Phase Controlled Rectifier, Half Controlled

RMS values

(%i1) Vrms: Vm / sqrt(2);

$$(\%o1) \quad \frac{Vm}{\sqrt{2}}$$

current, 1st harmonic

(%i2) assume(Iout > 0);

$$(\%o2) \quad [Iout > 0]$$

(%i3) Irms: Iout * sqrt((%pi - alpha) / %pi);

$$(\%o3) \quad \frac{\sqrt{\pi - \alpha} \cdot Iout}{\sqrt{\pi}}$$

(%i4) I1s: 1 / %pi * (integrate(Iout * sin(x), x, alpha, %pi) +
           integrate(- Iout * sin(x), x, %pi + alpha, 2 * %pi));

$$(\%o4) \quad \frac{(\cos(\alpha) + 1) \cdot Iout - (-1 - \cos(\alpha)) \cdot Iout}{\pi}$$

(%i5) I1c: 1 / %pi * (integrate(Iout * cos(x), x, alpha, %pi) +
           integrate(- Iout * cos(x), x, %pi + alpha, 2 * %pi));

$$(\%o5) \quad -\frac{2 \cdot \sin(\alpha) \cdot Iout}{\pi}$$

(%i6) I1: sqrt(I1c^2 + I1s^2);

$$(\%o6) \quad \sqrt{\frac{((\cos(\alpha) + 1) \cdot Iout - (-1 - \cos(\alpha)) \cdot Iout)^2}{\pi^2} + \frac{4 \cdot \sin(\alpha)^2 \cdot Iout^2}{\pi^2}}$$
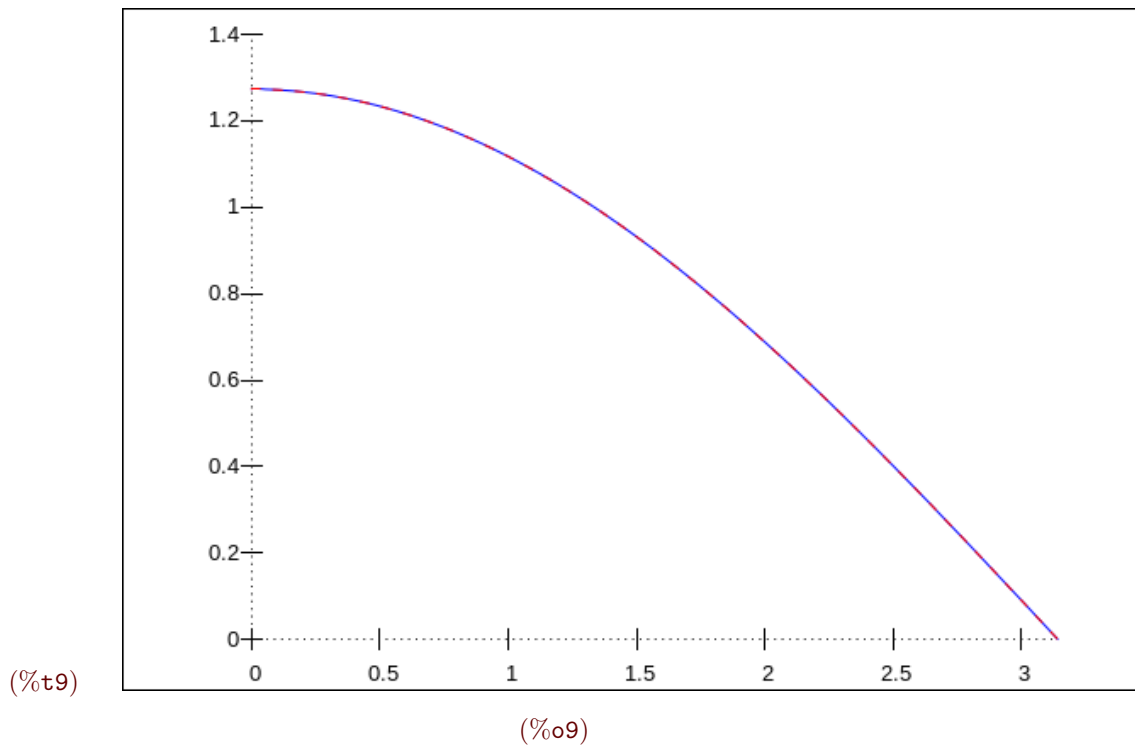
(%i7) I1: trigsimp(I1);

