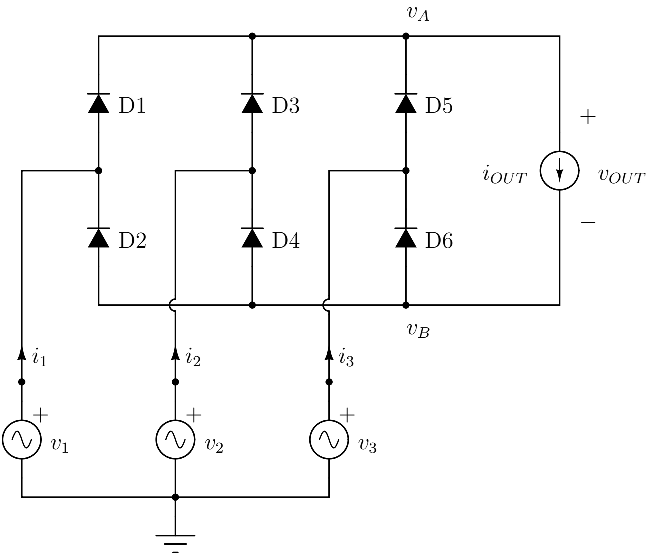
Introduction

— three-phase diode bridge rectifier —

what is this all about?



input voltages

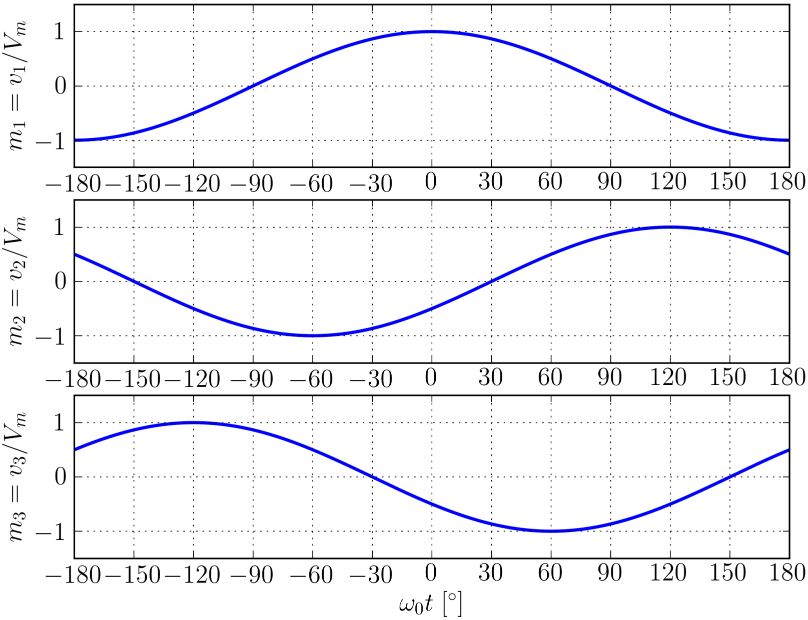
$$v_1 = V_m \cos\left(\omega_0 t\right)$$

$$v_2 = V_m \cos\left(\omega_0 t - \frac{2\pi}{3}\right)$$

$$v_3 = V_m \cos\left(\omega_0 t - \frac{4\pi}{3}\right)$$

$$v_k = V_m \cos\left(\omega_0 t - (k-1)\frac{2\pi}{3}\right), \quad k \in \{1,\ 2,\ 3\}$$

input voltages, waveforms



normalization of voltages

$$\boxed{m_X \triangleq \frac{v_X}{V_m}}$$

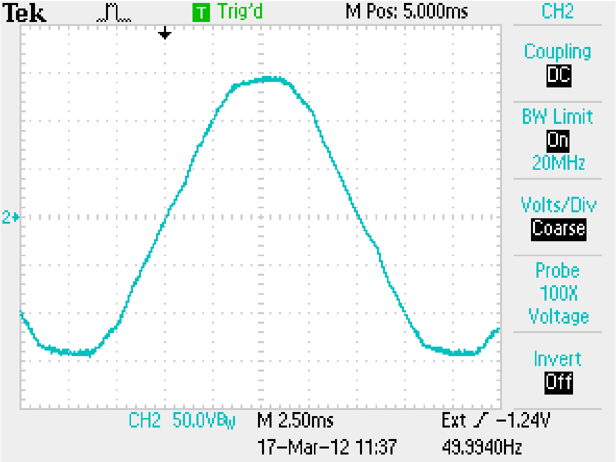
$$m_1 = \cos\left(\omega_0 t\right)$$

$$m_2 = \cos\left(\omega_0 t - \frac{2\pi}{3}\right)$$

$$m_3 = \cos\left(\omega_0 t - \frac{4\pi}{3}\right)$$

voltages?

$$v_k = V_m \cos\left(\omega_0 t - (k-1)\,\frac{2\pi}{3}\right), \qquad k \in \{\mathbf{1},\, 2,\, 3\}$$



voltages?

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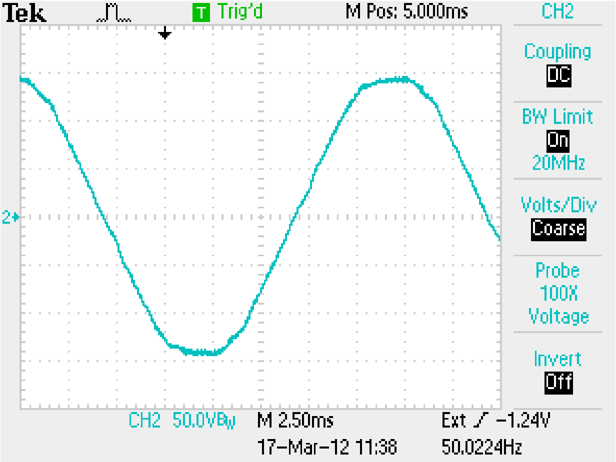

voltages?

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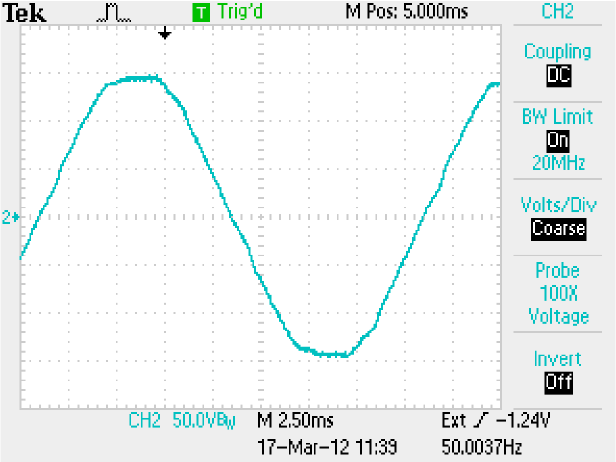

voltages?

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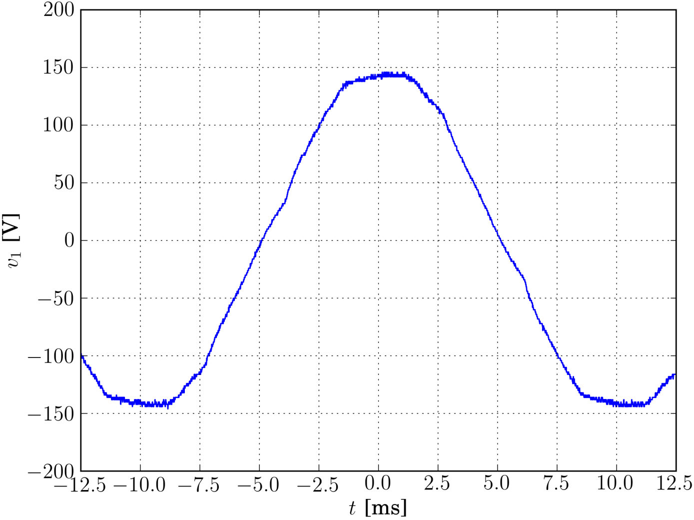
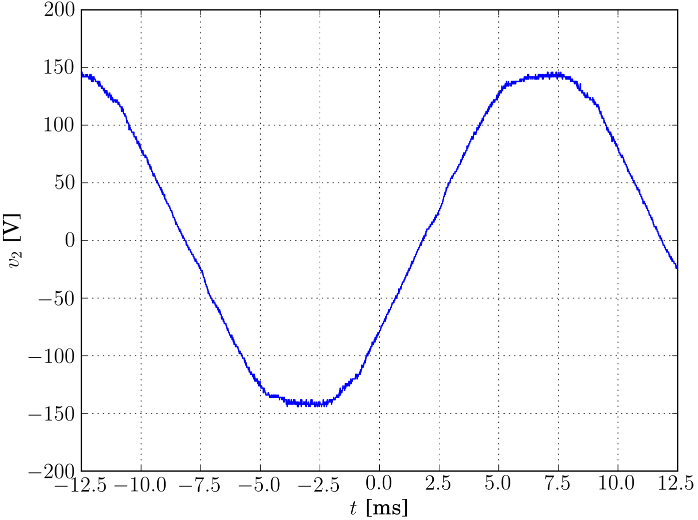


