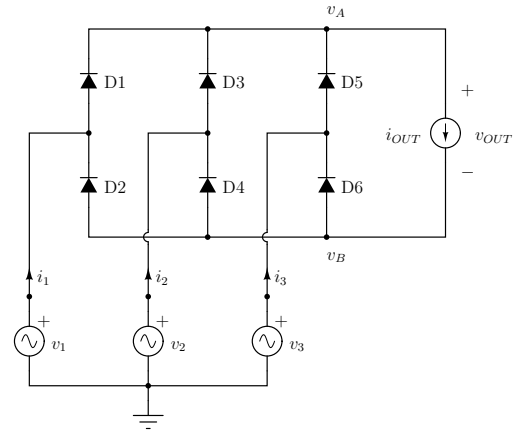


what is this all about?

Introduction

— three-phase diode bridge rectifier —



input voltages

$$v_1 = V_m \cos(\omega_0 t)$$

$$v_2 = V_m \cos\left(\omega_0 t - \frac{2\pi}{3}\right)$$

$$v_3 = V_m \cos\left(\omega_0 t - \frac{4\pi}{3}\right)$$

$$v_k = V_m \cos\left(\omega_0 t - (k-1)\frac{2\pi}{3}\right), \quad k \in \{1, 2, 3\}$$

normalization of voltages

$$m_X \triangleq \frac{v_X}{V_m}$$

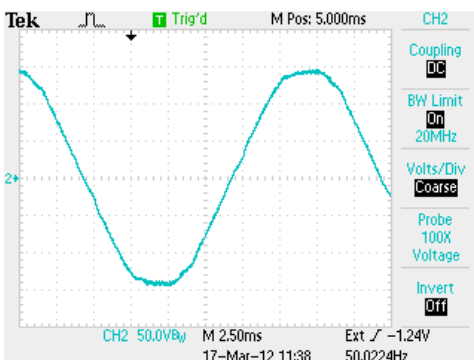
$$m_1 = \cos(\omega_0 t)$$

$$m_2 = \cos\left(\omega_0 t - \frac{2\pi}{3}\right)$$

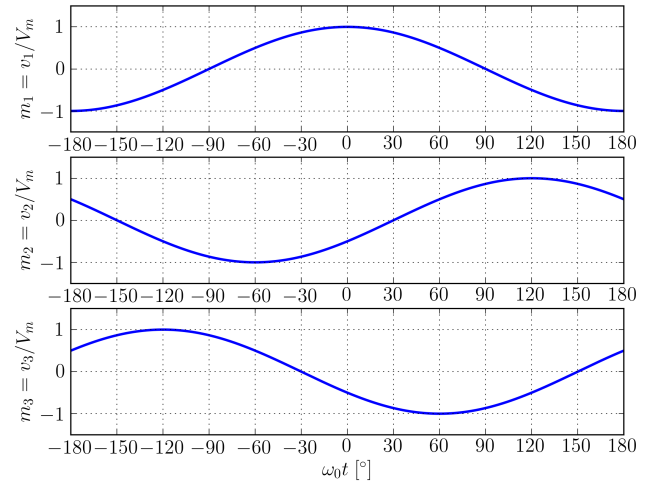
$$m_3 = \cos\left(\omega_0 t - \frac{4\pi}{3}\right)$$

voltages?

$$v_k = V_m \cos\left(\omega_0 t - (k-1)\frac{2\pi}{3}\right), \quad k \in \{1, 2, 3\}$$

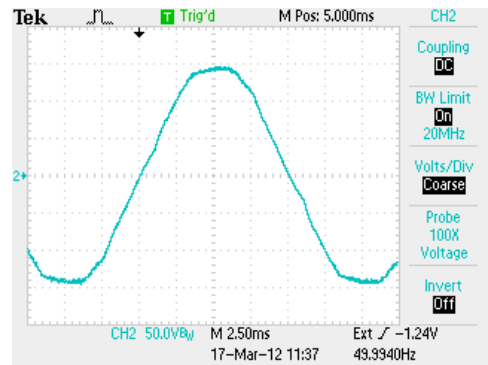


input voltages, waveforms



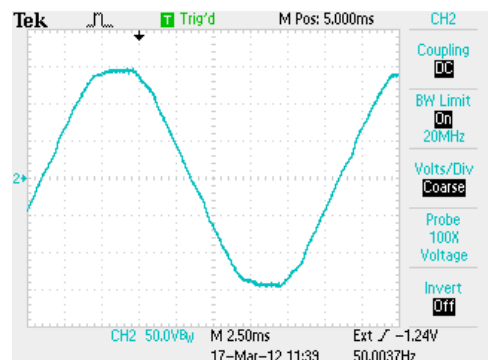
voltages?

$$v_k = V_m \cos\left(\omega_0 t - (k-1)\frac{2\pi}{3}\right), \quad k \in \{1, 2, 3\}$$



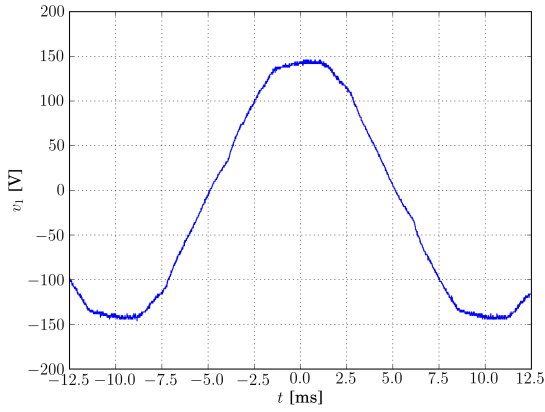
voltages?