$$(\%o7) \quad \frac{\sqrt{8 \cdot \cos(\alpha) + 8} \cdot Iout}{\pi}$$

(%i8) I1a: 4 / %pi * cos(alpha / 2) * Iout;

$$(\%o8) \quad \frac{4 \cdot \cos\left(\frac{\alpha}{2}\right) \cdot Iout}{\pi}$$

(%i9) wxplot2d([I1 / Iout, I1a / Iout], [alpha, 0, %pi-1e-3], [box, false], [legend, false]);

(%t9)

(%o9)

(%i10) I1: I1a;

$$(\%o10) \quad \frac{4 \cdot \cos\left(\frac{\alpha}{2}\right) \cdot Iout}{\pi}$$

(%i11) I1rms: I1 / sqrt(2);

$$(\%o11) \quad \frac{2^{\frac{3}{2}} \cdot \cos\left(\frac{\alpha}{2}\right) \cdot Iout}{\pi}$$

DPF

(%i12) DPF: I1s / I1;

$$(\%o12) \quad \frac{(\cos\left(\alpha\right) + 1) \cdot Iout - (-1 - \cos\left(\alpha\right)) \cdot Iout}{4 \cdot \cos\left(\frac{\alpha}{2}\right) \cdot Iout}$$

(%i13) DPF: ratsimp(DPF);

$$(\%o13) \quad \frac{1 + \cos\left(\alpha\right)}{2 \cdot \cos\left(\frac{\alpha}{2}\right)}$$

(%i14) DPF: radcan(DPF);

$$(\%o14) \quad \frac{1 + \cos\left(\alpha\right)}{2 \cdot \cos\left(\frac{\alpha}{2}\right)}$$

(%i15) (cos(alpha / 2))^2;

$$(\%o15) \quad \cos\left(\frac{\alpha}{2}\right)^2$$

(%i16) trigrat(%);

$$(\%o16) \quad \frac{1 + \cos\left(\alpha\right)}{2}$$

(%i17) DPF^2 - cos(alpha/2)^2;

$$(\%\text{o}17) \quad \frac{(1 + \cos(\alpha))^2}{4 \cdot \cos\left(\frac{\alpha}{2}\right)^2} - \cos\left(\frac{\alpha}{2}\right)^2$$

(%i18) trigrat(%);

$$(\%\text{o}18) \quad 0$$

(%i19) DPF: cos(alpha / 2);

$$(\%\text{o}19) \quad \cos\left(\frac{\alpha}{2}\right)$$

P

(%i20) P: Vrms * I1rms * DPF;

