DIMMER

Normalized voltage

(%i28) m: sin(x);

$$\sin(x) \tag{m}$$

RMS

(%i2) Mrms: sqrt(1 / %pi * integrate(m^2, x, alpha, %pi));

$$\frac{\sqrt{\frac{\sin(2\alpha) - 2\alpha}{4} + \frac{\pi}{2}}}{\sqrt{\pi}} \tag{Mrms}$$

(%i3) Mrms: ratsimp(Mrms);

$$\frac{\sqrt{\sin(2\alpha) - 2\alpha + 2\pi}}{2\sqrt{\pi}} \tag{Mrms}$$

First Harmonic

(%i4) M1s: 1 / %pi * (integrate(m * sin(x), x, alpha, %pi) + integrate(m * sin(x), x, %pi + alpha, 2 * %pi));

$$\frac{\frac{3\pi}{2} + \frac{\sin(2\alpha) - 2\alpha}{4} + \frac{-2\pi - 2\alpha + \sin(2\alpha)}{4}}{\pi} \tag{M1s}$$

(%i5) M1s: ratsimp(M1s);

$$\frac{2\pi - 2\alpha + \sin(2\alpha)}{2\pi} \tag{M1s}$$

(%i6) ev(M1s, alpha = 0);

$$1 \tag{%o6}$$

(%i7) M1c: 1 / %pi * (integrate(m * cos(x), x, alpha, %pi) + integrate(m * cos(x), x, %pi + alpha, 2 * %pi));

$$\frac{\cos(\alpha)^2 - 1}{\pi} \tag{M1c}$$

(%i8) ev(M1c, alpha = 0);

$$0 \tag{%o8}$$

(%i9) M1: sqrt(M1s^2 + M1c^2);

$$\sqrt{\frac{(2\pi - 2\alpha + \sin(2\alpha))^2}{4\pi^2} + \frac{\left(\cos(\alpha)^2 - 1\right)^2}{\pi^2}} \tag{M1}$$

(%i10) M1: ratsimp(M1);

$$\frac{\sqrt{\sin(2\alpha)^2 + (4\pi - 4\alpha)\sin(2\alpha) + 4\cos(\alpha)^4 - 8\cos(\alpha)^2 + 4\alpha^2 - 8\pi\alpha + 4\pi^2 + 4}}{2\pi} \tag{M1}$$

(%i11) M1: trigsimp(M1);

$$\frac{\sqrt{\sin(2\alpha)^2 + (4\pi - 4\alpha)\sin(2\alpha) + 4\sin(\alpha)^4 + 4\alpha^2 - 8\pi\alpha + 4\pi^2}}{2\pi} \tag{M1}$$
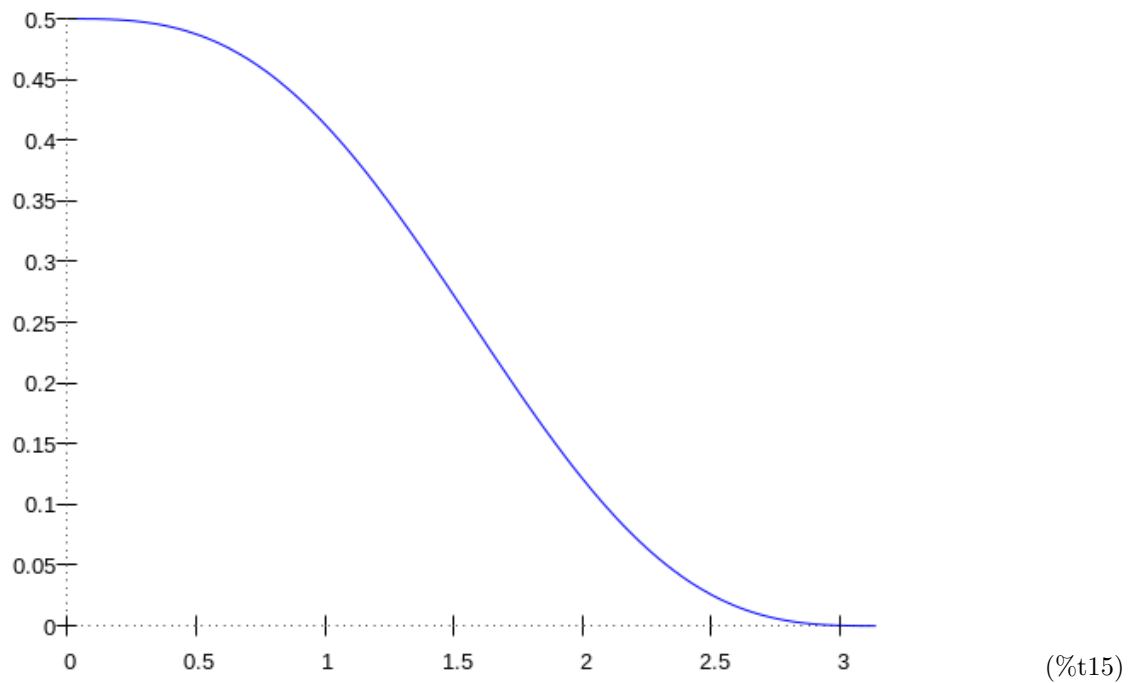
1