$$v_k = V_m \cos\left(\omega_0 t - (k-1)\frac{2\pi}{3}\right), \quad k \in \{1, 2, 3\}$$



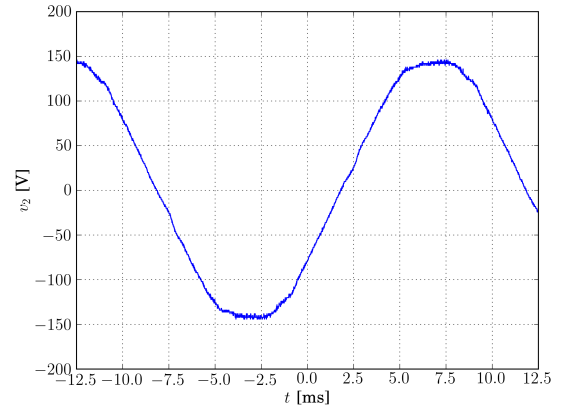
voltages?

$$v_k = V_m \cos(\omega_0 t - (k-1) \frac{2\pi}{3}), \quad k \in \{1, 2, 3\}$$



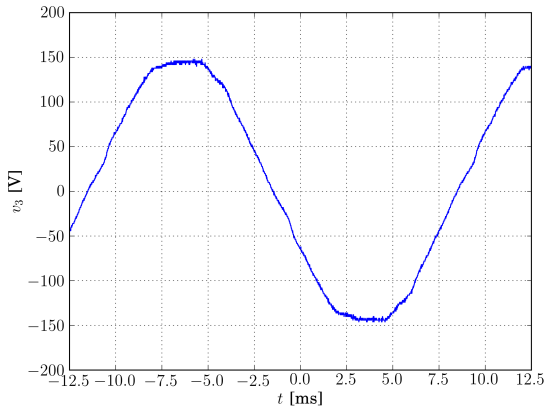
voltages?

$$v_k = V_m \cos(\omega_0 t - (k-1) \frac{2\pi}{3}), \quad k \in \{1, 2, 3\}$$

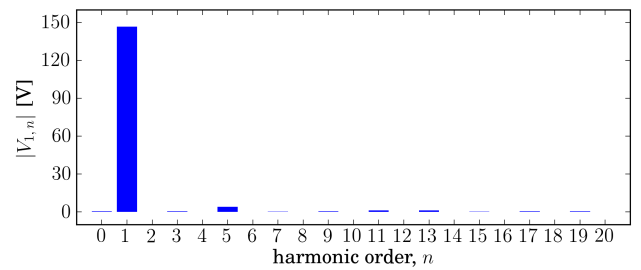


voltages?

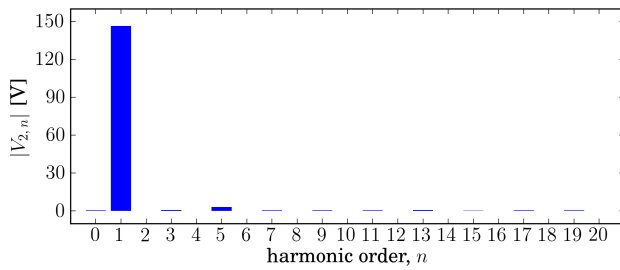
$$v_k = V_m \cos(\omega_0 t - (k-1) \frac{2\pi}{3}), \quad k \in \{1, 2, 3\}$$



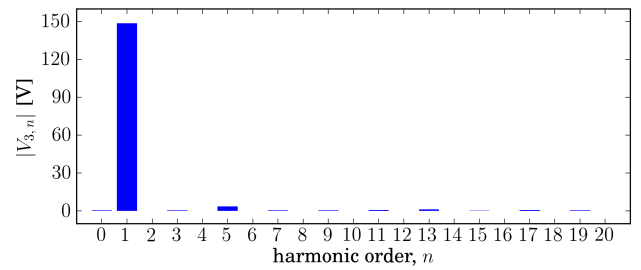
v1, spectrum



v2, spectrum



v3, spectrum



voltages, quantitative characterization

k	$V_{k,RMS}$	$THD(v_k)$
1	103.83 V	3.34 %
2	103.70 V	2.77 %
3	105.12 V	3.06 %

THD

And what is THD?

$$THD \triangleq \frac{\sqrt{\sum_{k=2}^{\infty} I_{k,RMS}^2}}{I_{1,RMS}}$$

Parseval's identity:

$$I_{RMS}^2 = \sum_{k=1}^{\infty} I_{k,RMS}^2 \quad \text{assumed } I_0 = 0$$

results in

$$THD \triangleq \frac{\sqrt{I_{RMS}^2 - I_{1,RMS}^2}}{I_{1,RMS}}$$

simple, but important
computational issues, finite sums ...

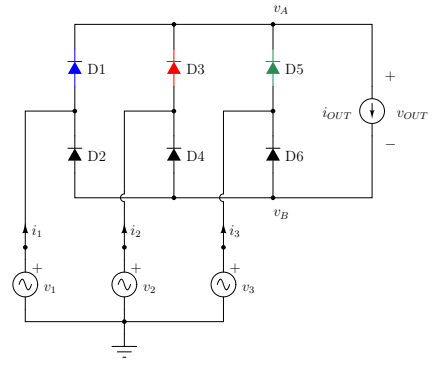
$$jX \triangleq \frac{i_X}{I_{OUT}}$$

unless otherwise noted

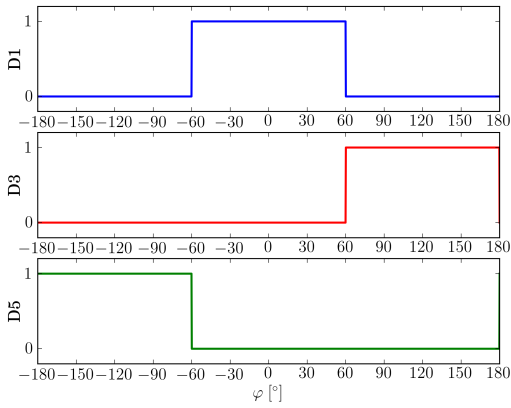
$$\varphi \triangleq \omega_0 t$$

good: physical dimensions lost, reduced number of variables, results are generalized, core of the problem focused