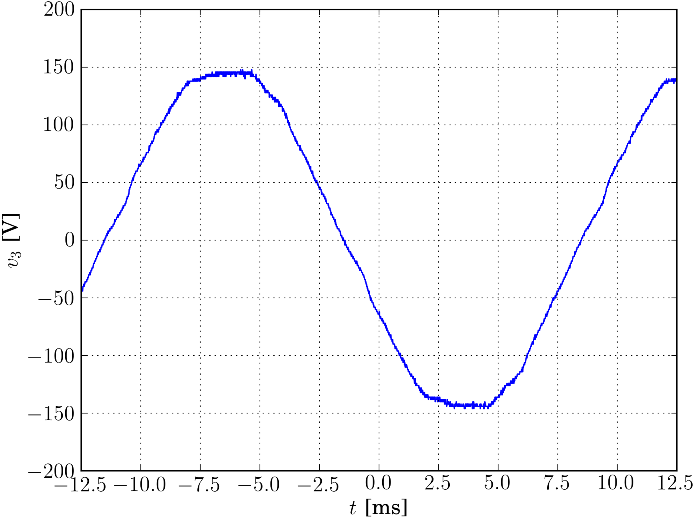
voltages?

$$v_k = V_m \cos\left(\omega_0 t - (k-1)\,\frac{2\pi}{3}\right), \qquad k \in \{1,\, \mathbf{2},\, 3\}$$

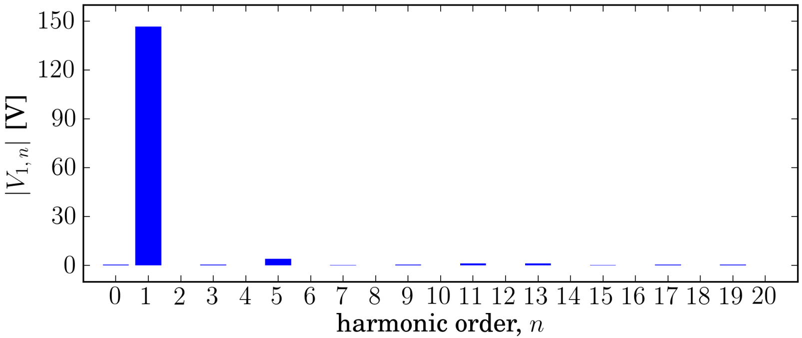


voltages?

$$v_k = V_m \cos\left(\omega_0 t - (k-1)\,\frac{2\pi}{3}\right), \qquad k \in \{1,\, 2,\, \mathbf{3}\}$$

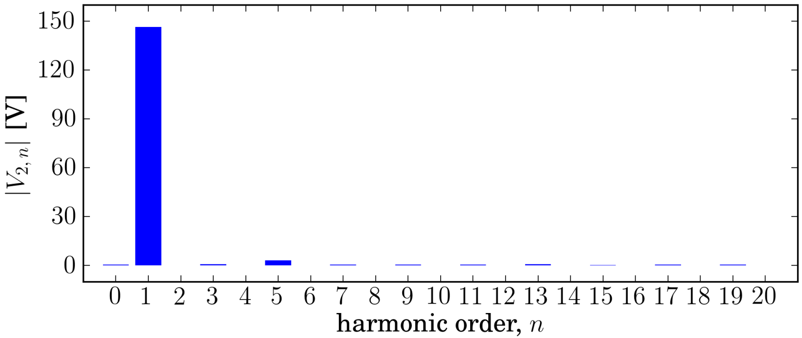


v_1, spectrum



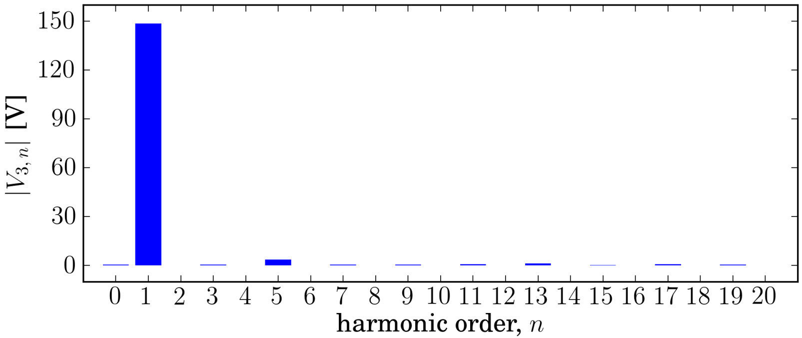
150
120
90
60
30
0
$|V_{1,n}|$ [V]
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
harmonic order, n

v_2, spectrum



150
120
90
60
30
0
$|V_{2,n}|$ [V]
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
harmonic order, n

v_3, spectrum



150
120
90
60
30
0
$|V_{3,n}|$ [V]
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
harmonic order, n

voltages, quantitative characterization

k	$V_{k\,RMS}$	$THD(v_k)$
1	103.83 V	3.34 %
2	103.70 V	2.77 %
3	105.12 V	3.06 %

all graphs and data PyLab processed

THD

And what is THD?

$$THD \triangleq \frac{\sqrt{\sum_{k=2}^{\infty} I_{k\,RMS}^2}}{I_{1\,RMS}}$$

Parseval's identity:

$$I_{RMS}^2 = \sum_{k=1}^{\infty} I_{k\,RMS}^2 \quad \text{assumed} \quad I_0 = 0$$

results in

$$THD \triangleq \frac{\sqrt{I_{RMS}^2 - I_{1\,RMS}^2}}{I_{1\,RMS}}$$

simple, but important
computational issues, finite sums ...

normalization of currents and time

$$\boxed{j_X \triangleq \frac{i_X}{I_{OUT}}}$$

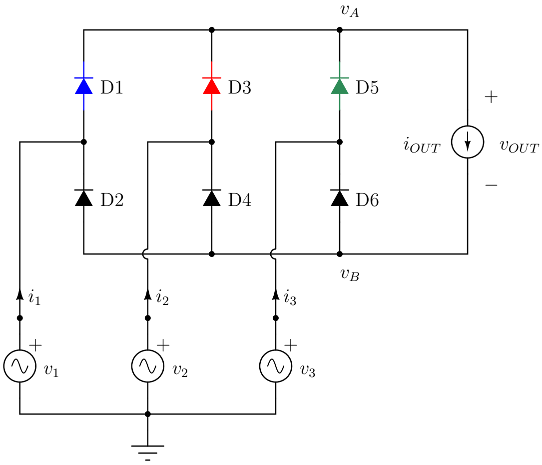
unless otherwise noted

$$\boxed{\varphi \triangleq \omega_0 t}$$

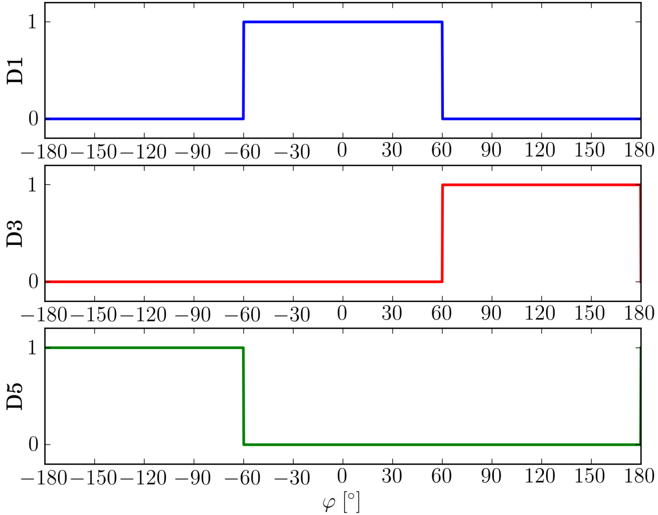
good: **physical dimensions lost**, reduced number of variables, results are generalized, core of the problem focused