(%i12) M1rms: M1 / sqrt(2);

$$\frac{\sqrt{\sin\left(2\alpha\right)^2 + \left(4\pi - 4\alpha\right)\sin\left(2\alpha\right) + 4\sin\left(\alpha\right)^4 + 4\alpha^2 - 8\pi\alpha + 4\pi^2}}{2^{\frac{3}{2}}\pi} \tag{M1rms}$$

Power

(%i13) P: Vm^2 / R * Mrms^2;

$$\frac{\left(2\pi - 2\alpha + \sin\left(2\alpha\right)\right)\,Vm^2}{4\pi R} \tag{P}$$

(%i14) ev(P, alpha = 0);

$$\frac{Vm^2}{2R} \tag{%o14}$$

(%i15) wxplot2d(P * R / Vm^2, [alpha, 0, %pi-1e-3], [box, false]);
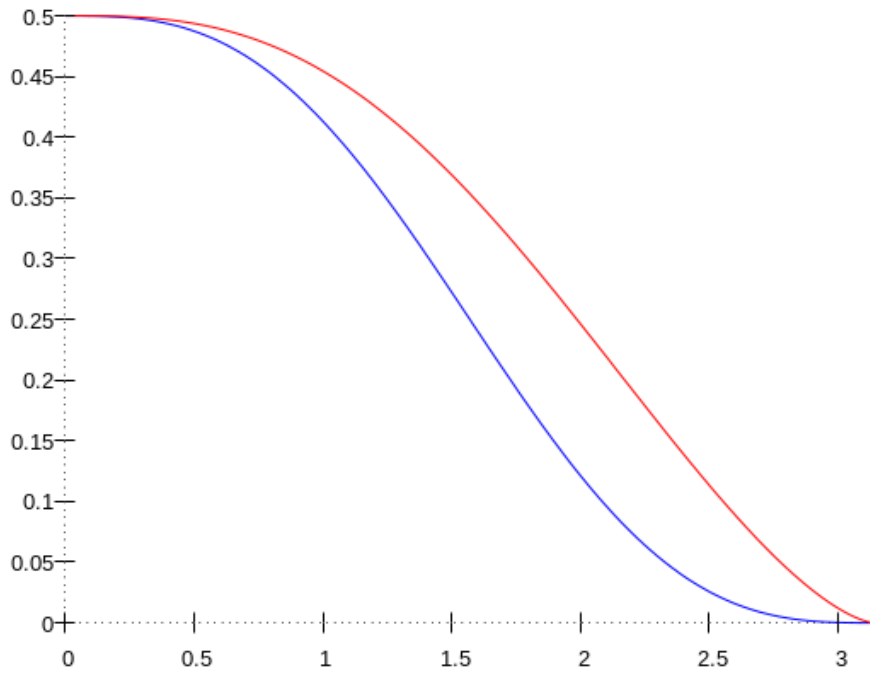