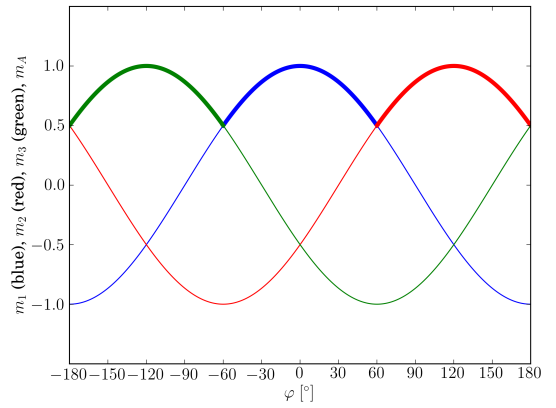
bad: physical dimensions lost, perfect double-check tool is lost



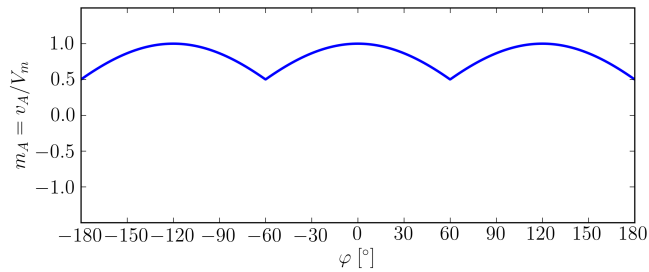
one of the three: D1, D3, D5



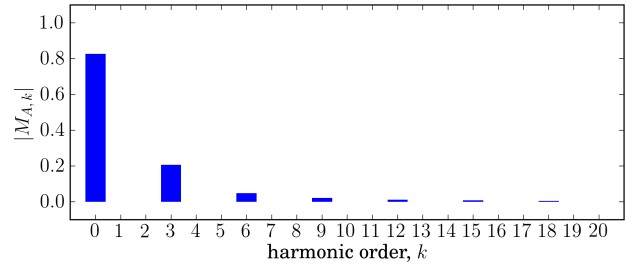
v_A



v_A , analytical



v_A , spectrum

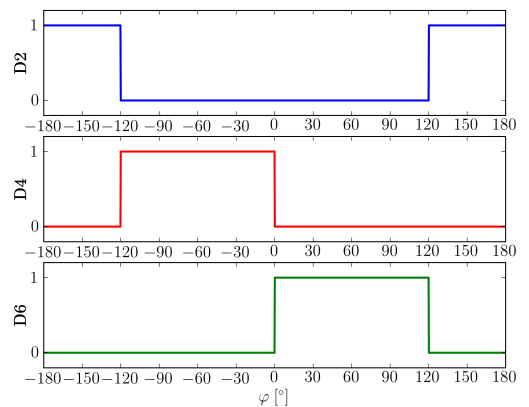
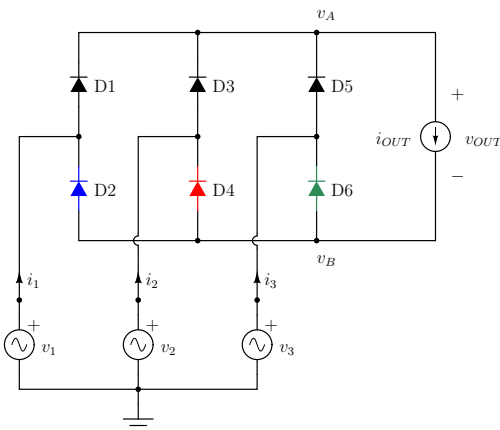


$$m_A = \max(m_1, m_2, m_3)$$

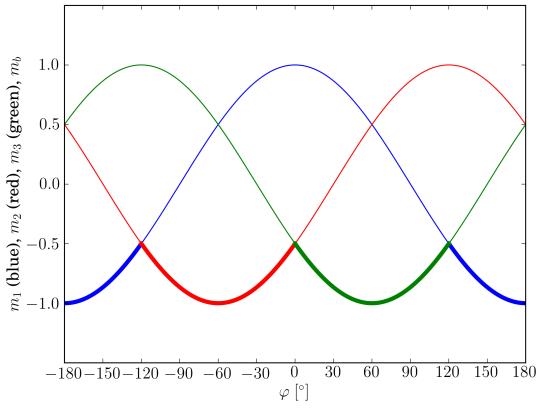
$$m_A = \frac{3\sqrt{3}}{2\pi} \left(1 + 2 \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{9k^2 - 1} \cos(3k\omega_0 t) \right)$$

what about v_B ?