bad: **physical dimensions lost**, perfect double-check tool is lost

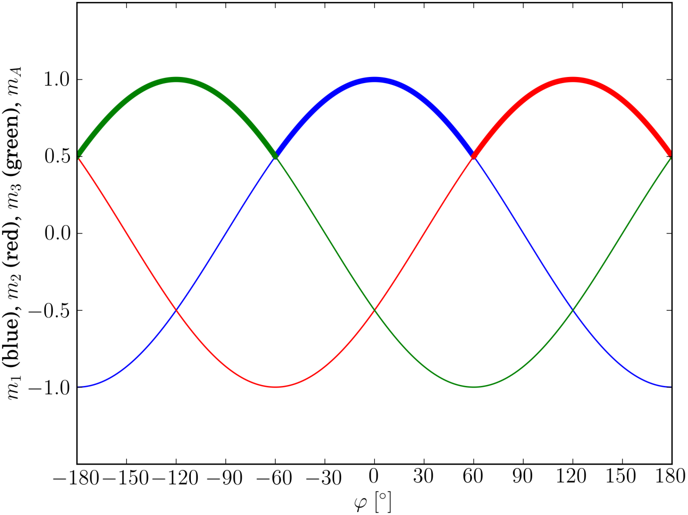
how does it work? part 1: theory



one of the three: D1, D3, D5

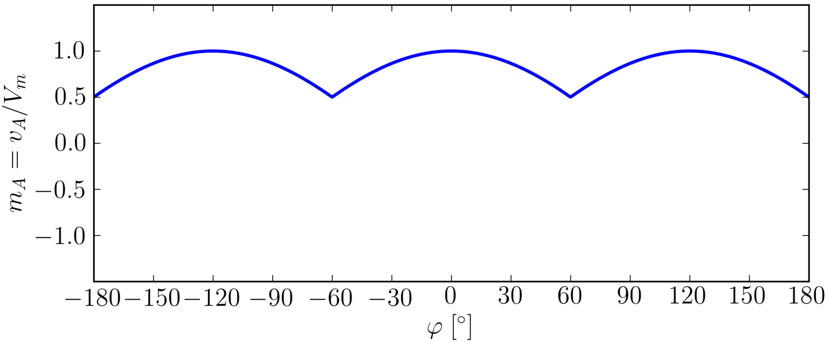


v_A



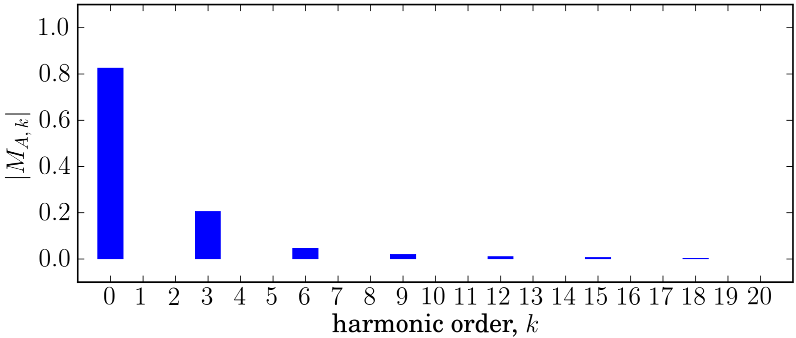
m_1 (blue), m_2 (red), m_3 (green), m_A
1.0
0.5
0.0
−0.5
−1.0
−180 −150 −120 −90 −60 −30 0 30 60 90 120 150 180
φ [°]

v_A, analytical



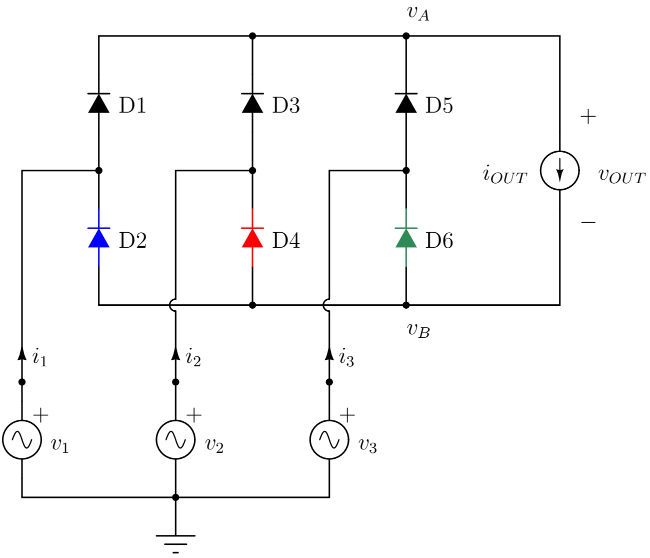
$$m_A = \max\left(m_1, m_2, m_3\right)$$

v_A, spectrum

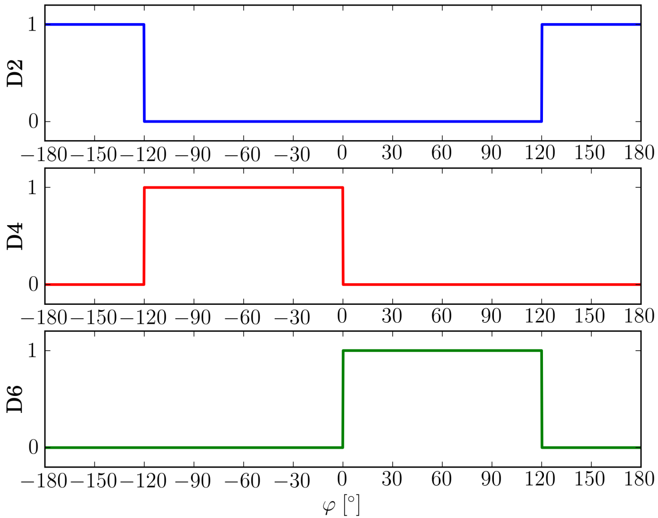


$$m_A = \frac{3\sqrt{3}}{2\pi}\left(1 + 2\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{9k^2-1}\cos\left(3k\omega_0 t\right)\right)$$

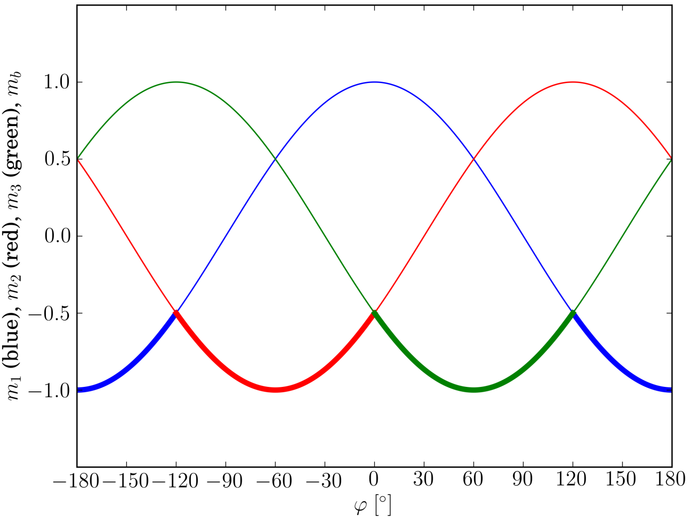
what about v_B?