$$\tag{%t15}$$

$$\tag{%o15}$$

Apparent Power

(%i16) S: Vm / sqrt(2) * Vm / R * Mrms;

$$\frac{\sqrt{\sin\left(2\alpha\right) - 2\alpha + 2\pi}\,Vm^2}{2^{\frac{3}{2}}\sqrt{\pi}R} \tag{S}$$

2

(%i17) wxplot2d([P * R / Vm^2, S * R / Vm^2], [alpha, 0, %pi-1e-3], [box, false], [legend, false]);

(%t17)

(%o17)

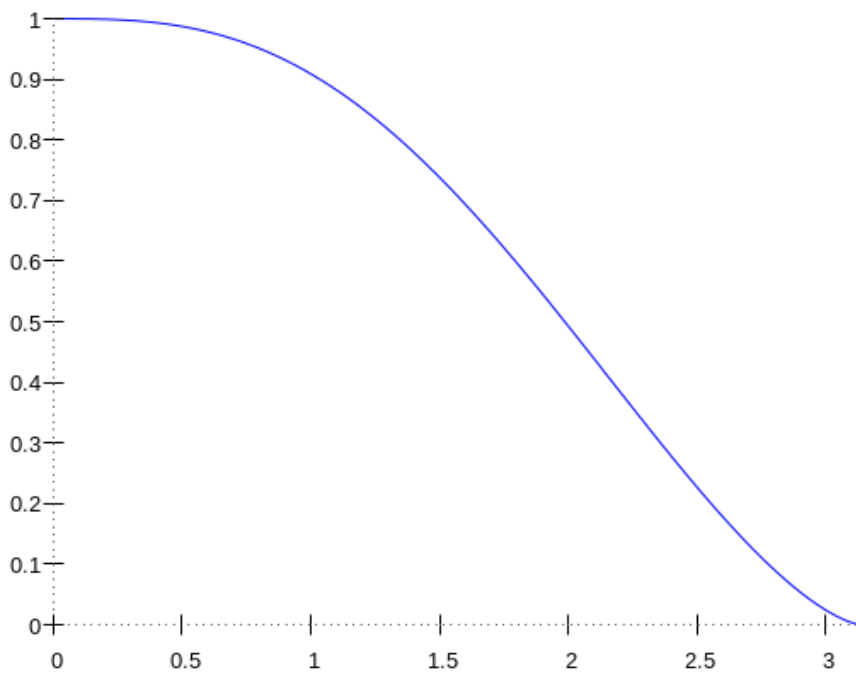Power Factor

(%i18) PF: P / S;