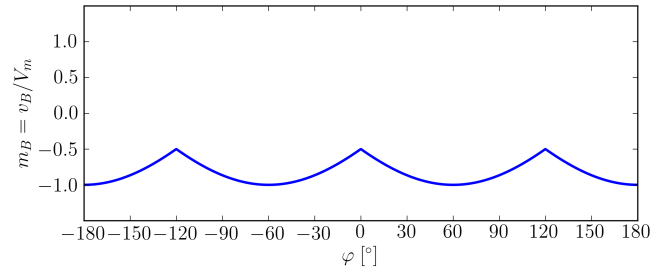
one of the three, again: D2, D4, D6



v_B

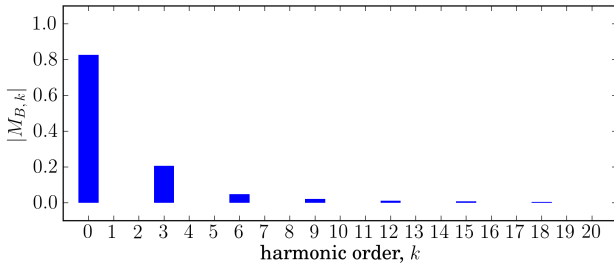


v_B , analytical



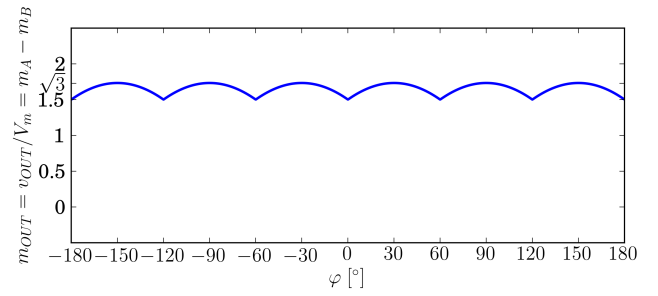
$$m_B = \min(m_1, m_2, m_3)$$

v_B , spectrum



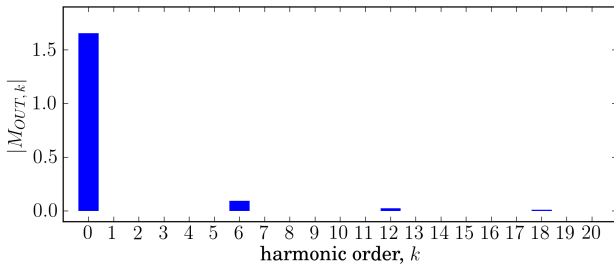
$$m_B = \frac{3\sqrt{3}}{2\pi} \left(-1 + 2 \sum_{k=1}^{\infty} \frac{1}{9k^2 - 1} \cos(3k\omega_0 t) \right)$$

the output voltage, v_{OUT}



$$m_{OUT} = m_A - m_B = \max(m_1, m_2, m_3) - \min(m_1, m_2, m_3)$$

v_{OUT} , spectrum



$$m_{OUT} = \frac{3\sqrt{3}}{\pi} \left(1 - 2 \sum_{k=1}^{\infty} \frac{1}{36k^2 - 1} \cos(6k\omega_0 t) \right)$$

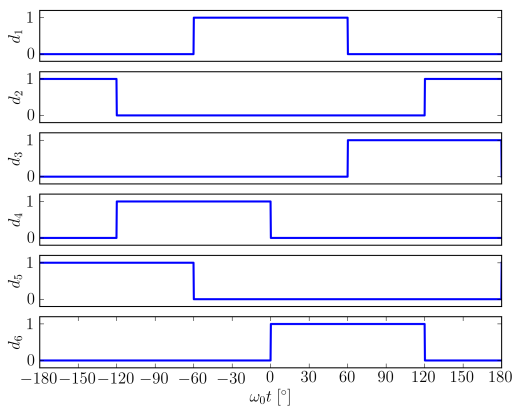
currents?

$$i_1(t) = (d_1(t) - d_2(t)) I_{OUT}$$

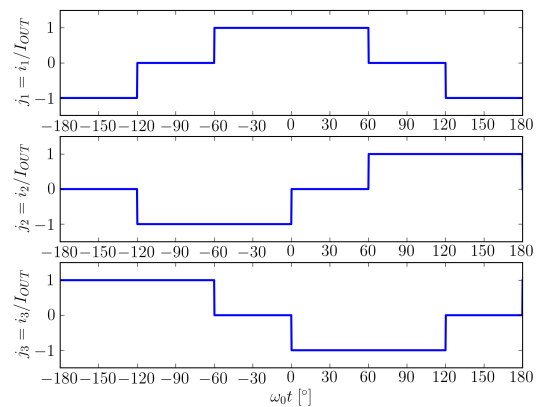
$$i_2(t) = (d_3(t) - d_4(t)) I_{OUT}$$

$$i_3(t) = (d_5(t) - d_6(t)) I_{OUT}$$

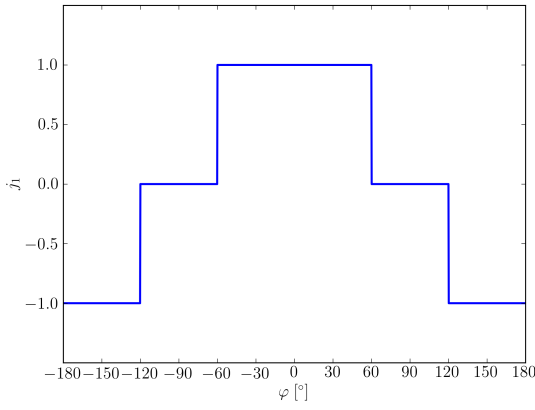
states of the diodes



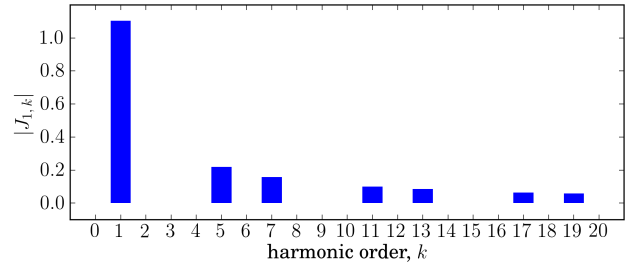
the input currents



consider i_1



spectra of the input currents



spectra of the input currents, analytical

$$j_1(t) = \sum_{k=1}^{+\infty} J_{1C,k} \cos(k\omega_0 t)$$

$$J_{1C,k} = \frac{2\sqrt{3}}{\pi} \begin{cases} -\frac{1}{k}, & k = 6n - 1 \\ \frac{1}{k}, & k = 6n + 1 \\ 0, & \text{otherwise} \end{cases} \quad \text{for } n \in \mathbb{N}_0, k > 0$$