one of the three, again: D2, D4, D6

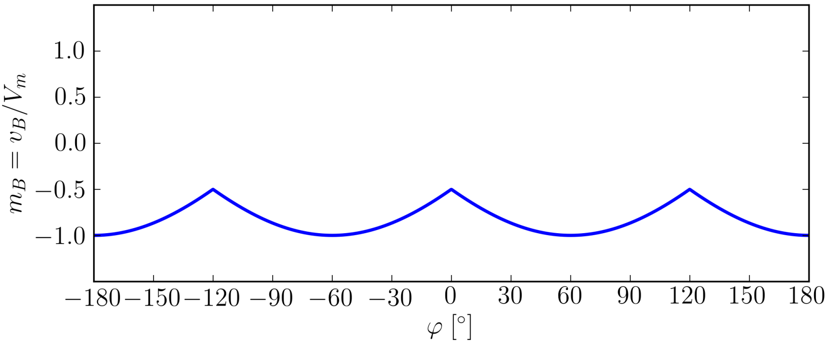


v_B



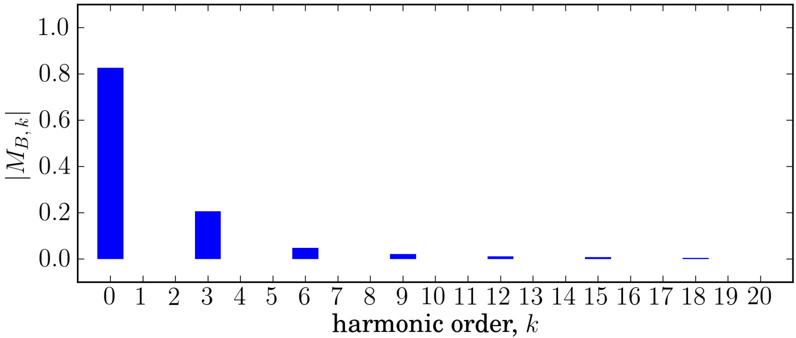
1.0
0.5
0.0
−0.5
−1.0
−180
−150
−120
−90
−60
−30
0
30
60
90
120
150
180
m_1 (blue), m_2 (red), m_3 (green), m_b
φ [°]

v_B, analytical



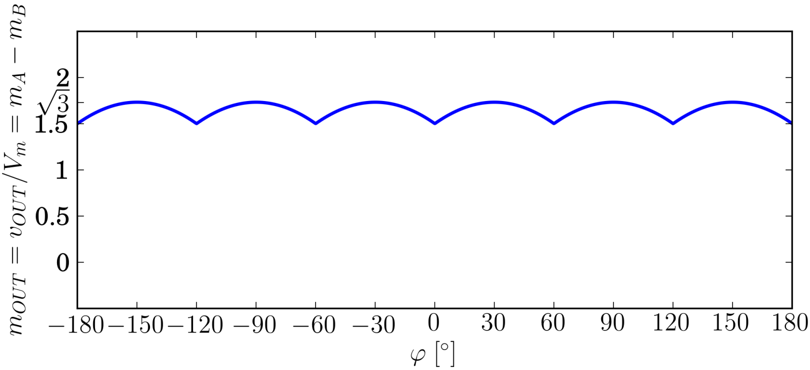
$$m_B = \min(m_1, m_2, m_3)$$

v_B, spectrum



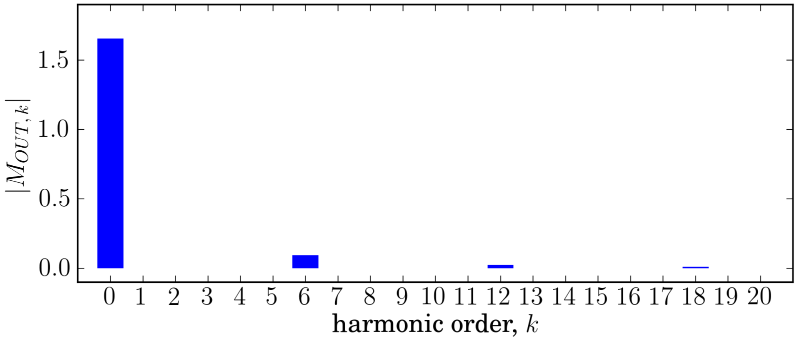
$$m_B = \frac{3\sqrt{3}}{2\pi}\left(-1 + 2\sum_{k=1}^{\infty}\frac{1}{9k^2-1}\cos\left(3k\omega_0 t\right)\right)$$

the output voltage, v_{OUT}



$$m_{OUT} = m_A - m_B = \max(m_1, m_2, m_3) - \min(m_1, m_2, m_3)$$

v_{OUT}, spectrum



$$m_{OUT} = \frac{3\sqrt{3}}{\pi}\left(1 - 2\sum_{k=1}^{\infty}\frac{1}{36k^2-1}\cos\left(6k\omega_0 t\right)\right)$$

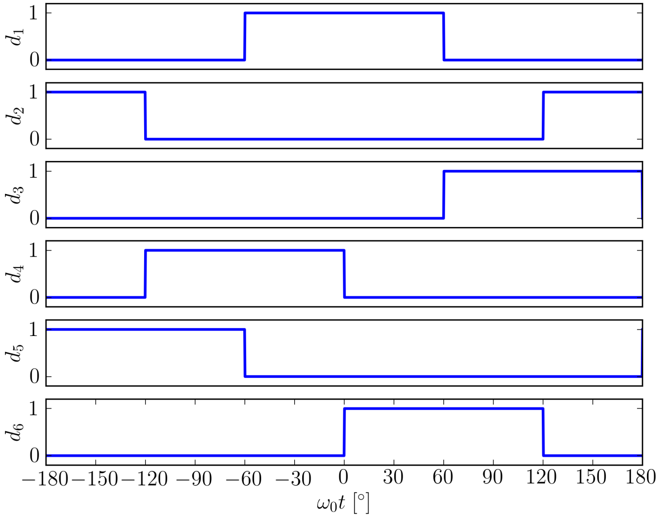
currents?

$$i_1(t) = (d_1(t) - d_2(t))\, I_{OUT}$$

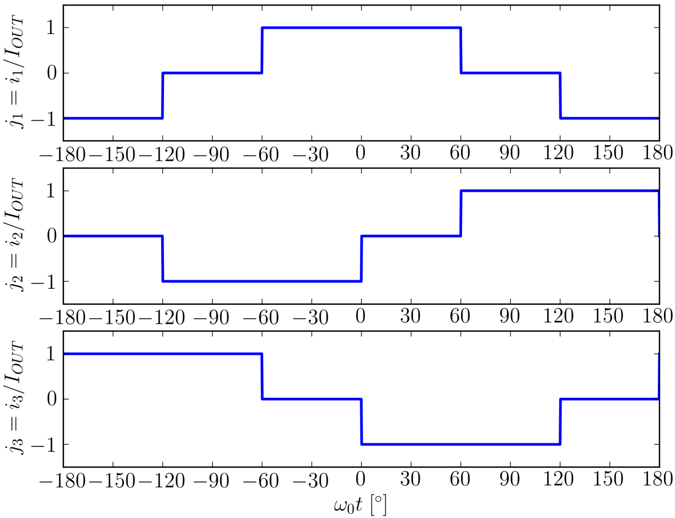
$$i_2(t) = (d_3(t) - d_4(t))\, I_{OUT}$$