$$\frac{\sqrt{\sin(2\alpha) - 2\alpha + 2\pi}}{\sqrt{2}\,\sqrt{\pi}} \tag{PF}$$

(%i19) wxplot2d(PF, [alpha, 0, %pi-1e-3], [box, false], [legend, false]);
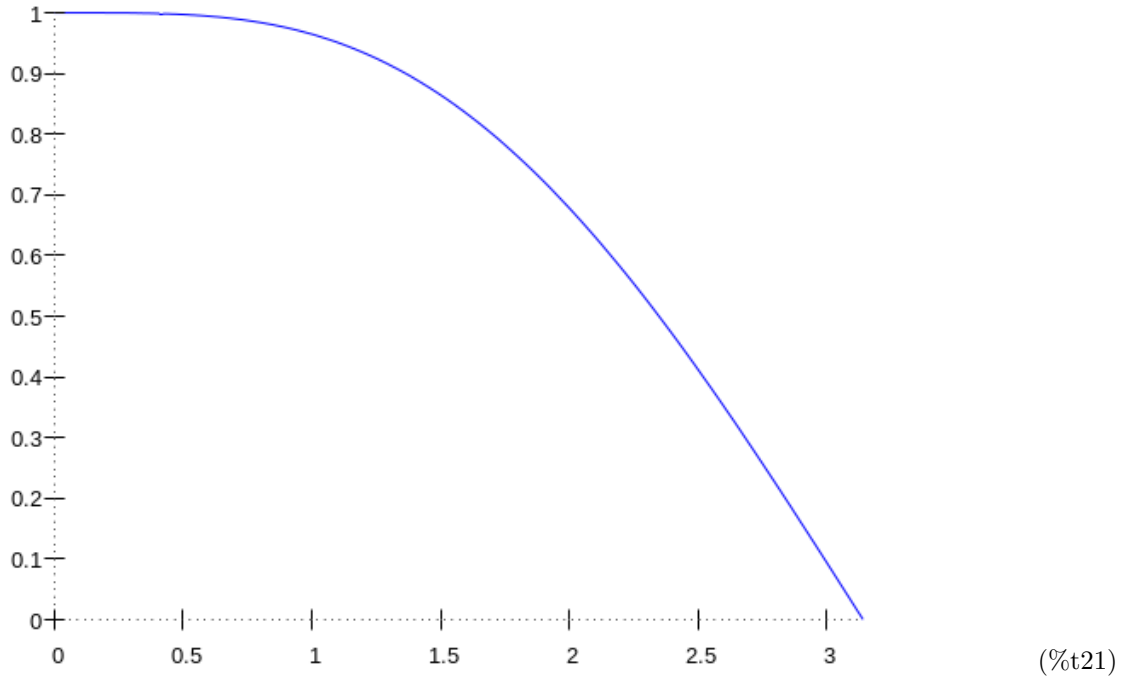

(%t19)

(%o19)

Displacement Power Factor

(%i20) DPF: M1s / M1;

$$\frac{2\pi - 2\alpha + \sin\left(2\alpha\right)}{\sqrt{\sin\left(2\alpha\right)^2 + \left(4\pi - 4\alpha\right)\sin\left(2\alpha\right) + 4\sin\left(\alpha\right)^4 + 4\alpha^2 - 8\pi\alpha + 4\pi^2}} \tag{DPF}$$

(%i21) wxplot2d(DPF, [alpha, 0, %pi-1e-3], [box, false], [legend, false]);
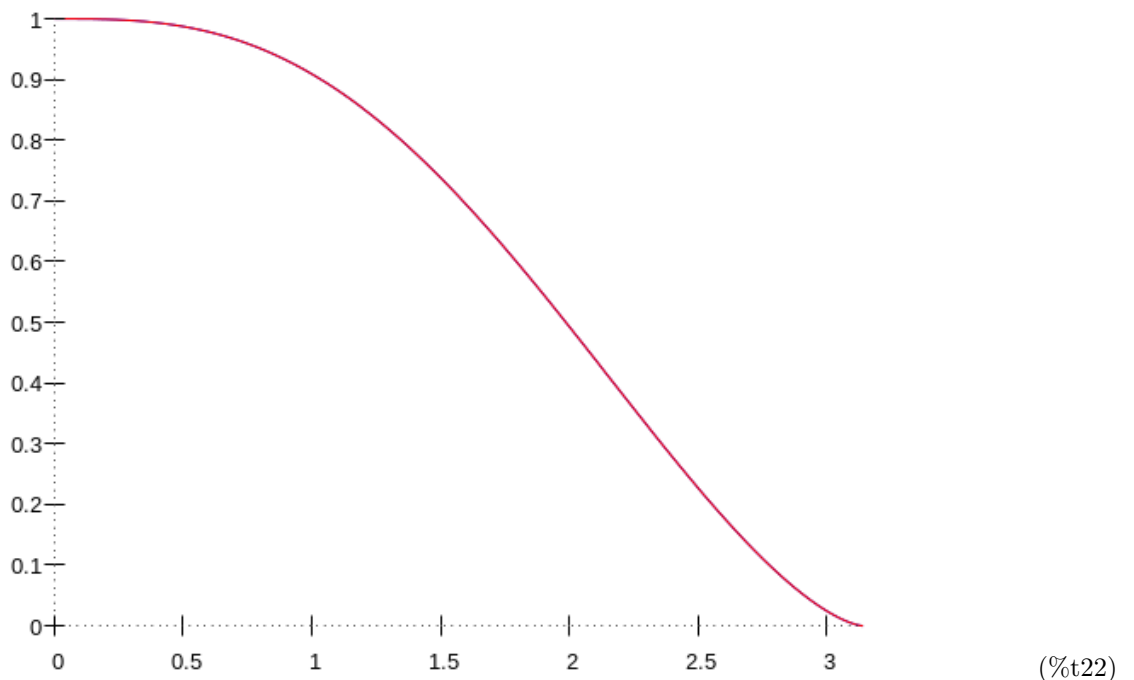