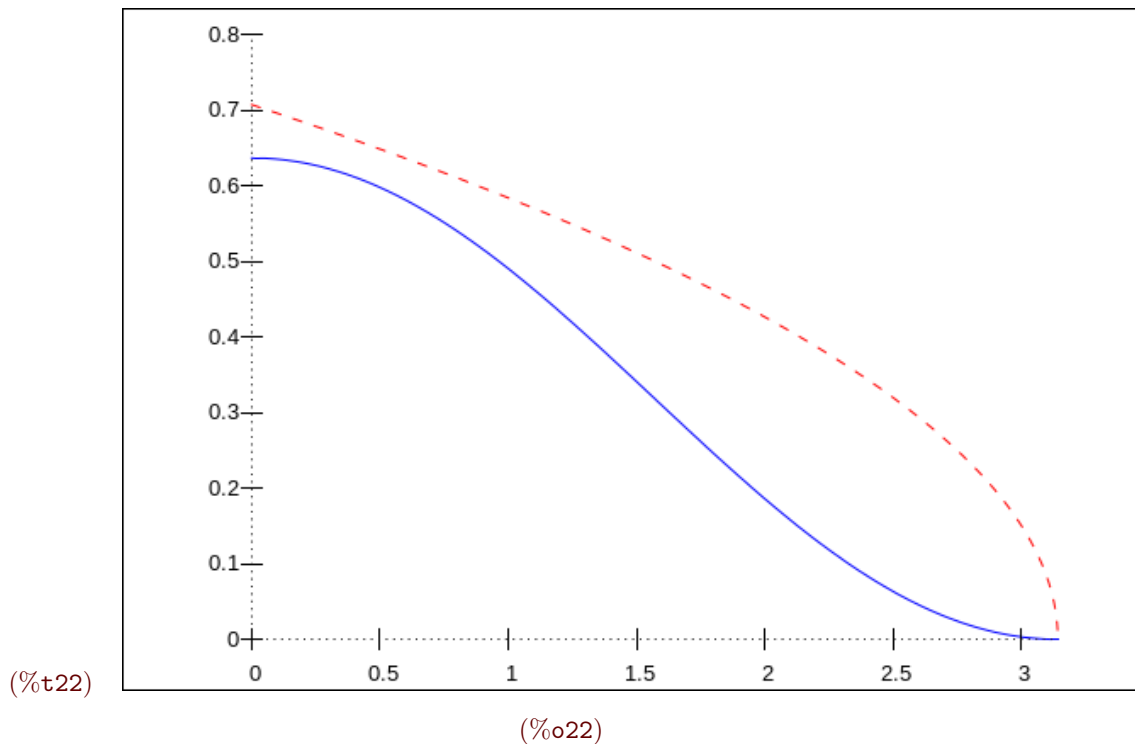$$(\%\text{o}20) \quad \frac{2 \cdot \cos\left(\frac{\alpha}{2}\right)^2 \cdot Iout \cdot Vm}{\pi}$$

S

(%i21) S: Vrms * Irms;

$$(\%\text{o}21) \quad \frac{\sqrt{\pi - \alpha} \cdot Iout \cdot Vm}{\sqrt{2} \cdot \sqrt{\pi}}$$

(%i22) wxplot2d([P / (Vm * Iout), S / (Vm * Iout)], [alpha, 0, %pi-1e-3], [box, false], [legend, false]);

(%t22)

$$(\%\text{o}22)$$

PF

(%i23) PF: P / S;

$$(\%\text{o}23) \quad \frac{2^{\frac{3}{2}} \cdot \cos\left(\frac{\alpha}{2}\right)^2}{\sqrt{\pi} \cdot \sqrt{\pi - \alpha}}$$
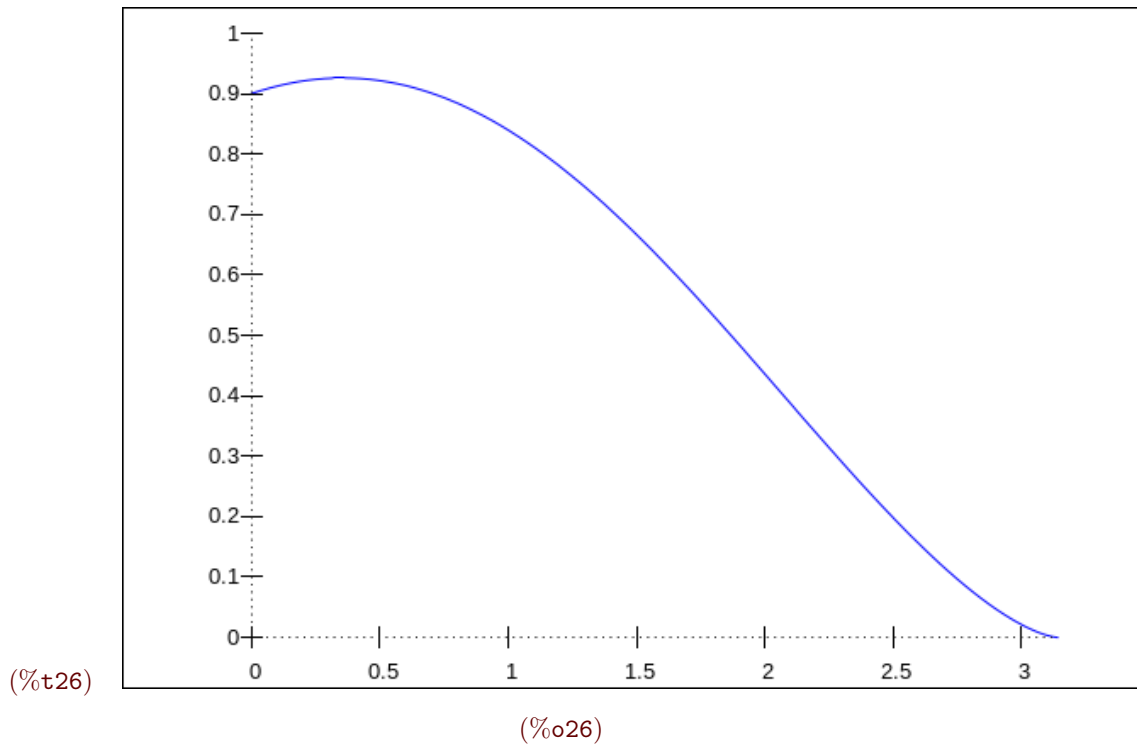
(%i24) ev(PF, alpha = 0);