$$i_3(t) = (d_5(t) - d_6(t))\, I_{OUT}$$

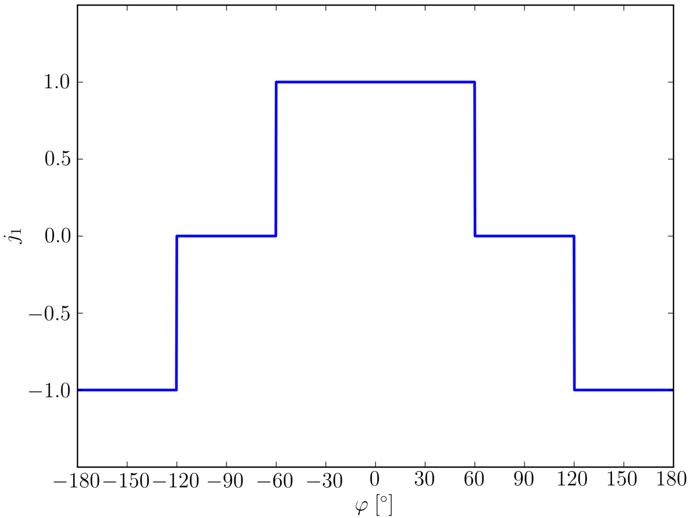
states of the diodes



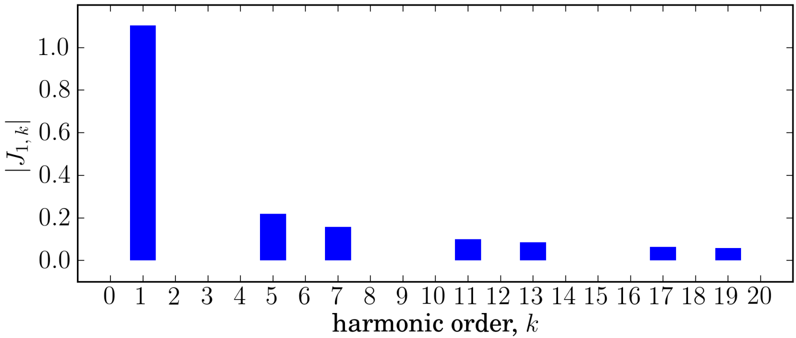
the input currents



consider i_1



spectra of the input currents



spectra of the input currents, analytical

$$j_1(t) = \sum_{k=1}^{+\infty} J_{1C,\,k} \cos\left(k\omega_0 t\right)$$

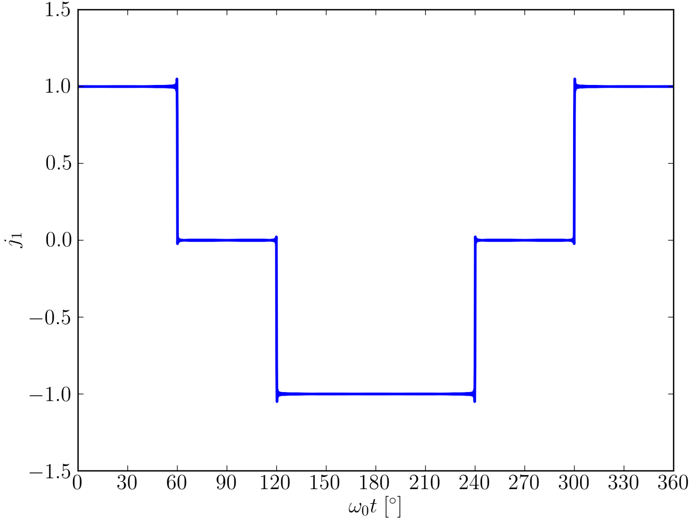
$$J_{1C,\,k} = \frac{2\sqrt{3}}{\pi} \begin{cases} -\dfrac{1}{k}, & k = 6n - 1 \\ \dfrac{1}{k}, & k = 6n + 1 \\ 0, & \text{otherwise} \end{cases} \qquad \text{for } n \in \mathbb{N}_0,\ k > 0$$

double-check:

$$P_{IN} = \frac{3}{2} \times 1 \times \frac{2\sqrt{3}}{\pi} = \frac{3\sqrt{3}}{\pi} = P_{OUT}$$

obtained using wxMaxima

numerical verification, Gibbs phenomenon



THD of the input currents

$$I_{k\,RMS} = \sqrt{\frac{2}{3}}\, I_{OUT}$$

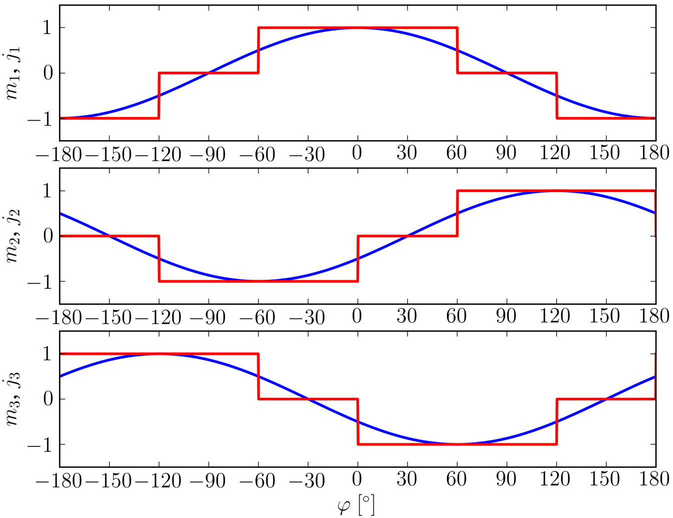
$$I_{k\,RMS,\,1} = \frac{\sqrt{6}}{\pi}\, I_{OUT}$$

$$THD \triangleq \frac{\sqrt{I^2_{k\,RMS} - I^2_{k\,RMS,\,1}}}{I_{k\,RMS,\,1}}$$

$$THD = \sqrt{\frac{\pi^2}{9} - 1} \approx 31.08\,\%$$

Parseval's identity based formula turned out to be useful

voltages and currents



some more parameters

$$X_{RMS} \triangleq \sqrt{\frac{1}{2\pi}\int_0^{2\pi} \left(x(\omega_0 t)\right)^2 \, d(\omega_0 t)}, \quad x \in \{i,\, v\}$$

already used for the THD

$$S \triangleq I_{RMS}\, V_{RMS}$$

$$P \triangleq \frac{1}{2\pi}\int_0^{2\pi} v(\omega_0 t)\, i(\omega_0 t)\, d(\omega_0 t)$$

$$PF \triangleq \frac{P}{S}$$

$$DPF \triangleq \cos\phi_1$$

and ϕ_1 is ...

and if the voltages are sinusoidal . . .

$$S = V_{RMS}\, I_{RMS}$$

$$P = V_{RMS}\, I_{1,\,RMS}\ \cos\phi_1$$

$$PF = \frac{P}{S} = \frac{I_{1,\,RMS}}{I_{RMS}}\ \cos\phi_1 = \frac{I_{1,\,RMS}}{I_{RMS}}\, DPF$$

$$DPF = \cos\varphi_1$$

$$THD = \frac{\sqrt{I_{RMS}^2 - I_{1,\,RMS}^2}}{I_{1,\,RMS}} = \sqrt{\left(\frac{I_{RMS}}{I_{1,\,RMS}}\right)^2 - 1}$$

i.e. everything depends on the current waveform and its position

some more parameters, plain rectifier

$$I_{k\,RMS} = \sqrt{\frac{2}{3}}\, I_{OUT} \quad V_{k\,RMS} = \frac{1}{\sqrt{2}}\, V_m$$

$$S = 3 \times \sqrt{\frac{2}{3}}\, I_{OUT} \times \frac{1}{\sqrt{2}}\, V_m = \sqrt{3}\, V_m\, I_{OUT}$$

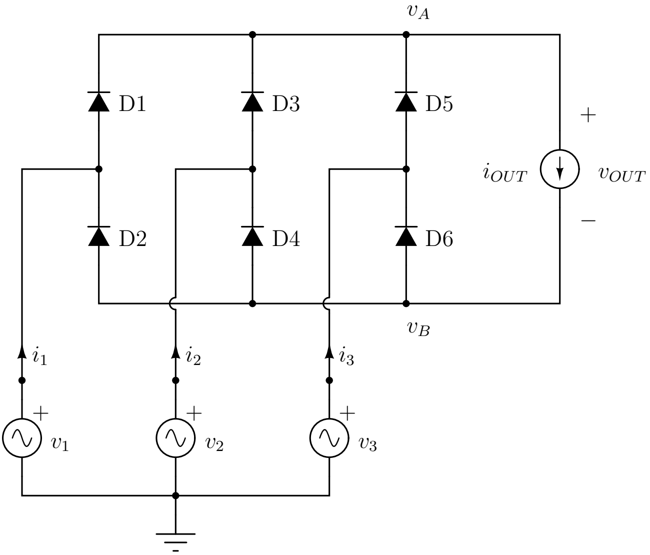
$$P = V_{OUT}\, I_{OUT} = \frac{3\sqrt{3}}{\pi}\, V_m\, I_{OUT}$$

