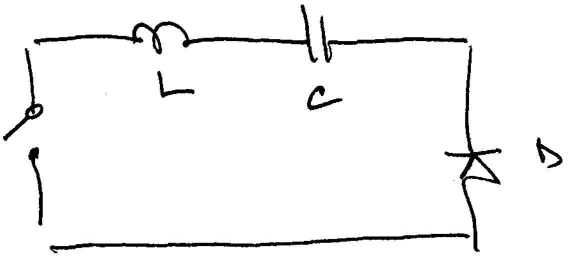
1 zero voltage switching
2 максималне струје прекидача исте као код PWM
3 повећан напон на прекидачу
4 анализа дуална type-a прекидачу
- своди се на



Type C Resonant Switch

1 zero current switching
2 [illegible] [illegible] [illegible] ca PWM
3 unilateral blocking voltage

- [illegible] ce [illegible]



Type d resonant switch

- zero voltage switching
- максимални напон као код конвенционалног PWM
- максималне струје прекидача повећане
- дуално to type c switch

- своди се на