double-check:

$$P_{IN} = \frac{3}{2} \times 1 \times \frac{2\sqrt{3}}{\pi} = \frac{3\sqrt{3}}{\pi} = P_{OUT}$$

obtained using wxMaxima

THD of the input currents

$$I_{k,RMS} = \sqrt{\frac{2}{3}} I_{OUT}$$

$$I_{k,RMS,1} = \frac{\sqrt{6}}{\pi} I_{OUT}$$

$$THD \triangleq \frac{\sqrt{I_{k,RMS}^2 - I_{k,RMS,1}^2}}{I_{k,RMS,1}}$$

$$THD = \sqrt{\frac{\pi^2}{9} - 1} \approx 31.08\%$$

Parseval's identity based formula turned out to be useful

some more parameters

$$X_{RMS} \triangleq \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (x(\omega_0 t))^2 d(\omega_0 t)}, \quad x \in \{i, v\}$$

already used for the THD

$$S \triangleq I_{RMS} V_{RMS}$$

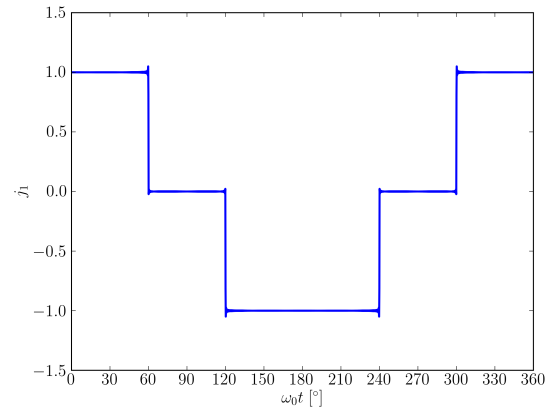
$$P \triangleq \frac{1}{2\pi} \int_0^{2\pi} v(\omega_0 t) i(\omega_0 t) d(\omega_0 t)$$

$$PF \triangleq \frac{P}{S}$$

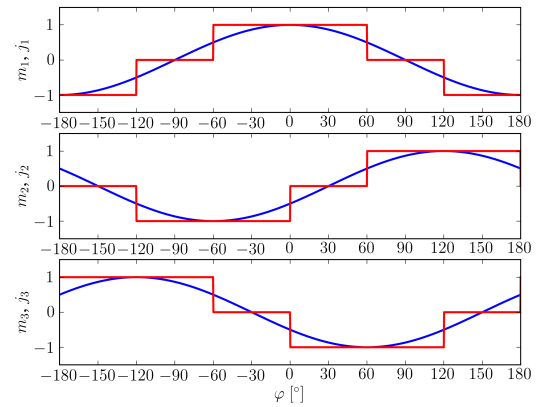
$$DPF \triangleq \cos \phi_1$$

and ϕ_1 is ...

numerical verification, Gibbs phenomenon



voltages and currents



and if the voltages are sinusoidal ...

$$S = V_{RMS} I_{RMS}$$

$$P = V_{RMS} I_{1,RMS} \cos \phi_1$$

$$PF = \frac{P}{S} = \frac{I_{1,RMS}}{I_{RMS}} \cos \phi_1 = \frac{I_{1,RMS}}{I_{RMS}} DPF$$

$$DPF = \cos \phi_1$$

$$THD = \frac{\sqrt{I_{RMS}^2 - I_{1,RMS}^2}}{I_{1,RMS}} = \sqrt{\left(\frac{I_{RMS}}{I_{1,RMS}}\right)^2 - 1}$$

i.e. everything depends on the current waveform and its position

some more parameters, plain rectifier

$$I_{kRMS} = \sqrt{\frac{2}{3}} I_{OUT} \quad V_{kRMS} = \frac{1}{\sqrt{2}} V_m$$

$$S = 3 \times \sqrt{\frac{2}{3}} I_{OUT} \times \frac{1}{\sqrt{2}} V_m = \sqrt{3} V_m I_{OUT}$$

$$P = V_{OUT} I_{OUT} = \frac{3\sqrt{3}}{\pi} V_m I_{OUT}$$

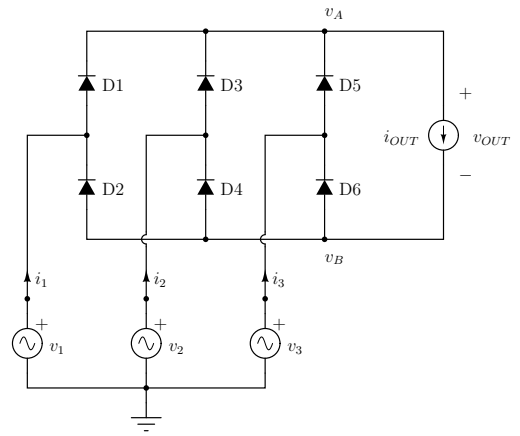
$$PF = \frac{3}{\pi} \approx 95.5\%$$

$$DPF = 1$$

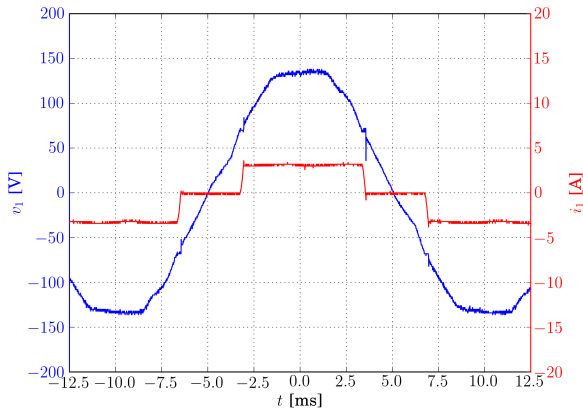
actually, not so bad; THD is the problem

back to the rectifier:

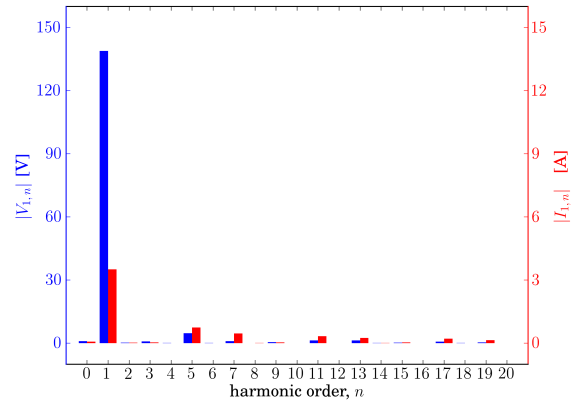
how does it work? part 2: experiment



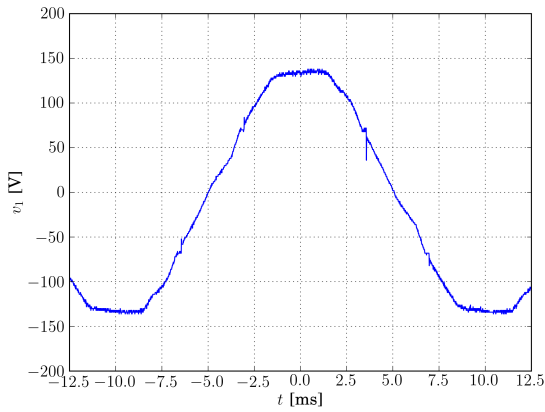
input, at $I_{OUT} = 3$ A



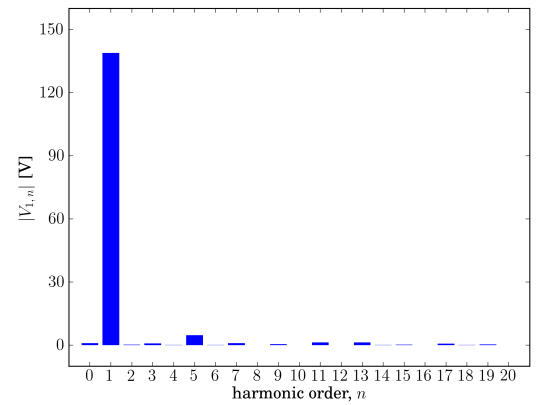
input, at $I_{OUT} = 3$ A



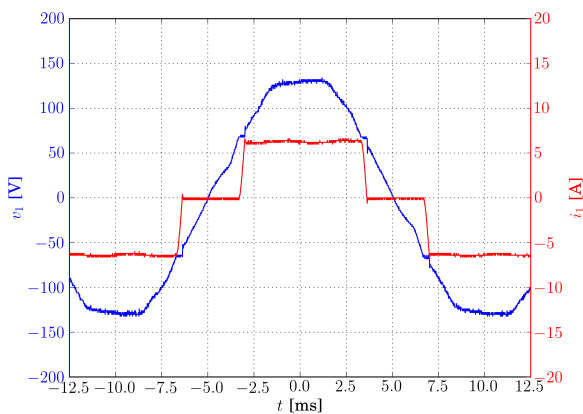
input, at $I_{OUT} = 3$ A



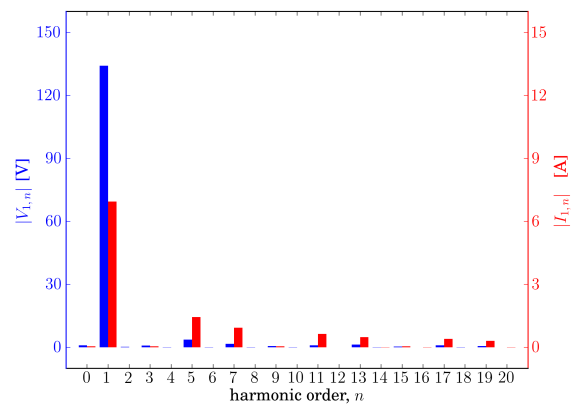
input, at $I_{OUT} = 3$ A



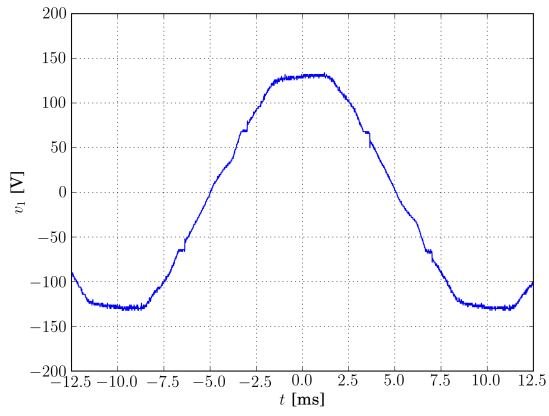
input, at $I_{OUT} = 6$ A



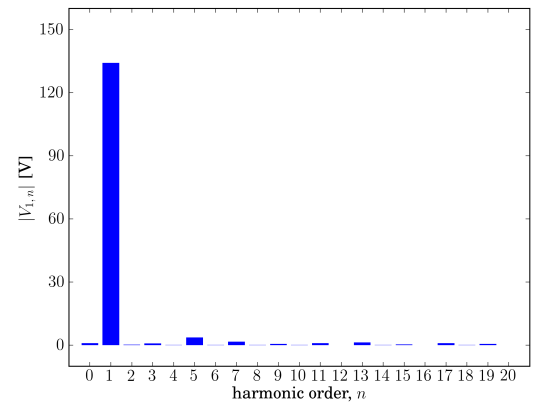
input, at $I_{OUT} = 6$ A



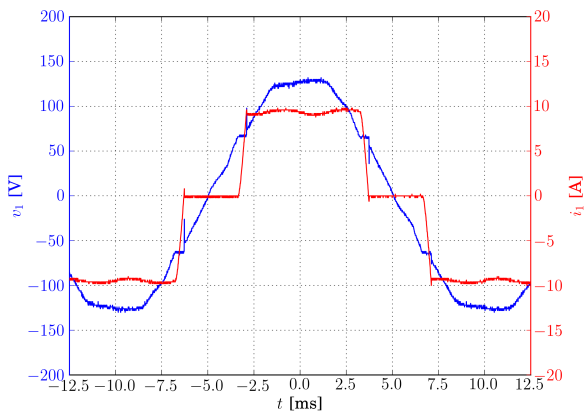
input, at $I_{OUT} = 6$ A



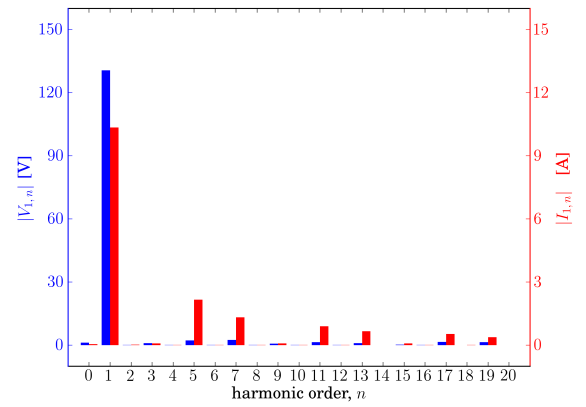
input, at $I_{OUT} = 6$ A



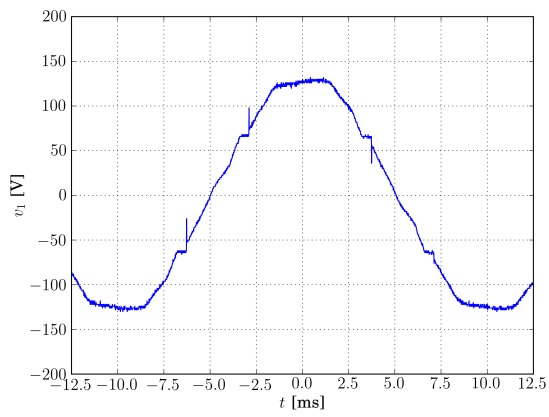
input, at $I_{OUT} = 9$ A



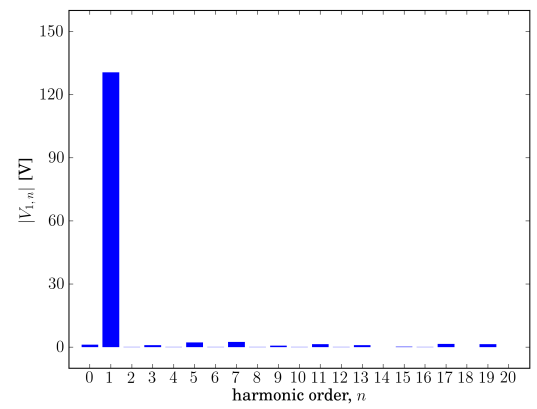
input, at $I_{OUT} = 9$ A



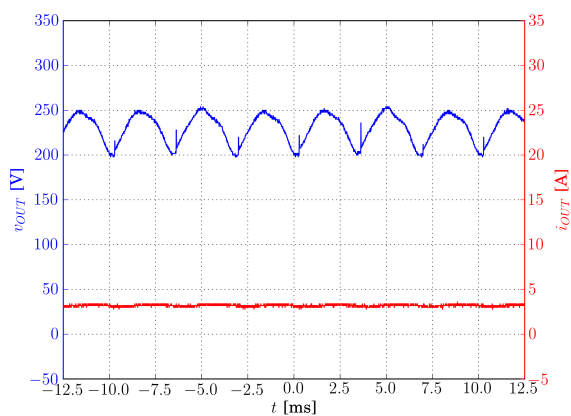
input, at $I_{OUT} = 9$ A