$$PF = \frac{3}{\pi} \approx 95.5\%$$

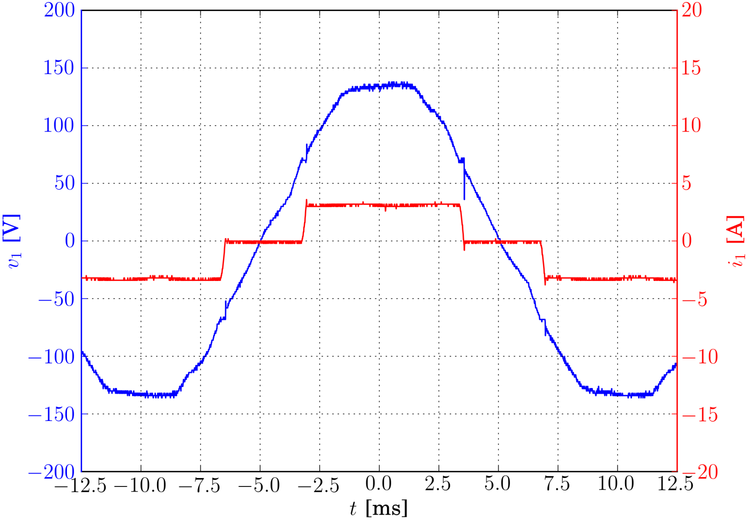
$$DPF = 1$$

actually, not so bad; THD is the problem

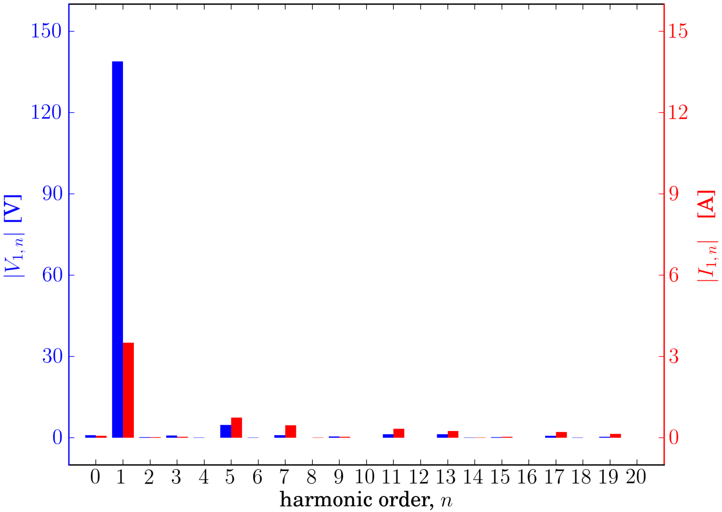
back to the rectifier:
how does it work? part 2: experiment