$$(\%\text{o}24) \quad \frac{2^{\frac{3}{2}}}{\pi}$$
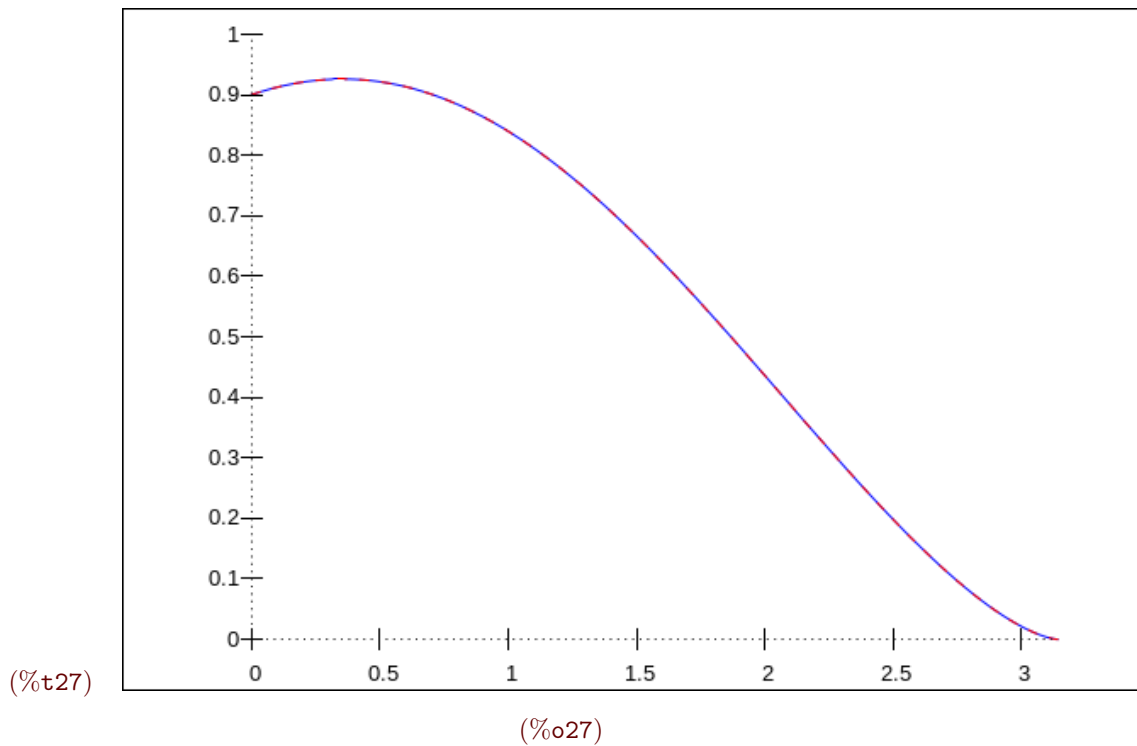
(%i25) `ev(PF, alpha = 0, numer);`