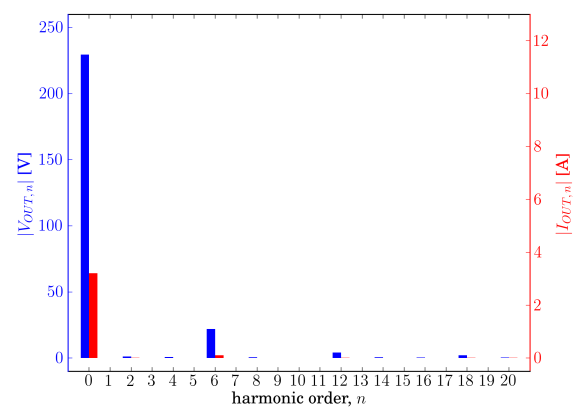
input, at $I_{OUT} = 9$ A



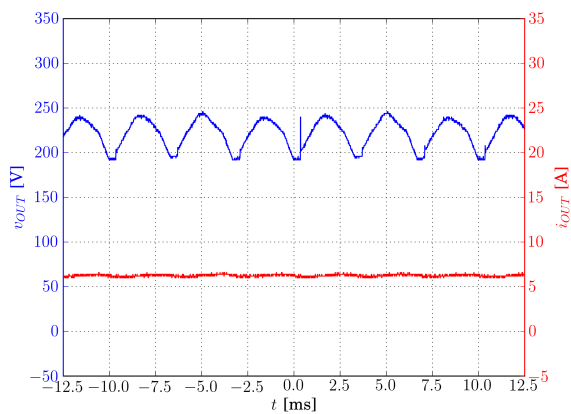
output, at $I_{OUT} = 3$ A



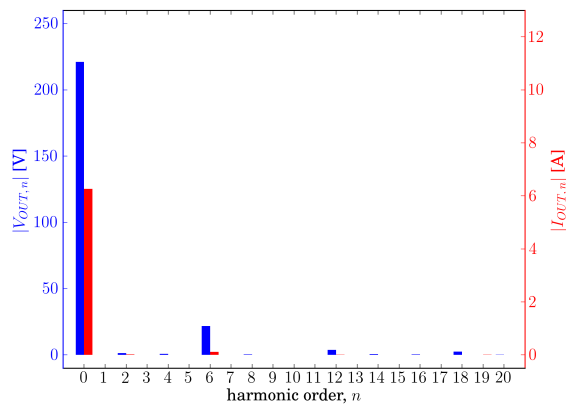
output, at $I_{OUT} = 3$ A



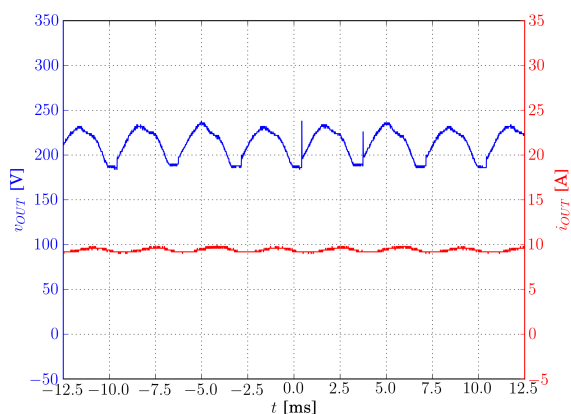
output, at $I_{OUT} = 6\text{ A}$



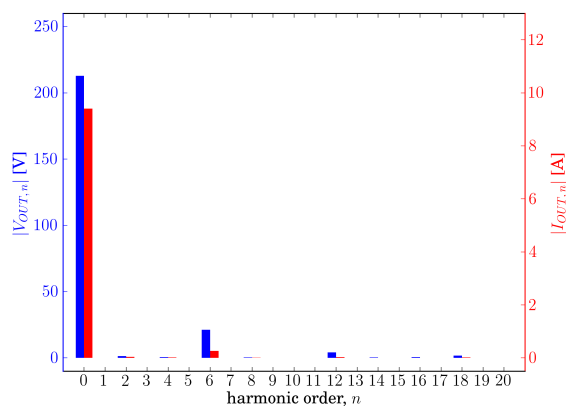
output, at $I_{OUT} = 6\text{ A}$



output, at $I_{OUT} = 9\text{ A}$



output, at $I_{OUT} = 9\text{ A}$



in quantitative terms, input, 1st

I_{OUT}	k	$I_{k\text{ RMS}}$ [A]	$V_{k\text{ RMS}}$ [V]	S_k [VA]	P_k [W]
0 A	1	—	101.29	—	—
	2	—	100.63	—	—
	3	—	102.40	—	—
3 A	1	2.60	98.23	255.01	245.16
	2	2.61	97.73	254.60	244.37
	3	2.63	98.82	259.71	251.00
6 A	1	5.12	94.87	485.41	466.87
	2	5.12	94.34	482.67	464.25
	3	5.13	96.80	496.95	477.08
9 A	1	7.59	92.38	701.53	673.86
	2	7.64	91.95	702.47	675.16