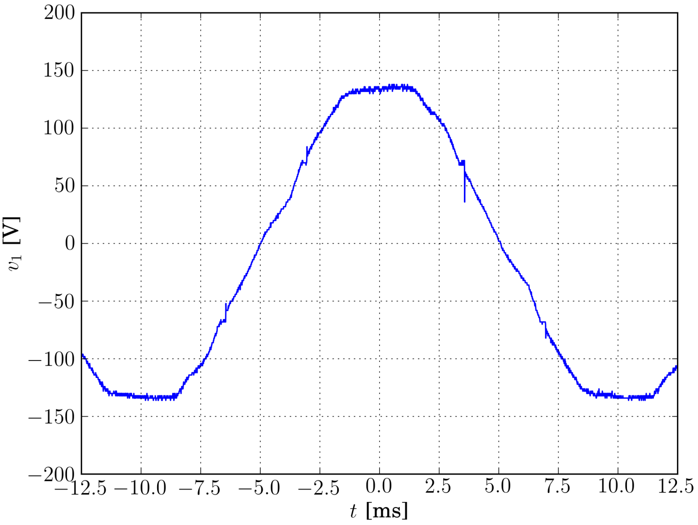
input, at $I_{OUT} = 3$ A



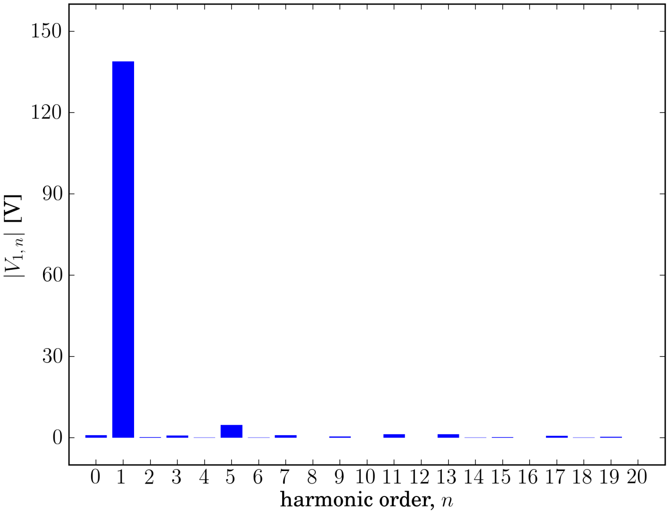
input, at $I_{OUT} = 3$ A



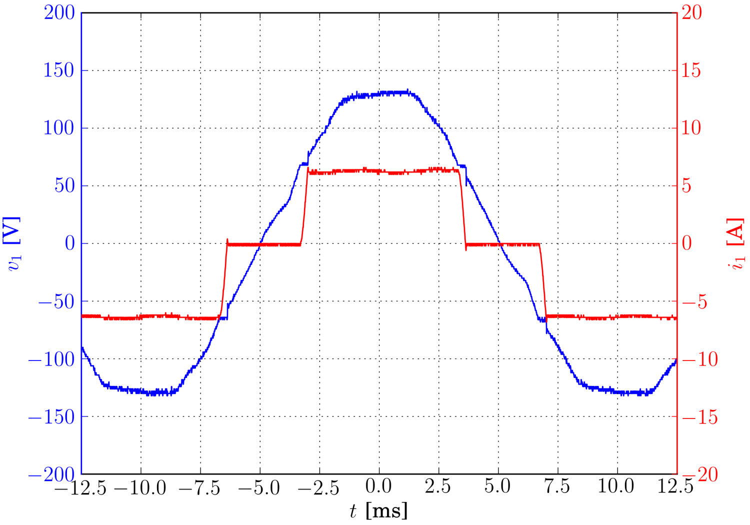
input, at $I_{OUT} = 3$ A



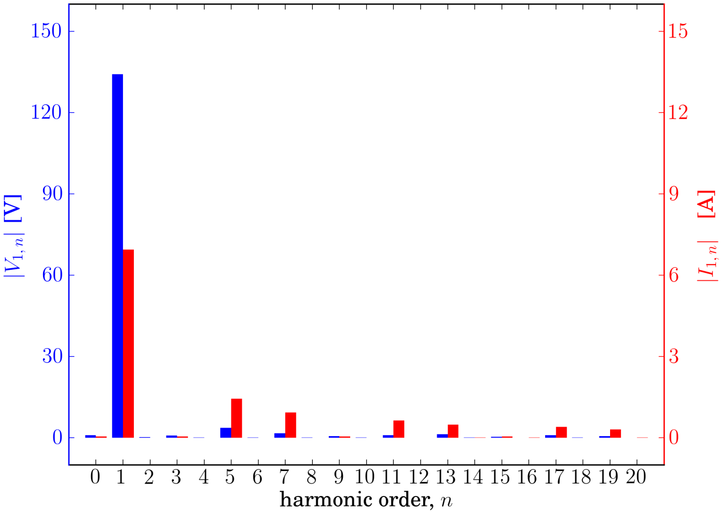
input, at $I_{OUT} = 3$ A



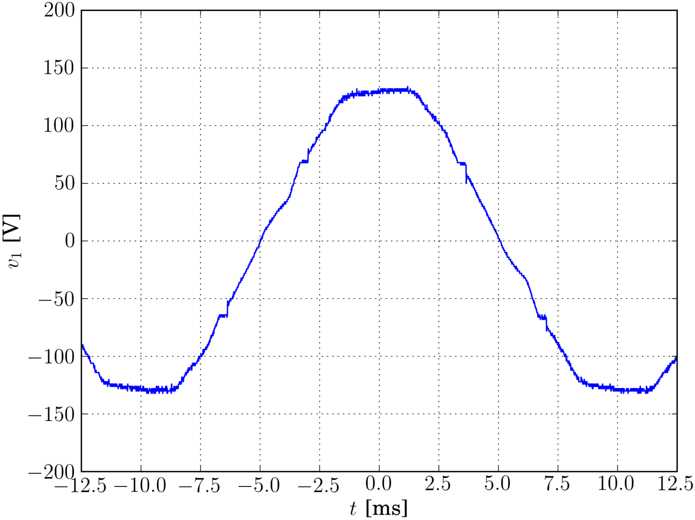
input, at $I_{OUT} = 6$ A



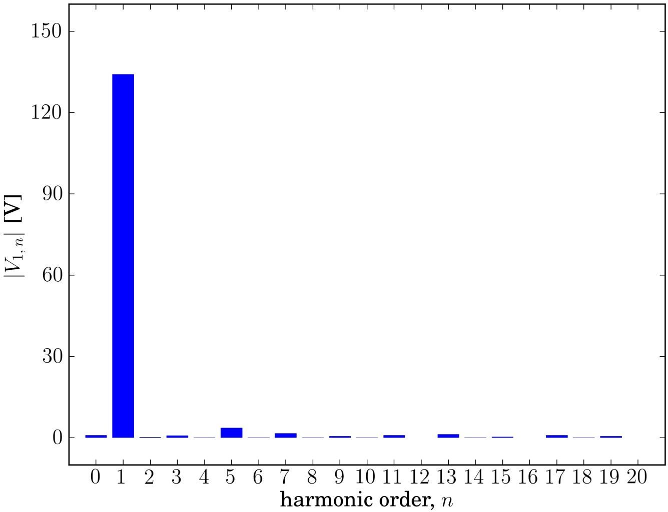
input, at $I_{OUT} = 6$ A



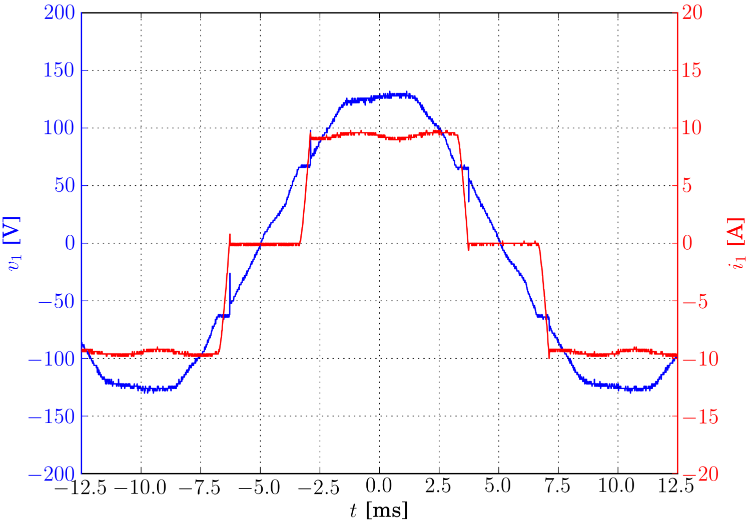
input, at $I_{OUT} = 6$ A



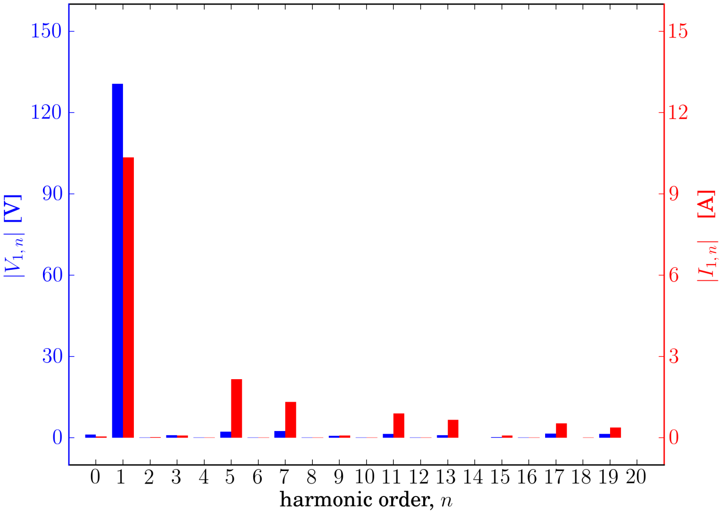
input, at $I_{OUT} = 6$ A



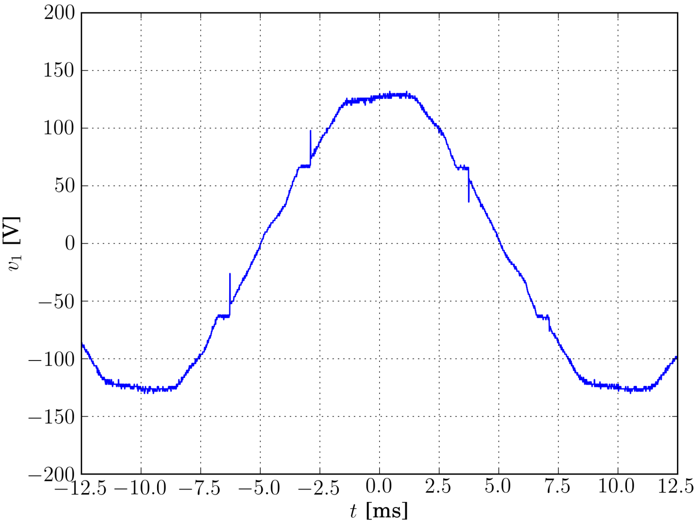
input, at $I_{OUT} = 9$ A



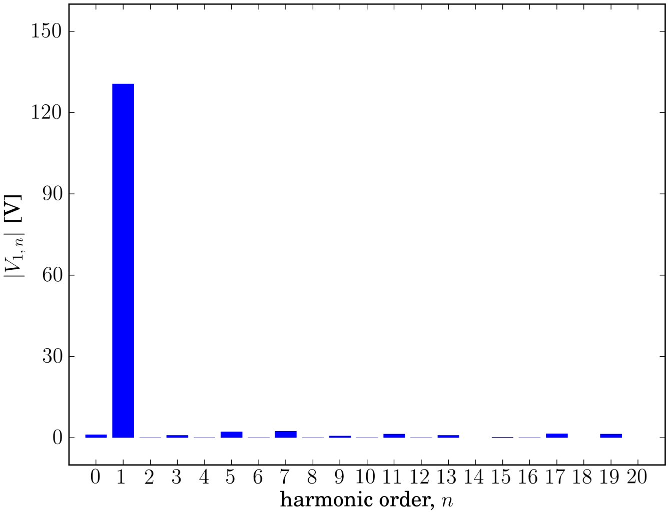
input, at $I_{OUT} = 9$ A



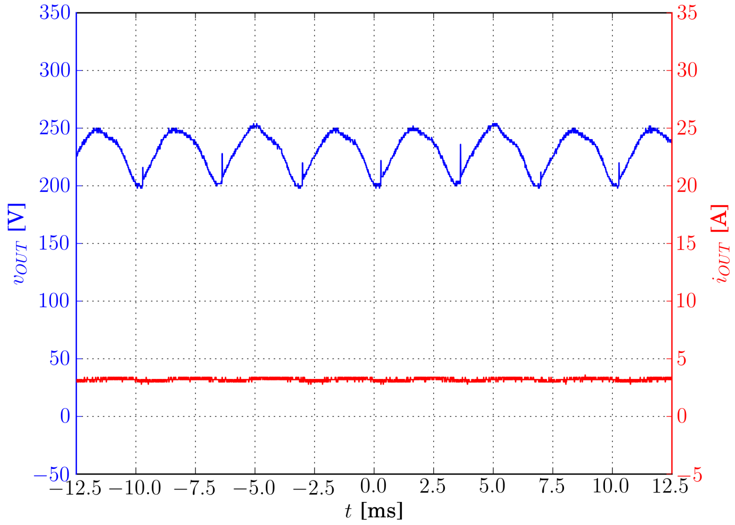
input, at $I_{OUT} = 9$ A



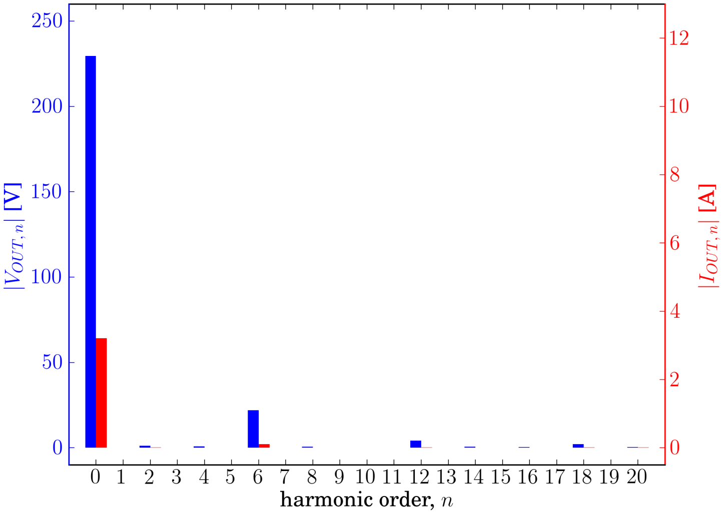
input, at $I_{OUT} = 9$ A



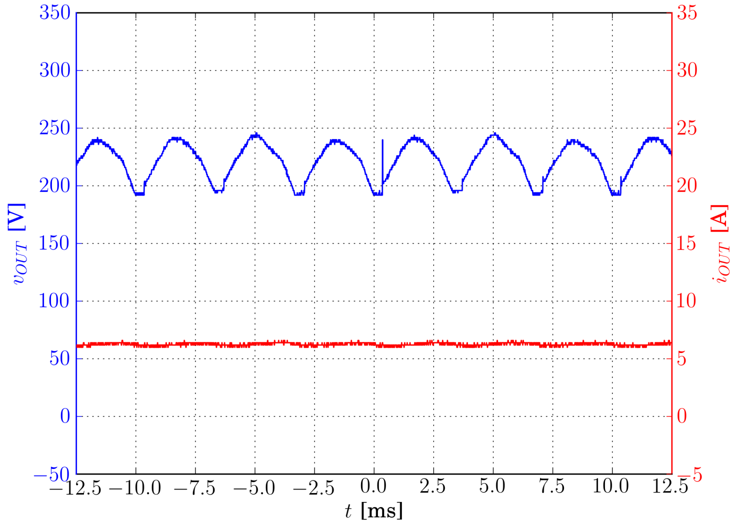
output, at $I_{OUT} = 3$ A



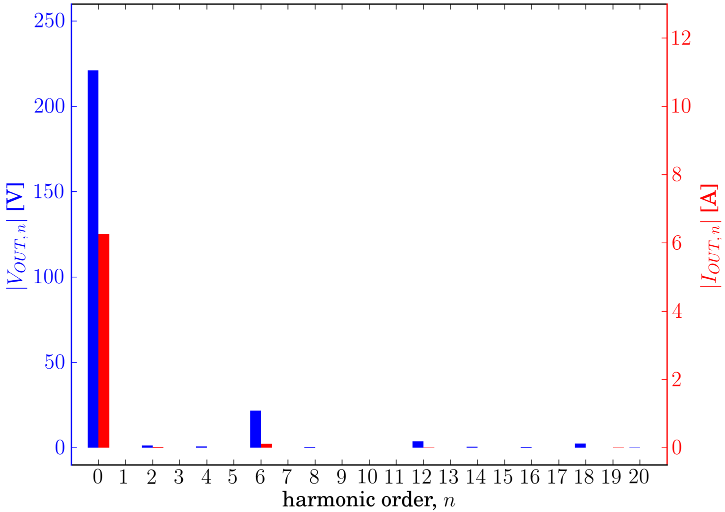
output, at $I_{OUT} = 3$ A



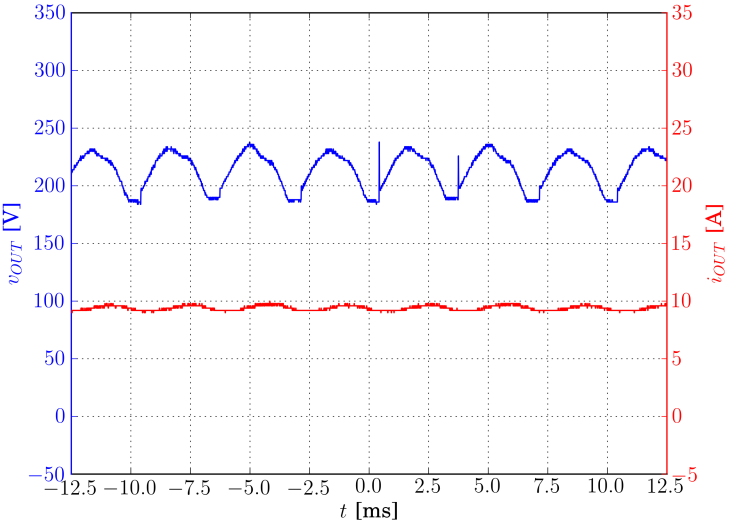
output, at $I_{OUT} = 6$ A



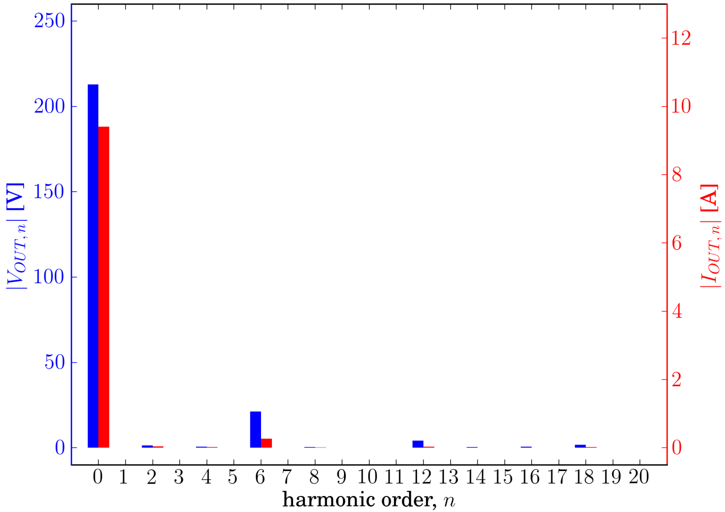
output, at $I_{OUT} = 6$ A



output, at $I_{OUT} = 9$ A



output, at $I_{OUT} = 9$ A



in quantitative terms, input, 1st