(%t21)

(%o21)

verify PF-DPF relation

(%i22) wxplot2d([PF, M1rms / Mrms * DPF], [alpha, 0, %pi-1e-3], [box, false], [legend, false]);
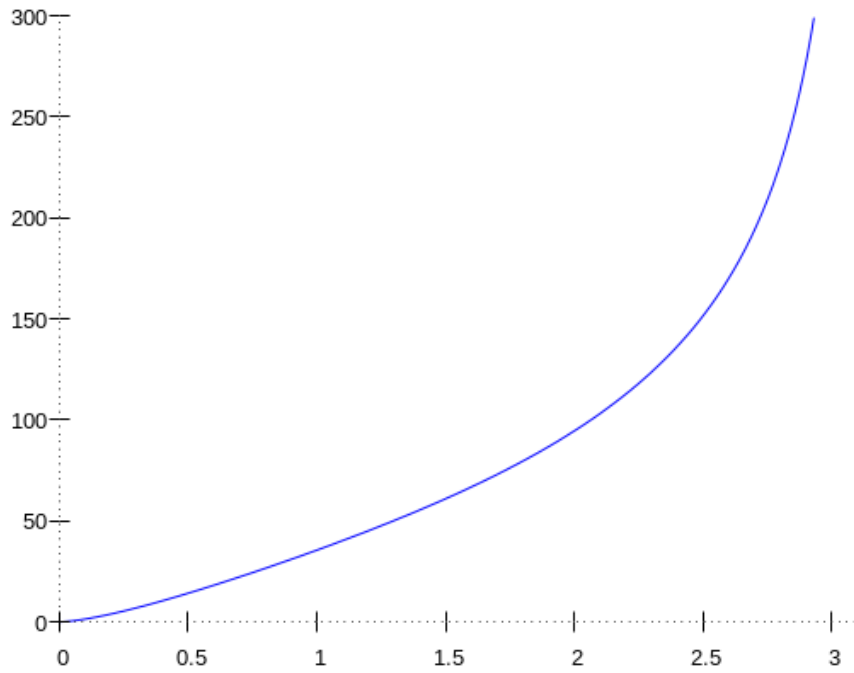

(%t22)

(%o22)

THD

(%i23) THD: sqrt(Mrms^2 / M1rms^2 - 1) * 100;

$$100\sqrt{\frac{2\pi\left(2\pi - 2\alpha + \sin\left(2\alpha\right)\right)}{\sin\left(2\alpha\right)^2 + \left(4\pi - 4\alpha\right)\sin\left(2\alpha\right) + 4\sin\left(\alpha\right)^4 + 4\alpha^2 - 8\pi\alpha + 4\pi^2} - 1} \qquad \text{(THD)}$$

(%i24) wxplot2d(THD, [alpha, 0, %pi-1e-3], [y, 0, 300], [box, false], [legend, false]);

plot2d: some values were clipped.

(%t24)

(%o24)