	3	7.66	94.04	720.00	692.30

in quantitative terms, input, 2nd

I_{OUT}	k	PF_k	$THD(i_k)$ [%]	$THD(v_k)$ [%]
0 A	1	—	—	4.33
	2	—	—	3.75
	3	—	—	4.75
3 A	1	0.9614	30.50	4.17
	2	0.9598	29.57	3.86
	3	0.9665	29.97	5.38
6 A	1	0.9618	29.26	3.87
	2	0.9618	28.37	3.66
	3	0.9600	28.31	3.87
9 A	1	0.9605	28.00	4.01
	2	0.9611	27.21	3.92
	3	0.9615	27.06	4.19

in quantitative terms, output

I_{OUT} [A]	V_{OUT} [V]	P_{OUT} [W]	P_{IN} [W]	η [%]
0.00	239.79	1.07	-0.81	—
3.21	229.51	736.72	740.53	99.49
6.27	221.23	1386.56	1408.20	98.46
9.41	212.91	2004.12	2041.32	98.18

overall impressions

- ▶ pretty good rectifier
- ▶ simple, robust, cheap
- ▶ good symmetry
- ▶ excellent DPF
- ▶ acceptable PF
- ▶ poor THD (but not that poor)
- ▶ up to this point:
 - ▶ diode bridge rectifier analyzed
 - ▶ measurement tools developed
- ▶ is there a way to do something with the THD ?