I_{OUT}	k	$I_{k\,RMS}$ [A]	$V_{k\,RMS}$ [V]	S_k [VA]	P_k [W]
0 A	1	—	101.29	—	—
	2	—	100.63	—	—
	3	—	102.40	—	—
3 A	1	2.60	98.23	255.01	245.16
	2	2.61	97.73	254.60	244.37
	3	2.63	98.82	259.71	251.00
6 A	1	5.12	94.87	485.41	466.87
	2	5.12	94.34	482.67	464.25
	3	5.13	96.80	496.95	477.08
9 A	1	7.59	92.38	701.53	673.86
	2	7.64	91.95	702.47	675.16
	3	7.66	94.04	720.00	692.30

in quantitative terms, input, 2nd

I_{OUT}	k	PF_k	$THD(i_k)$ [%]	$THD(v_k)$ [%]
0 A	1	—	—	4.33
	2	—	—	3.75
	3	—	—	4.75
3 A	1	0.9614	30.50	4.17
	2	0.9598	29.57	3.86
	3	0.9665	29.97	5.38
6 A	1	0.9618	29.26	3.87
	2	0.9618	28.37	3.66
	3	0.9600	28.31	3.87
9 A	1	0.9605	28.00	4.01
	2	0.9611	27.21	3.92
	3	0.9615	27.06	4.19

in quantitative terms, output

I_{OUT} [A]	V_{OUT} [V]	P_{OUT} [W]	P_{IN} [W]	η [%]
0.00	239.79	1.07	−0.81	—
3.21	229.51	736.72	740.53	99.49
6.27	221.23	1386.56	1408.20	98.46
9.41	212.91	2004.12	2041.32	98.18

overall impressions

- pretty good rectifier
- simple, robust, cheap
- good symmetry
- excellent DPF
- acceptable PF
- poor THD (but not that poor)

- up to this point:
  - diode bridge rectifier analyzed
  - measurement tools developed

- **is there a way to do something with the THD?**