(%o25)   0.9003163161571063

(%i26) `wxplot2d(PF, [alpha, 0, %pi-1e-3], [box, false], [legend, false]);`

(%t26)

(%o26)

verify PF-DPF relation

(%i27) `wxplot2d([PF, I1rms / Irms * DPF], [alpha, 0, %pi-1e-3], [box, false], [legend, false]);`
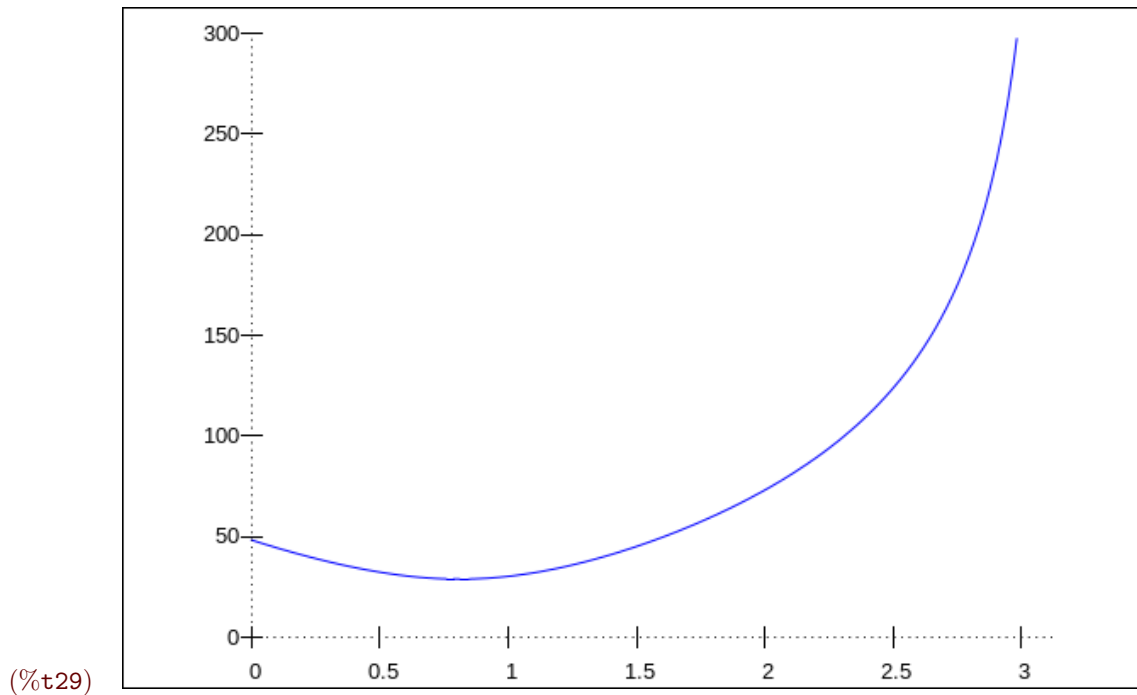
(%t27)

(%o27)

THD

(%i28) `THD: sqrt((Irms / I1rms)^2 - 1) * 100;`

$$(\%\text{o}28) \quad 100 \cdot \sqrt{\dfrac{\pi \cdot (\pi - \alpha)}{8 \cdot \cos\left(\frac{\alpha}{2}\right)^2} - 1}$$

(%i29) `wxplot2d(THD, [alpha, 0, %pi-1e-3], [y, 0, 300], [box, false], [legend, false]);`

plot2d: some values were clipped.

(%t29)

$$(\%\text{o}29)$$

(%i30) `ev(THD, alpha = 0);`

$$(\%\text{o}30) \quad 100 \cdot \sqrt{\dfrac{\pi^2}{8} - 1}$$

(%i31) `ev(THD, alpha = 0, numer);`

$$(\%\text{o}31) \quad 48.3425847608679$$